

**Far-infrared absorption of  
quantum dots;  
From single dots to arrays**

**Viðar Guðmundsson  
Ingibjörg Magnúsdóttir  
Sigurður I. Erlingsson.**

**Science Institute  
University of Iceland  
IS-107 Reykjavík  
[vidar@raunvis.hi.is](mailto:vidar@raunvis.hi.is)**

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**Cooperation**



- Daniela Pfannkuche
- Peter A. Maksym
- Rolf R. Gerhardt
- Detlef Heitmann
- Juan José Palacios
- Andrei Manolescu
- Arne Brataas
- Christoph Steinebach
- Ulrich Wulf

**How to break Kohn's theorem?**

## FIR-absorption - Time-dependent HFA

$$\hat{H}(t) = \hat{H}_{HF} + \delta\hat{V}e^{-i(\omega+i0^+)t}$$

$$\hat{\rho}(t \rightarrow -\infty) = \hat{\rho}^0 = f(\hat{H}_{HF})$$

### Linear response

$$\delta\rho_{\alpha,\beta}(t) = f^{\alpha,\beta}(\omega)\langle\alpha|\delta\hat{V}|\beta\rangle e^{-i(\omega+i0^+)t}$$

with

$$f^{\alpha,\beta}(\omega) = \left\{ \frac{f(\epsilon_\beta) - f(\epsilon_\alpha)}{\hbar\omega + \epsilon_\beta - \epsilon_\alpha + i\hbar0^+} \right\}$$

### Nonlocal exchange

$$\delta V_{\alpha,\beta} = (-e) \left\{ \langle\alpha|\phi_{ext}|\beta\rangle + \langle\alpha|\phi_{ind}^H|\beta\rangle + \langle\alpha|\phi_{ind}^F|\beta\rangle \right\}$$

$$\text{Self-consistency} \leftarrow \langle\alpha|\phi_{ind}^{H,F}|\beta\rangle \sim \delta\rho_{\alpha,\beta}$$

$$\sum_{\delta,\gamma} \epsilon_{\alpha\beta,\delta\gamma}(\omega) \langle\delta|\phi_{sc}|\gamma\rangle = \langle\alpha|\phi_{ext}|\beta\rangle$$

with

$$\epsilon_{\alpha\beta,\delta\gamma}(\omega) = \left\{ \delta_{\delta,\alpha}\delta_{\gamma,\beta} - (H_{\gamma\delta,\beta\alpha} - F_{\gamma\delta,\beta\alpha}) f^{\delta\gamma}(\omega) \right\}$$

$$H_{\gamma\delta,\beta\alpha} = \frac{e^2}{\kappa} \int d\mathbf{r}d\mathbf{r}' \frac{\psi_\gamma^*(\mathbf{r}')\psi_\delta(\mathbf{r}')\psi_\alpha^*(\mathbf{r})\psi_\beta(\mathbf{r})}{|\mathbf{r} - \mathbf{r}'|}$$

$$F_{\gamma\delta,\beta\alpha} = \frac{e^2}{\kappa} \int d\mathbf{r}d\mathbf{r}' \frac{\psi_\gamma^*(\mathbf{r}')\psi_\delta(\mathbf{r})\psi_\alpha^*(\mathbf{r}')\psi_\beta(\mathbf{r})}{|\mathbf{r} - \mathbf{r}'|}$$

Now one could use

$$\det \epsilon_{\alpha\beta,\delta\gamma}(\omega) = 0, \quad \alpha = (n, M, s)$$

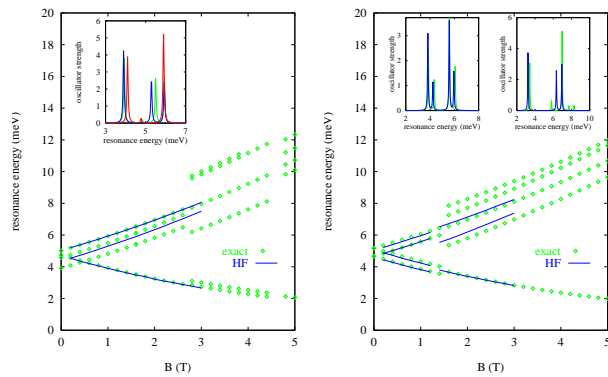
or calculate the absorption

$$\begin{aligned} P(\omega) &= \frac{1}{2} \int d\mathbf{r} \text{Re} [\delta\mathbf{j}(\mathbf{r}) \cdot \mathbf{E}_{sc}^*(\mathbf{r})] \\ &\sim \Im \sum_{\alpha\beta} \omega \langle\beta|\phi_{ext}|\alpha\rangle \langle\alpha|\phi_{sc}|\beta\rangle f^{\alpha\beta}(\omega) \end{aligned}$$

Few electrons

Radial deviations

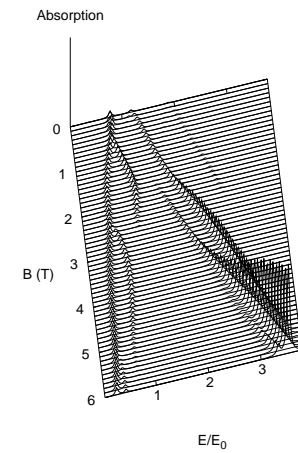
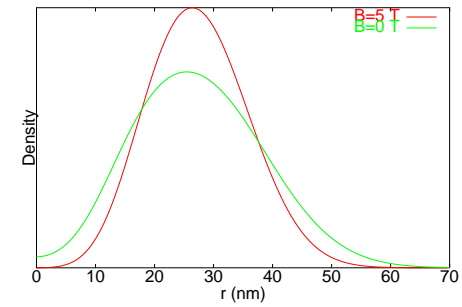
$$V_{conf}(r) = ar^2 + br^4 + cr^6$$

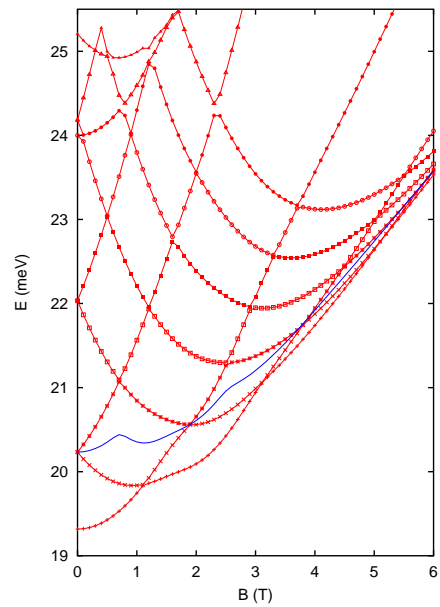


$N = 3$ , spin doublet, spin quartet

Fingerprints of few electron states

Circular dot with a soft hole in the center

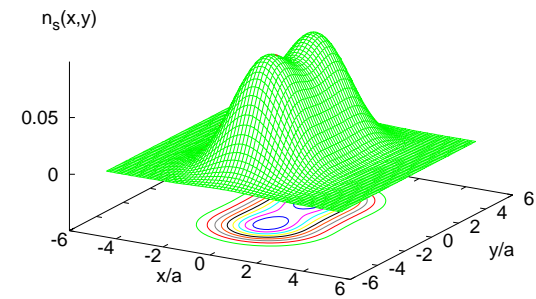




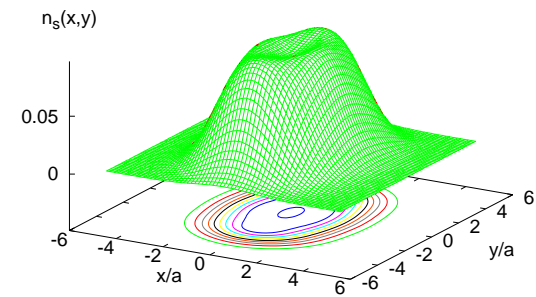
- A. Lorke and R. J. Luyken,  
Physica B 256-258, 424 (1998)

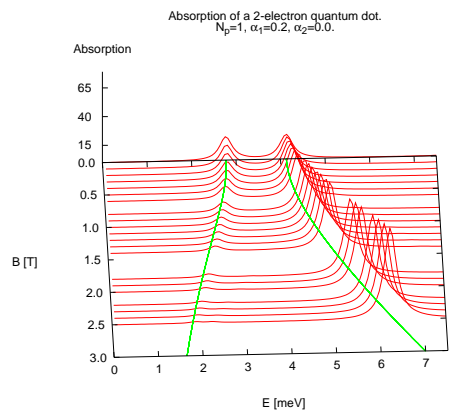
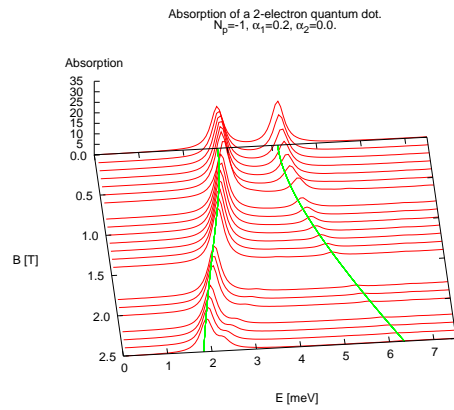
### Angular deviations, elliptic

$\alpha_1=0.2, \alpha_2=0.0, N_s=2, B=0 \text{ T.}$



$\alpha_1=0.2, \alpha_2=0.0, N_s=3, B=0 \text{ T.}$

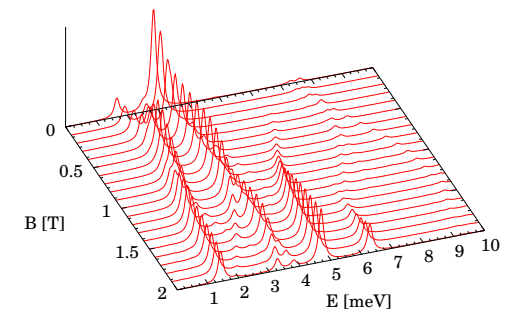




cm modes, Kohn's theorem holds

Angular deviations, square

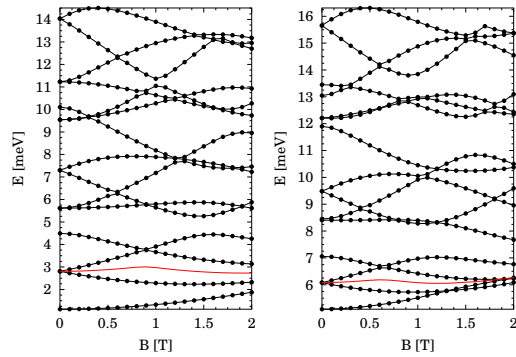
Hard wall square 100 nm × 100 nm



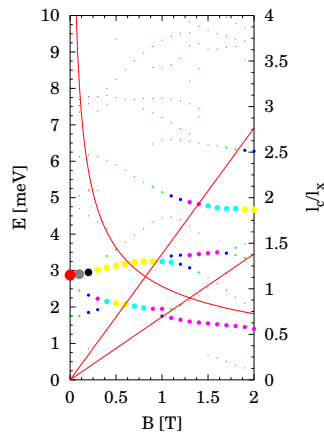
Also

- R. Ugajin, Phys. Rev. B **53**, 6963 (1996)

Noninteracting  $\leftrightarrow$  interacting



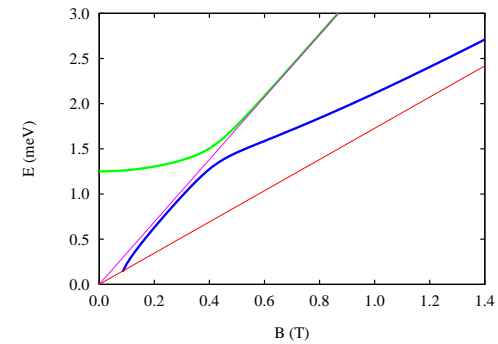
FIR dispersion, **not simple perturbed cm. modes**



**Many electrons**

**Bernstein modes**

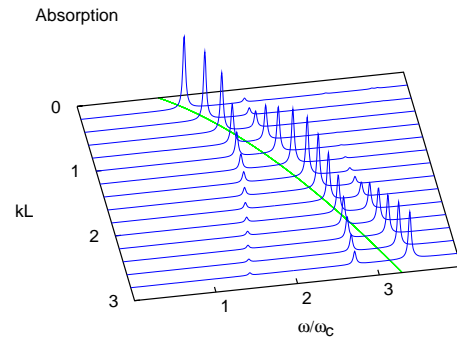
- Waves in a Plasma in a Magnetic Field, I. B. Bernstein, Phys. Rev. **109**, 10 (1958)
- N. J. M. Horing and M. M. Yildiz, Annals of Physics **97**, 216 (1976).



**Classical effect**, blocked by Kohn's theorem

## 2DEG - no modulation

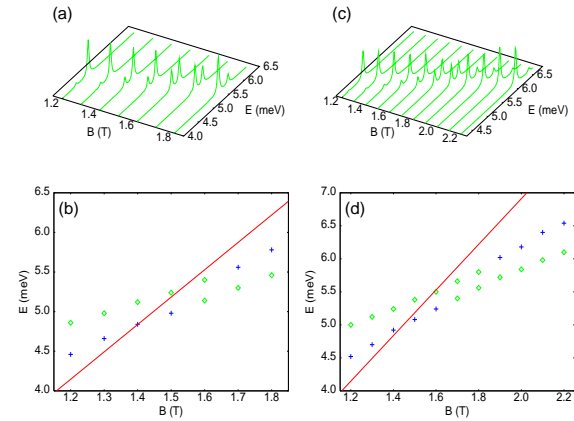
$L=200\text{nm}$ ,  $pq=1$ ,  $\hbar\omega_c=0.1786\text{meV}$ ,  $T=1\text{K}$ ,  $V=0.0\text{meV}$ ,  $N_s=1.00$



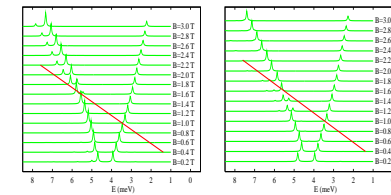
- $k \neq 0$  breaks Kohn's theorem
- Magnetoplasmon interacts with harmonics of the cyclotron resonance
- Cyclotron resonance seen in longitudinal excitation due to density modulation by the magnetoplasmon

Quantum dots  $3D \leftrightarrow 2D \leftrightarrow 1D \leftrightarrow 0D$

$br^4$ -deviation  $\leftrightarrow$   $cr^6$ -deviation



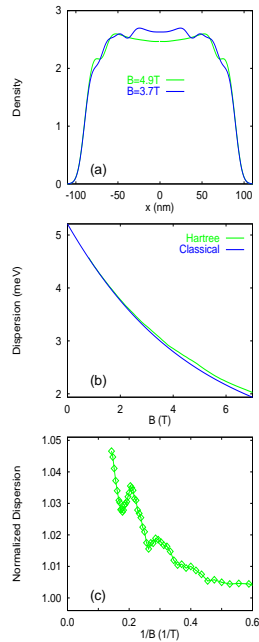
$N = 16$ , HA  $\leftrightarrow$  HFA



Not seen for few electrons!

## $\omega_-$ oscillations

$br^4$ -deviation



- K. Bollweg, T. Kurth, D. Heitmann, V. Gudmundsson, . . .  
Phys. Rev. Lett. **76**, 2774 (1996)
- T. Darnhofer, M. Suhrke, U. Rössler,  
Europhys. Lett. **35**, 591 (1996)

## Ground state

2DEG in a periodic potential

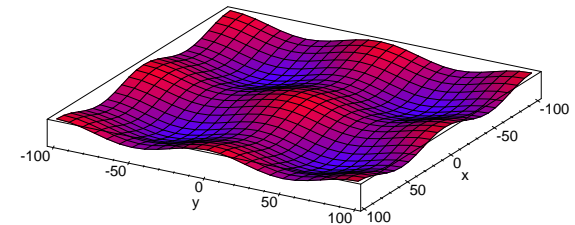
$$V(x, y) = V \{ \cos(gx) + \cos(gy) \}$$

$g = 2\pi/L$ , with the periodic length  $L$

Perpendicular magnetic field  $\mathbf{B} = B\hat{z}$

Integer number  $pq$  of flux units  $\Phi_0 = hc/e$   
flows through a lattice unit cell with area

$$A = L^2 \rightarrow B = pq\Phi_0/A$$





$$\phi_{n_i}^{\mu\nu}(\mathbf{r}) = \frac{1}{\sqrt{pq}} \sum_{\substack{m,n \\ =-\infty \\ \infty}} [S(\mathbf{c})e^{-i\mu}]^m [S(\mathbf{d})e^{-i\nu}]^n \phi_{n_i}(\mathbf{r})$$

$$\mu = \frac{\theta_1 + 2\pi n_1}{p}, \quad n_1 \in I_1 = \{0, \dots, p-1\}$$

$$\nu = \frac{\theta_2 + 2\pi n_2}{q}, \quad n_2 \in I_2 = \{0, \dots, q-1\}$$

$$\phi_{n_i}(\mathbf{r}) = \frac{1}{\sqrt{2\pi n_i! l^2}} \left( \frac{x+iy}{\sqrt{2}l} \right)^{n_i} \exp\left(-\frac{r^2}{4l^2}\right)$$

Complete orthogonal basis in  $\mathcal{H}_{\theta_1\theta_2}$  if  
 $(\mu, \nu) \neq (\pi, \pi)$  for all  $(n_1, n_2) \in I_1 \times I_2$

Periodic on the finer lattice

$$S(\mathbf{c})\phi_{n_i}^{\mu\nu} = e^{i\mu} \phi_{n_i}^{\mu\nu}, \quad S(\mathbf{d})\phi_{n_i}^{\mu\nu} = e^{i\nu} \phi_{n_i}^{\mu\nu}$$

$$\mathbf{c} = \frac{l_1}{p}, \quad \mathbf{d} = \frac{l_2}{q}, \quad pq \in \mathbf{N}$$

FIR-absorption, THFA

Self-consistent response to the in-field

$$\mathbf{E}_{ext}(\mathbf{r}, t) = -i\mathcal{E}_0 \frac{\mathbf{k} + \mathbf{G}}{|\mathbf{k} + \mathbf{G}|} \exp\{i(\mathbf{k} + \mathbf{G}) \cdot \mathbf{r} - i\omega t\}$$

System properties  $\rightarrow \epsilon_{\mathbf{G}, \mathbf{G}'}(\mathbf{k}, \omega) \rightarrow$   
self-consistent field  $-\nabla\phi_{sc}$

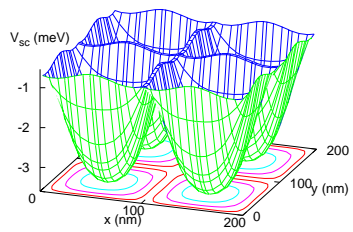
$$\sum_{\mathbf{G}'} \epsilon_{\mathbf{G}, \mathbf{G}'}(\mathbf{k}, \omega) \phi_{sc}(\mathbf{k} + \mathbf{G}', \omega) = \phi_{ext}(\mathbf{k} + \mathbf{G}, \omega)$$

Joule heating  $\rightarrow$  power absorption

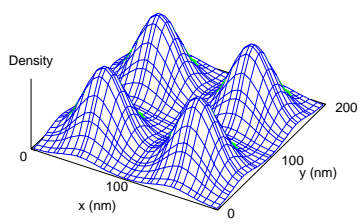
$$P(\mathbf{k} + \mathbf{G}, \omega) = -\frac{\omega}{4\pi} \Im\{\mathcal{E}_0 \phi_{sc}(\mathbf{k} + \mathbf{G}, \omega)\}$$

# Quantum dots

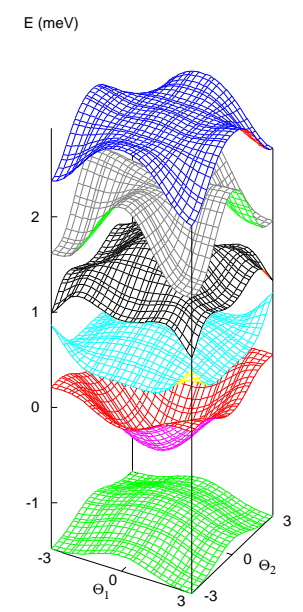
Potential



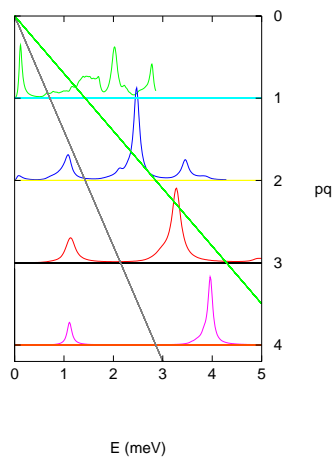
Density



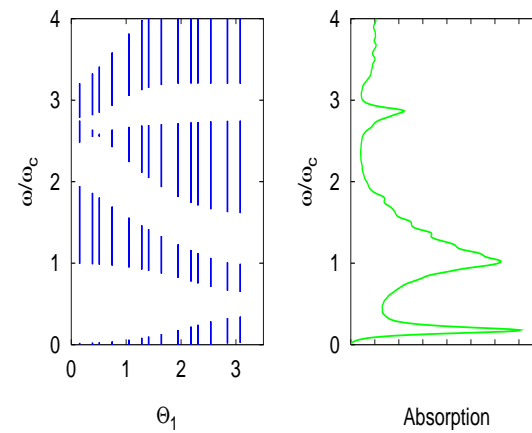
Energy spectrum,  $pq = 1$ ,  $N_s = \frac{1}{2}$ ,  $L = 100$  nm



Absorption,  $L = 100$ ,  $V = -5$  meV,  $N_s = \frac{1}{2}$

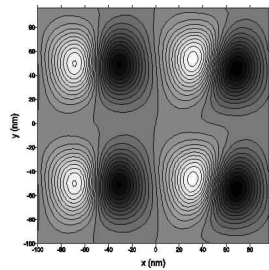


$pq = 1$

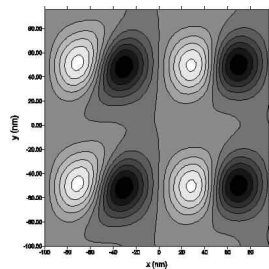


- $\hbar\omega_c = 0.714$  meV,  $V = -5$  meV
- Intra-band transitions
- Single-particle transitions

Induced density,  $pq = 3$ ,  $N_s = \frac{1}{2}$ ,  $l = 23$  nm



$$E = \hbar\omega = 1.13 \text{ meV}$$



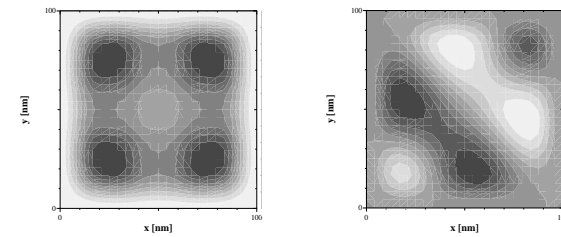
$$E = \hbar\omega = 3.27 \text{ meV}$$

Finite system,  $2 \times 2$  dots

Short "lattice" constant  $L = 50$  nm

Weak modulation  $V_0 = -5$  meV

Density  $\leftrightarrow$  Induced density



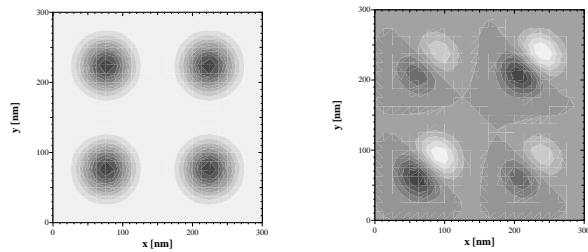
Overlapping density,  $B = 0$ T

### Finite system $2 \times 2$ dots

$$L = 150 \text{ nm}$$

$$V_0 = -10 \text{ meV}$$

Density  $\leftrightarrow$  Induced density



Vanishing density overlap, Coulomb coupling

Also found for the infinite periodic system

### Conclusion

- We can see inside quantum dots with FIR spectroscopy
- Experiments on dots with few electrons will be refined
- Coupled dots
- Microscopic picture of the Bernstein modes?
- Antidots