

# Magnetization of confined and extended 2DEG's

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## Cooperation

### Reykjavík:

- Ingibjörg Magnúsdóttir → COM Copenhagen
- Sigurður I. Erlingsson → TU Delft
- Andrei Manolescu → deCode Reykjavík

### Hamburg:

- Detlef Heitmann
- Dirk Grundler
- Ines Meinel

## Why magnetization?

- The electron **state** in a system is measured by **FIR-absorption**, **tunneling**, **Raman-scattering**, magnetization, and **transport**...
- FIR-absorption excites mostly **center-of-mass** modes, **correlation** effects block tunneling spectroscopy
- **Magnetization** is **not** limited by selection rules...
- In magnetization **many-electron** effects are seen...
- **Direct** access to the **ground state**
- Valuable addition, good experiments...

## Magnetization

Magnetization is defined in terms of **current-** and **spin-**density:

$$M_o + M_s = \frac{1}{2} \int_{\mathbf{R}^2} d\mathbf{r} (\mathbf{r} \times \langle \mathbf{J}(\mathbf{r}) \rangle) \cdot \hat{\mathbf{n}} - g\mu_B \int_{\mathbf{R}^2} d\mathbf{r} \langle \sigma_z(\mathbf{r}) \rangle,$$

or **total energy**:

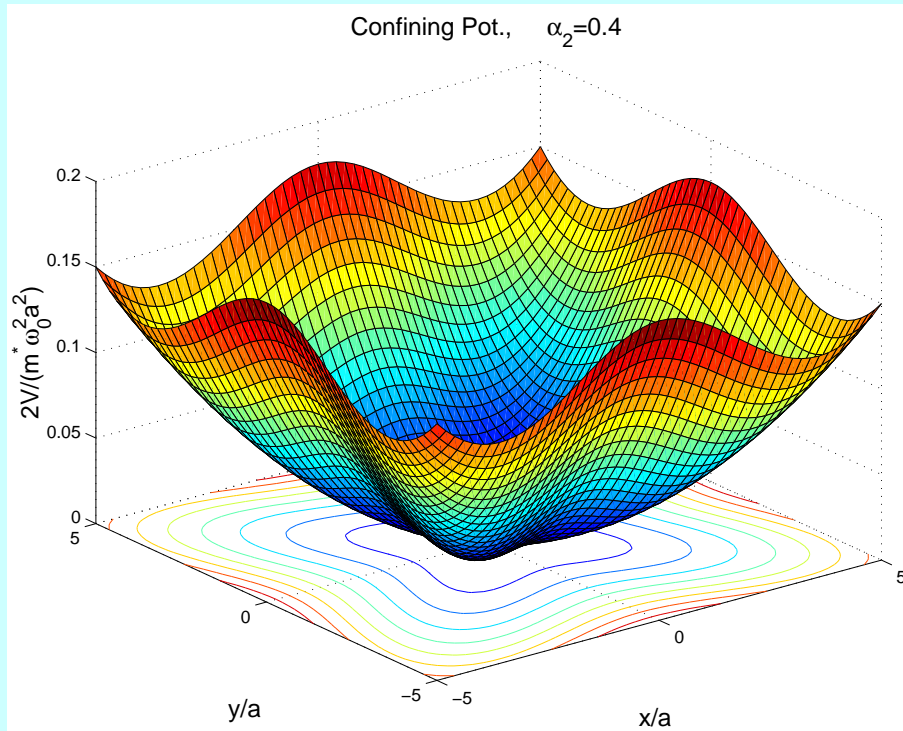
$$M_o + M_s = -\frac{\partial}{\partial B} (E_{\text{total}} - TS)$$

## Systems

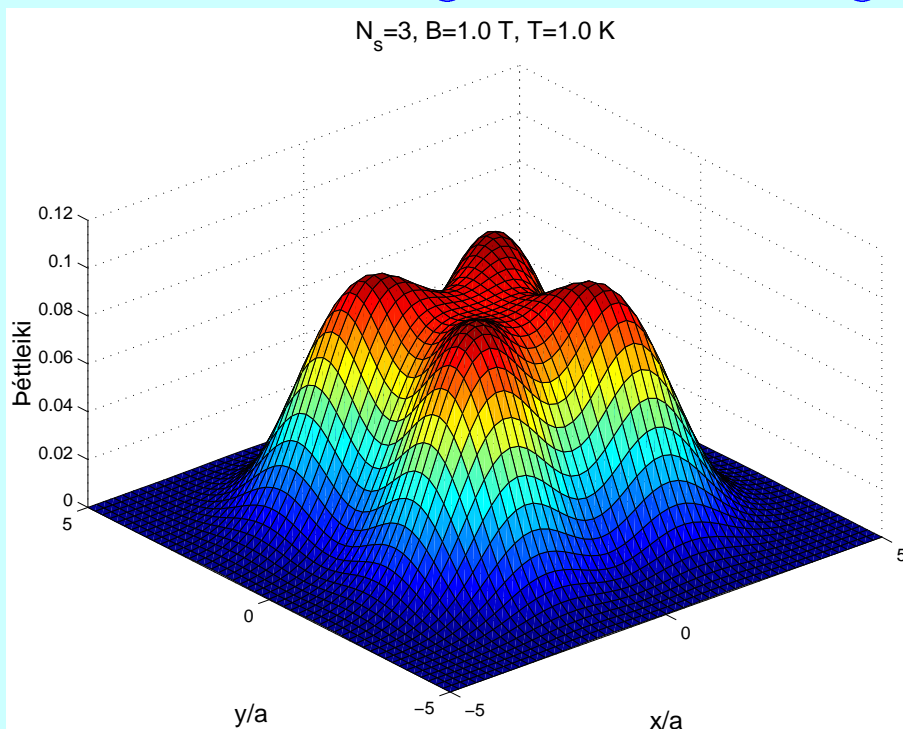
- **Quantum Dots**, noncircular
- **Finite system**, increasing size
- Infinite 2D-systems, **1D modulation**
- Infinite 2D-systems, **2D modulation**
- Infinite 2D-systems, **Hysteresis**

## Quantum dot, (IM)

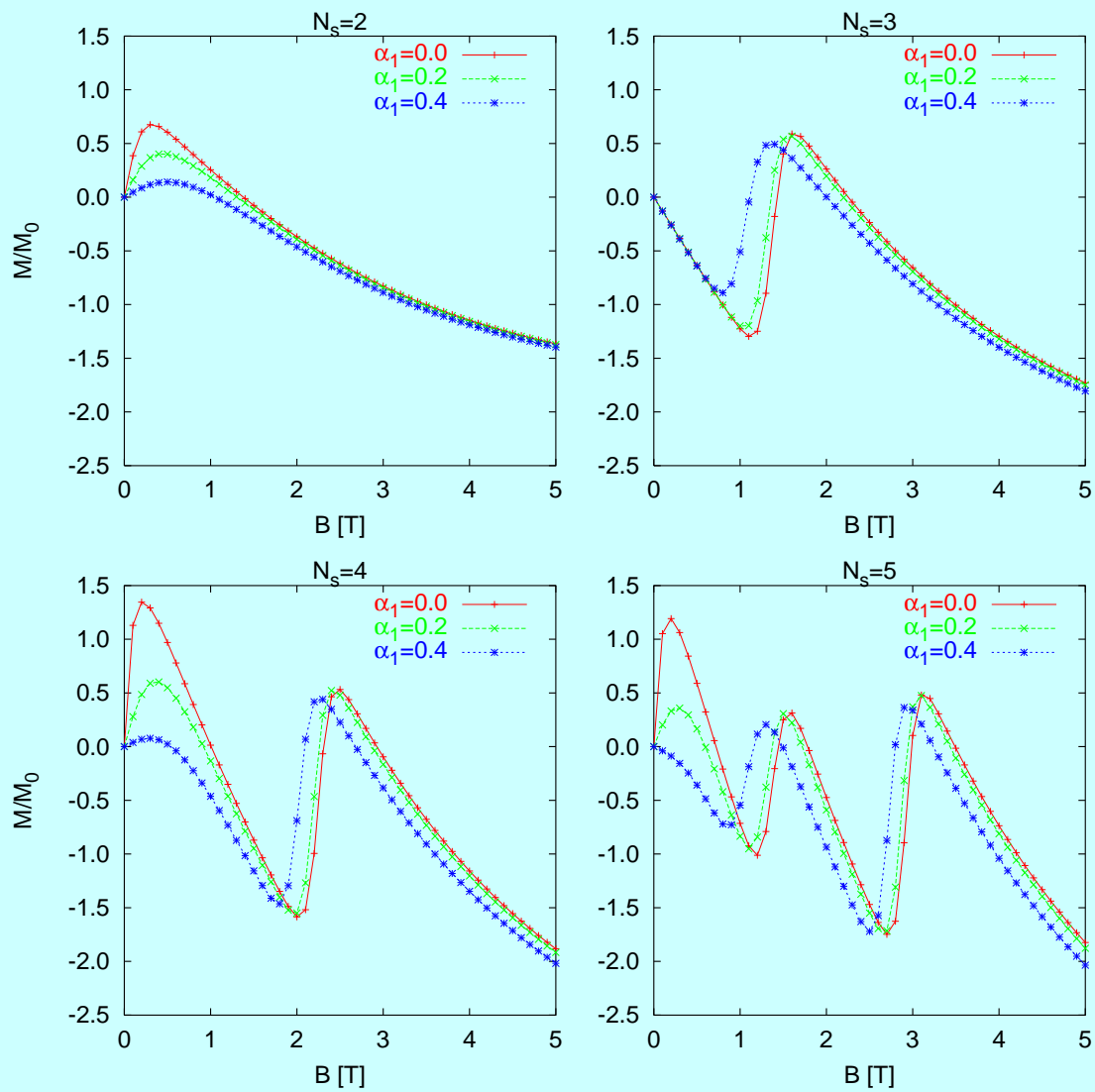
Confining potential for electrons in a dot



Density of three interacting electrons in magnetic field



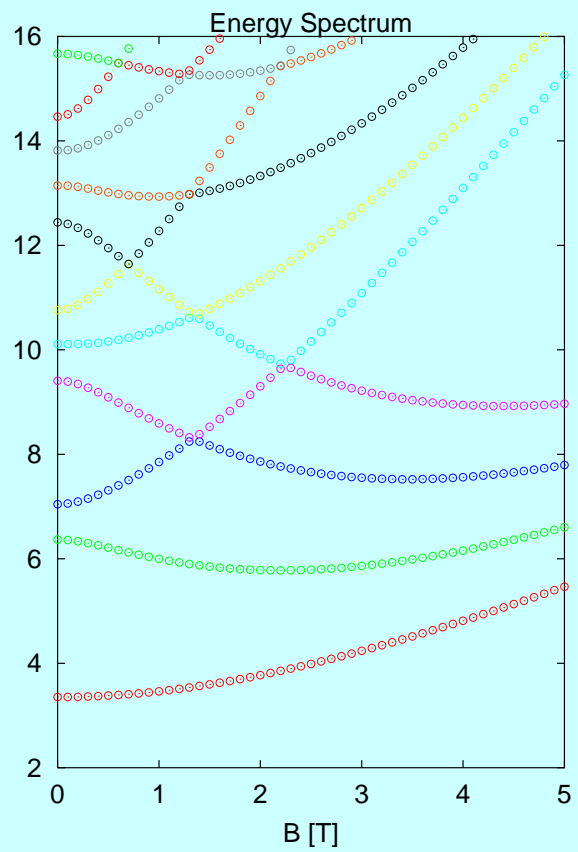
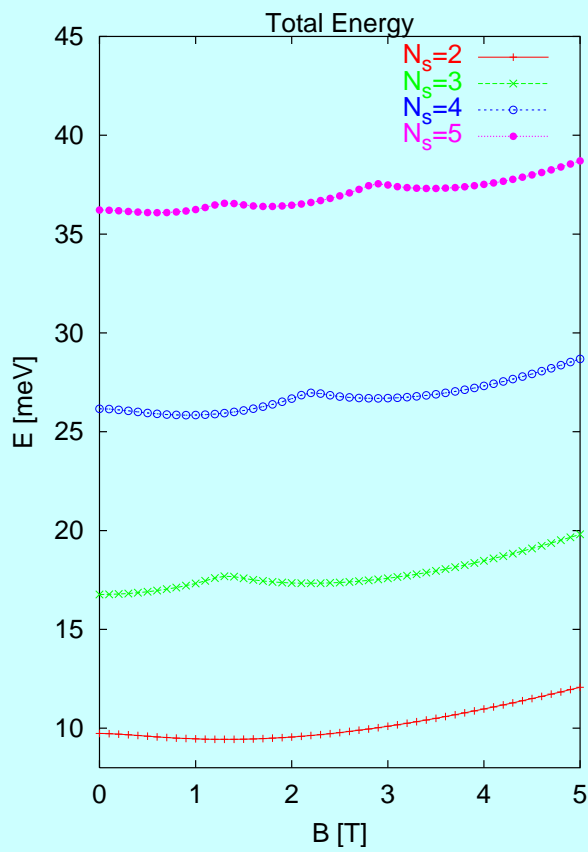
## Elliptic quantum dot, magnetization



Effects of increased number of electrons and deviation  
Noninteracting electrons

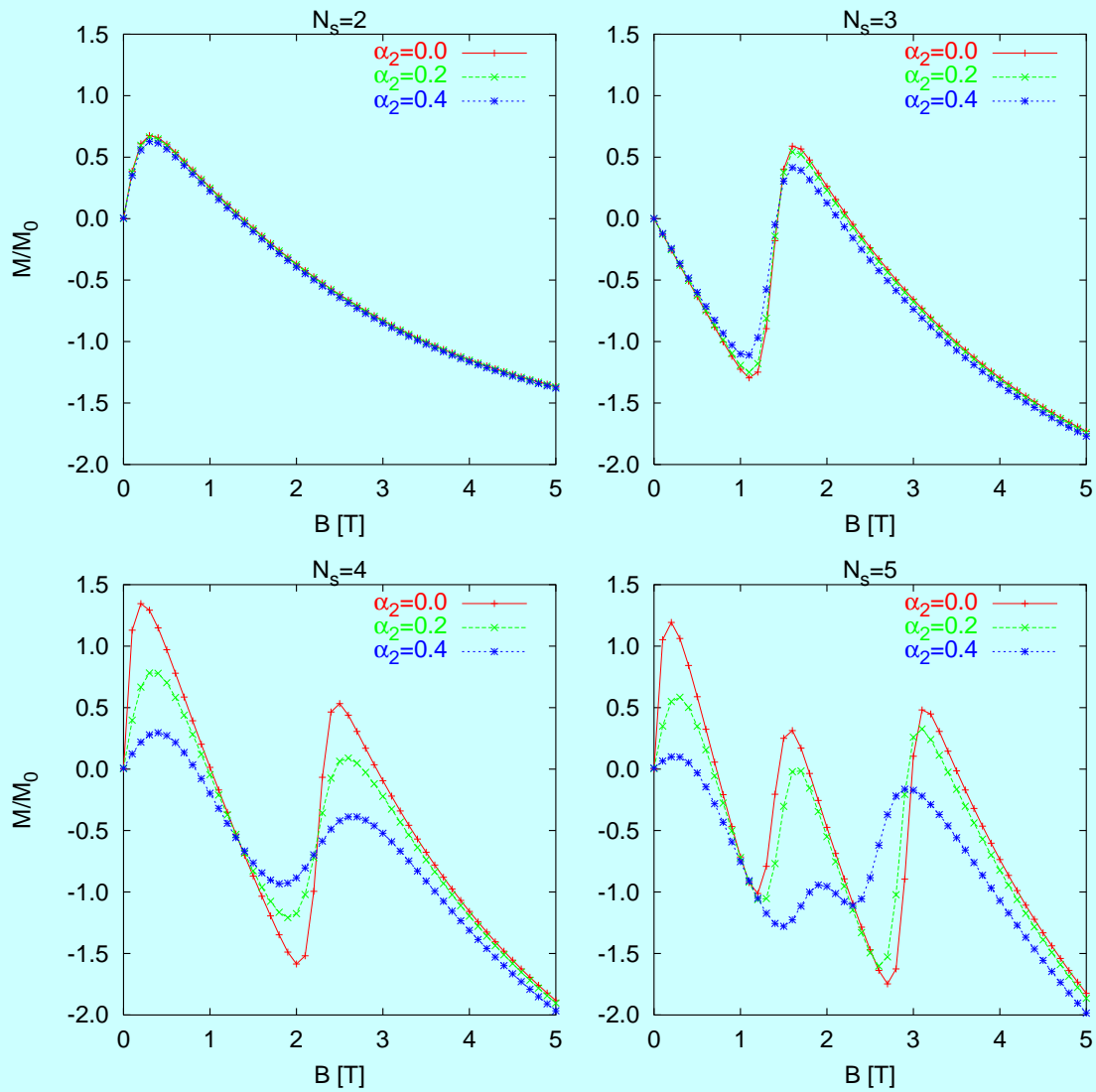
# Elliptic dot, energy spectrum

Total energy ↔ Energy spectrum



noninteracting electrons

## Square shaped dot, magnetization

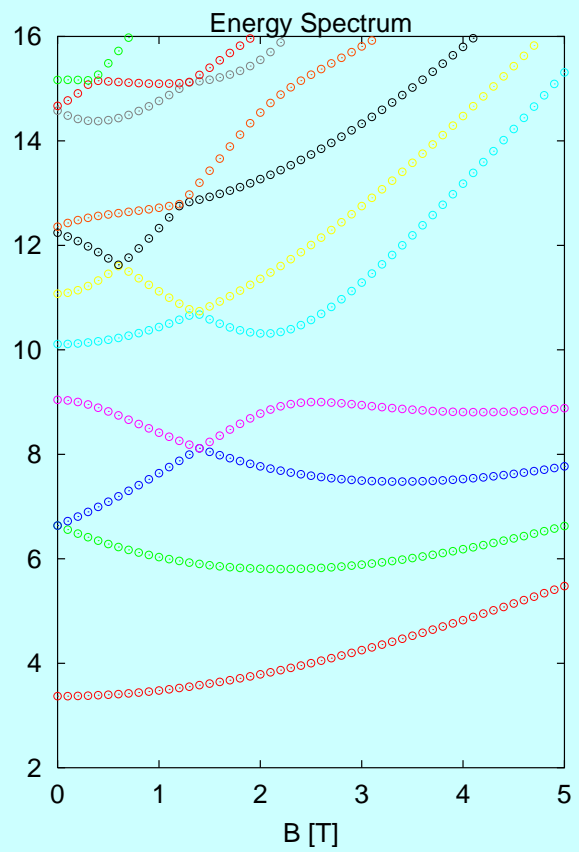
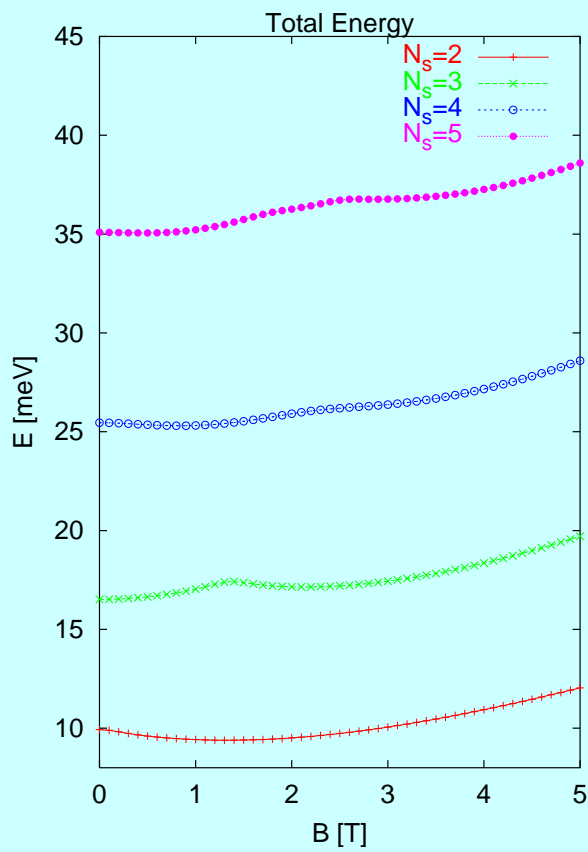


Effects of increased number of electrons and deviation  
Noninteracting electrons



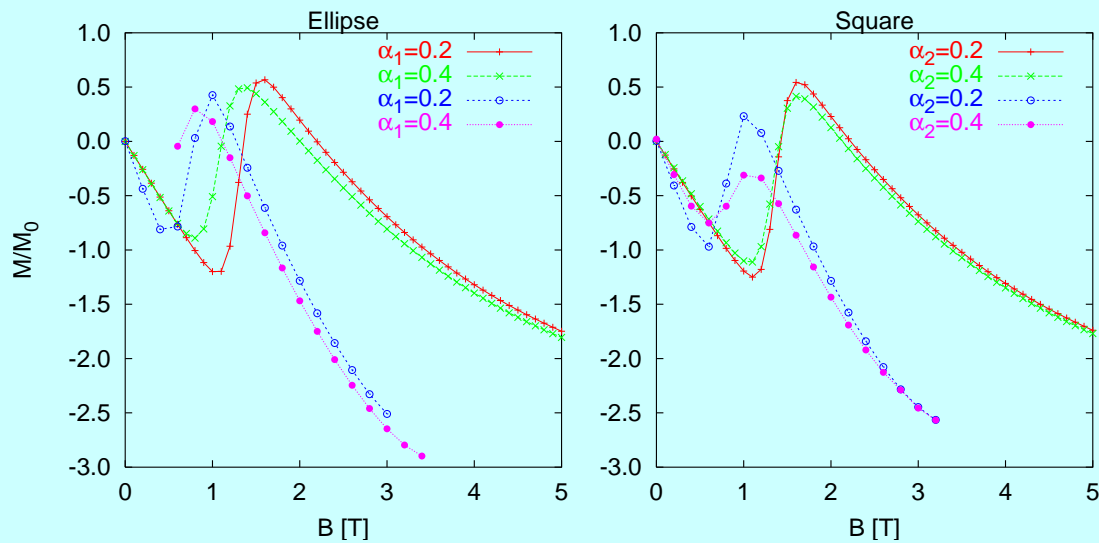
# Square shaped dot, energy spectrum

Total energy ↔ Energy spectrum



noninteracting electrons

## Interacting electrons, magnetization



## Hartree approximation for interaction

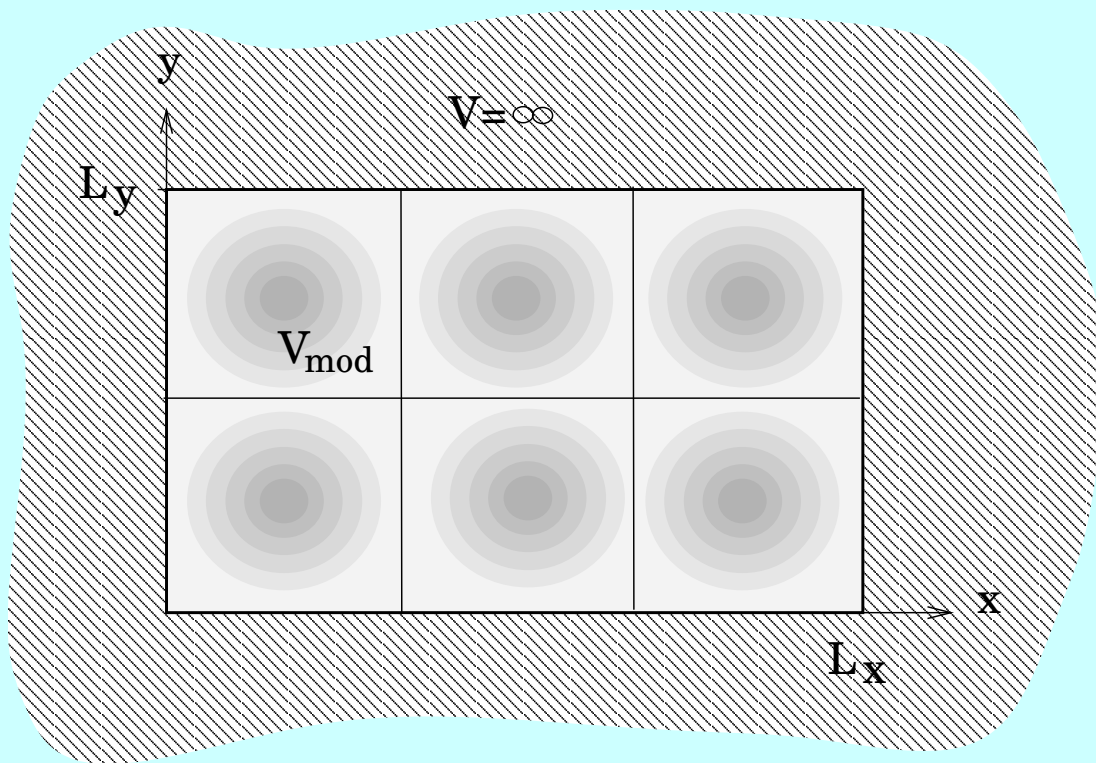
### Dots

- Increased  $B \rightarrow$  increased momentum of inertia, jumps between many-electron states. Magnified by Coulomb interaction
- For few electrons  $M$  depends strongly on  $N$ ,  $B$ , and deviation from circular shape
- Clear signature of many-electron states

Finite system, increased in size, (SIE)

$N_c = n_x \times n_y$  unit cells

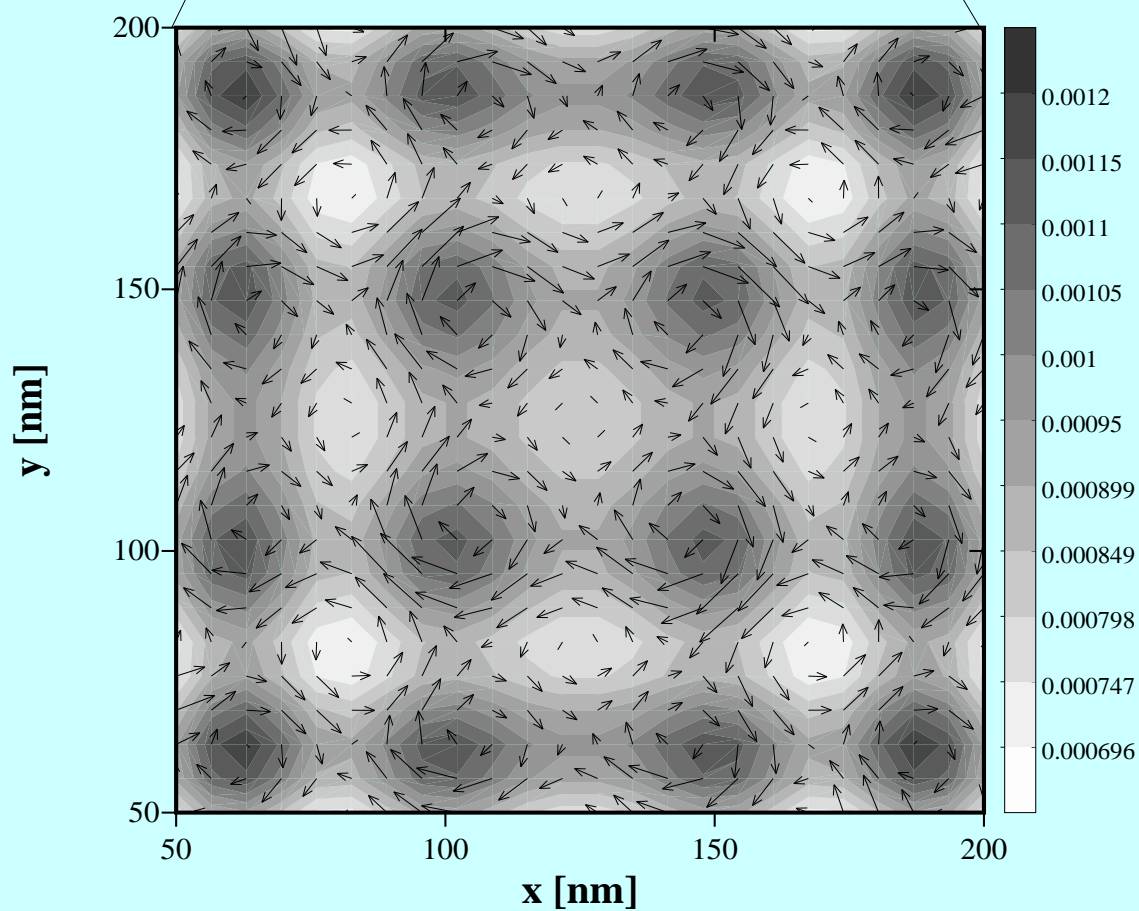
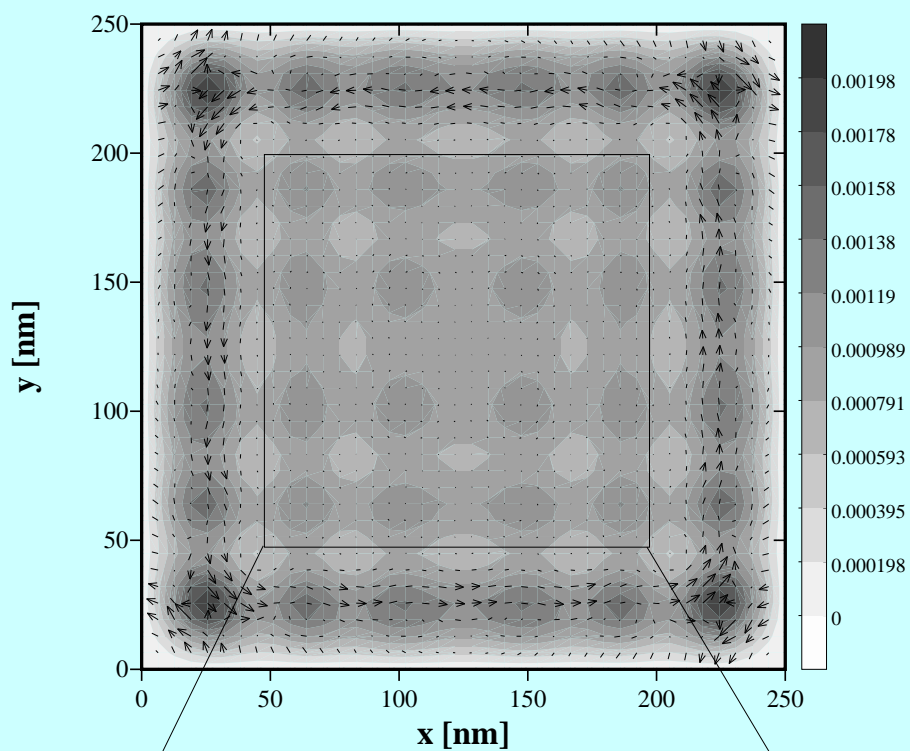
Hartree approximation for interaction



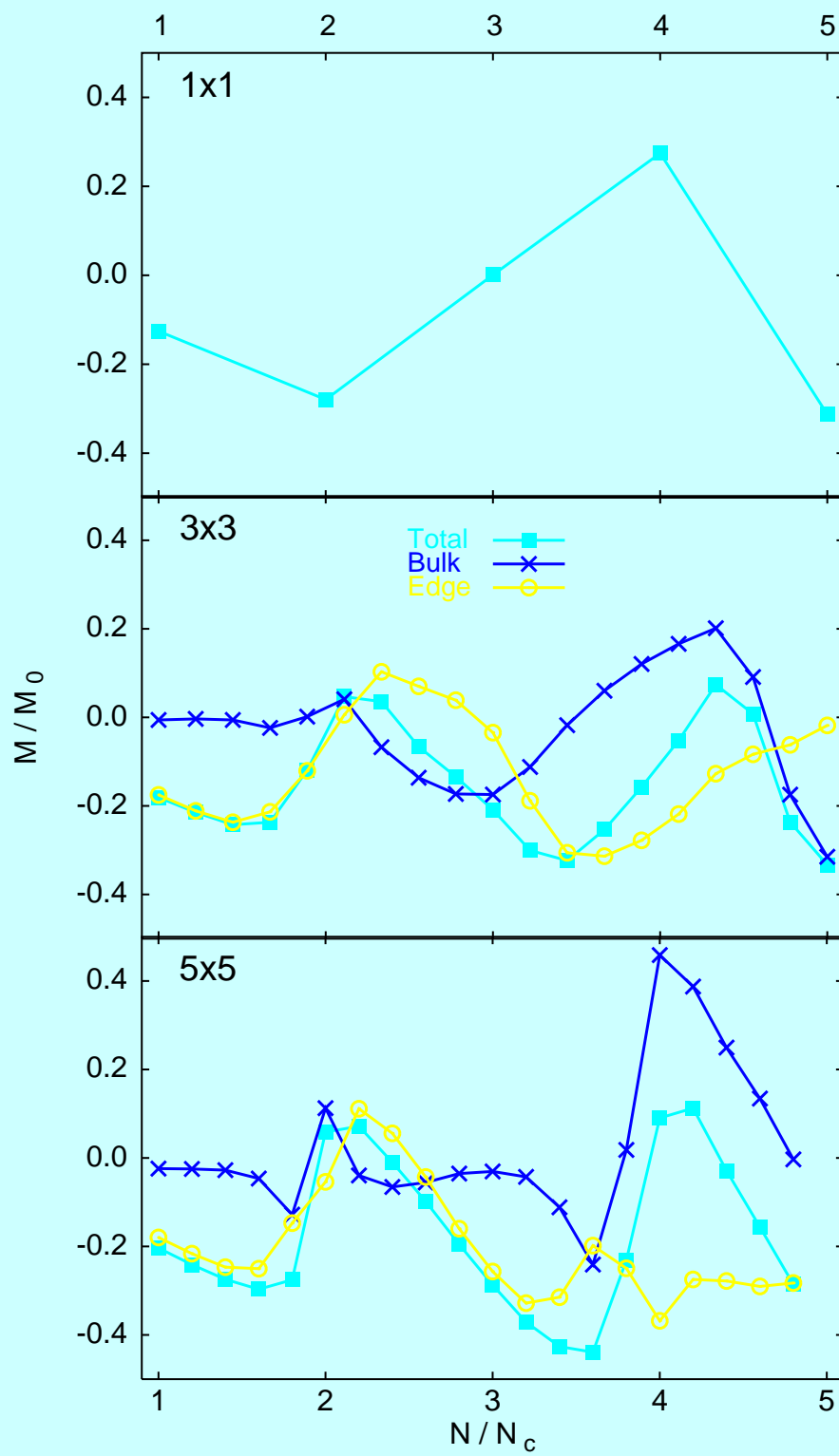
$$V_{\text{sq}}(\mathbf{r}) = V_0 \left\{ \sin \left( \frac{n_x \pi x}{L_x} \right) \sin \left( \frac{n_y \pi y}{L_y} \right) \right\}^2,$$

$V_0 = -1 \text{ meV}$ ,  $T = 1 \text{ K}$ ,  $B = 1.65 \text{ T}$ , ( $1 \times \Phi_0$  per cell)

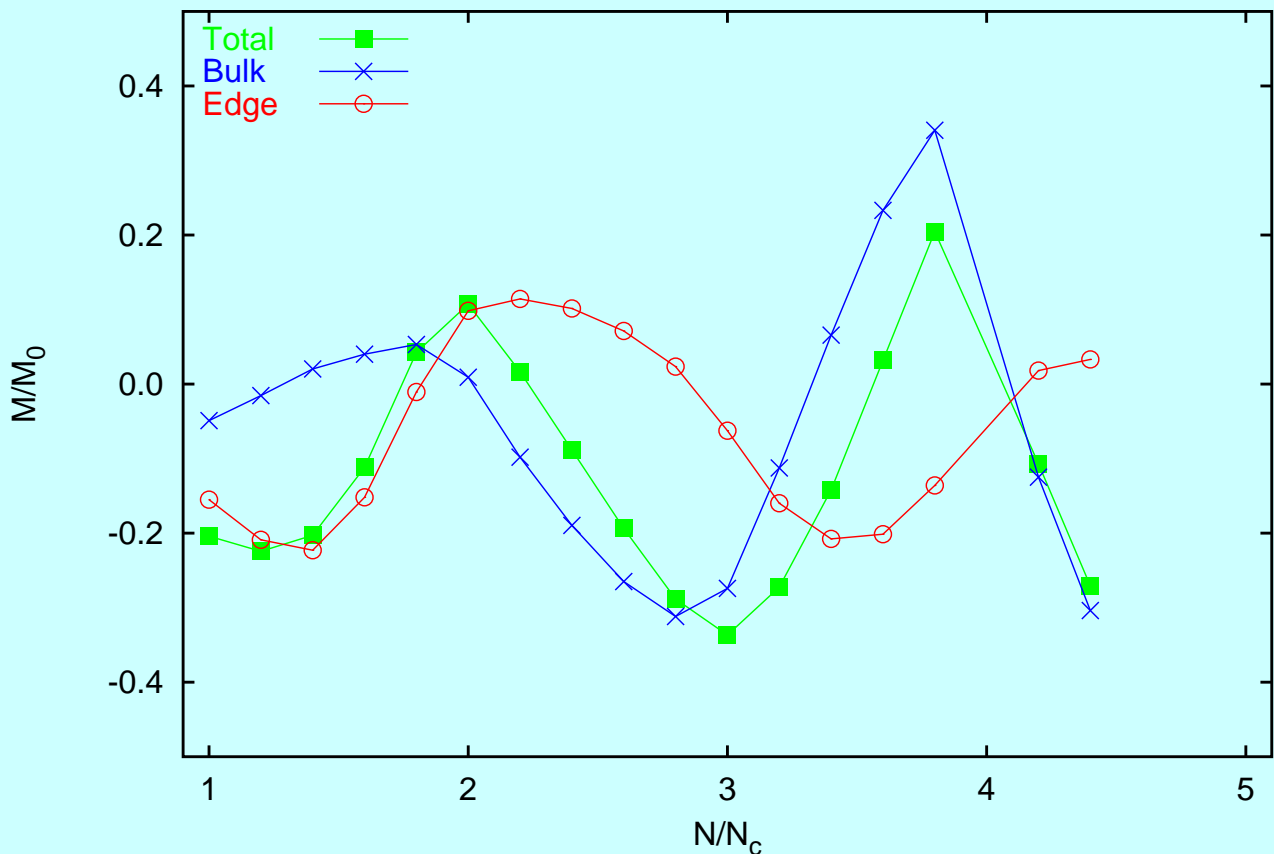
Total = edge + bulk, (arbitrary division)



$$\nu \approx N/N_c$$

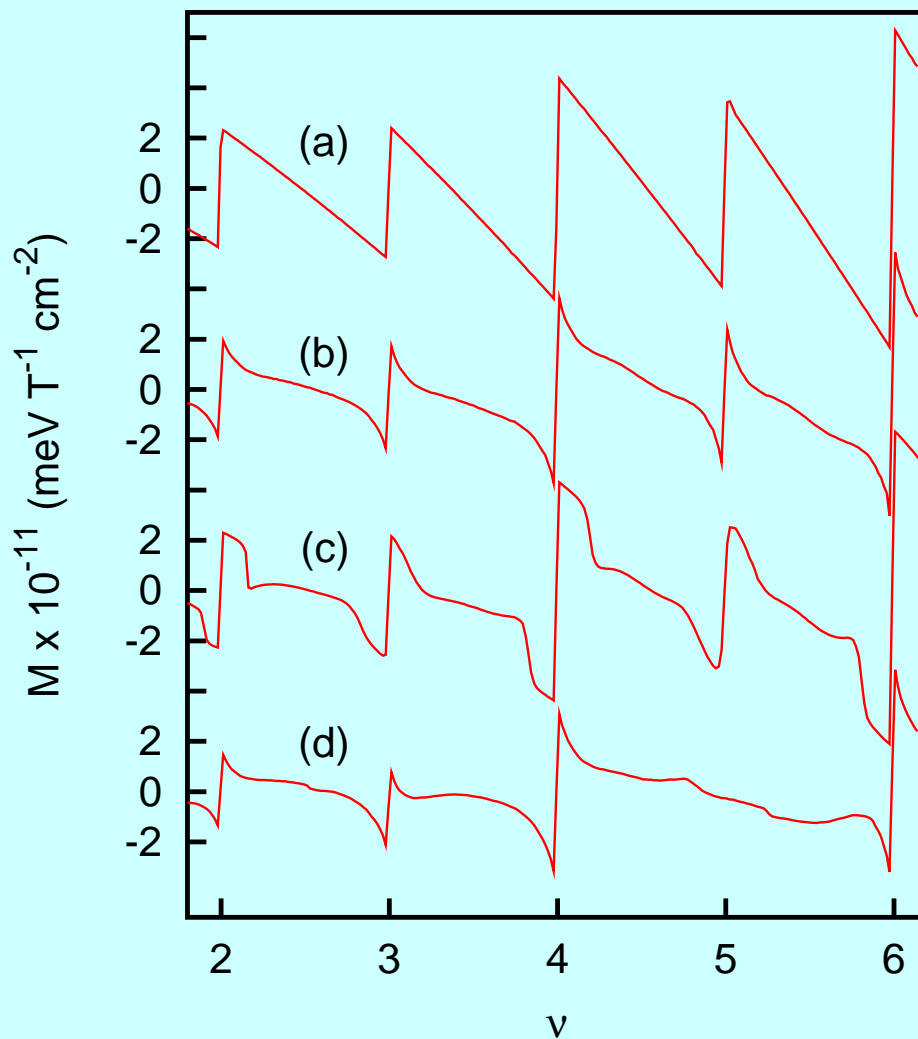


Without interaction,  $N_c = 5 \times 5$



- **Interaction** is important
- **Large system**  $\rightarrow$  strong screening, except for  $N/N_c \approx \nu = \text{integer}$
- $M$  is similar for weak 1D and 2D modulation
- **Bulk** contribution is similar to  $M$  for an infinite system

## Finite 2DEG, 1D modulation, (AM)



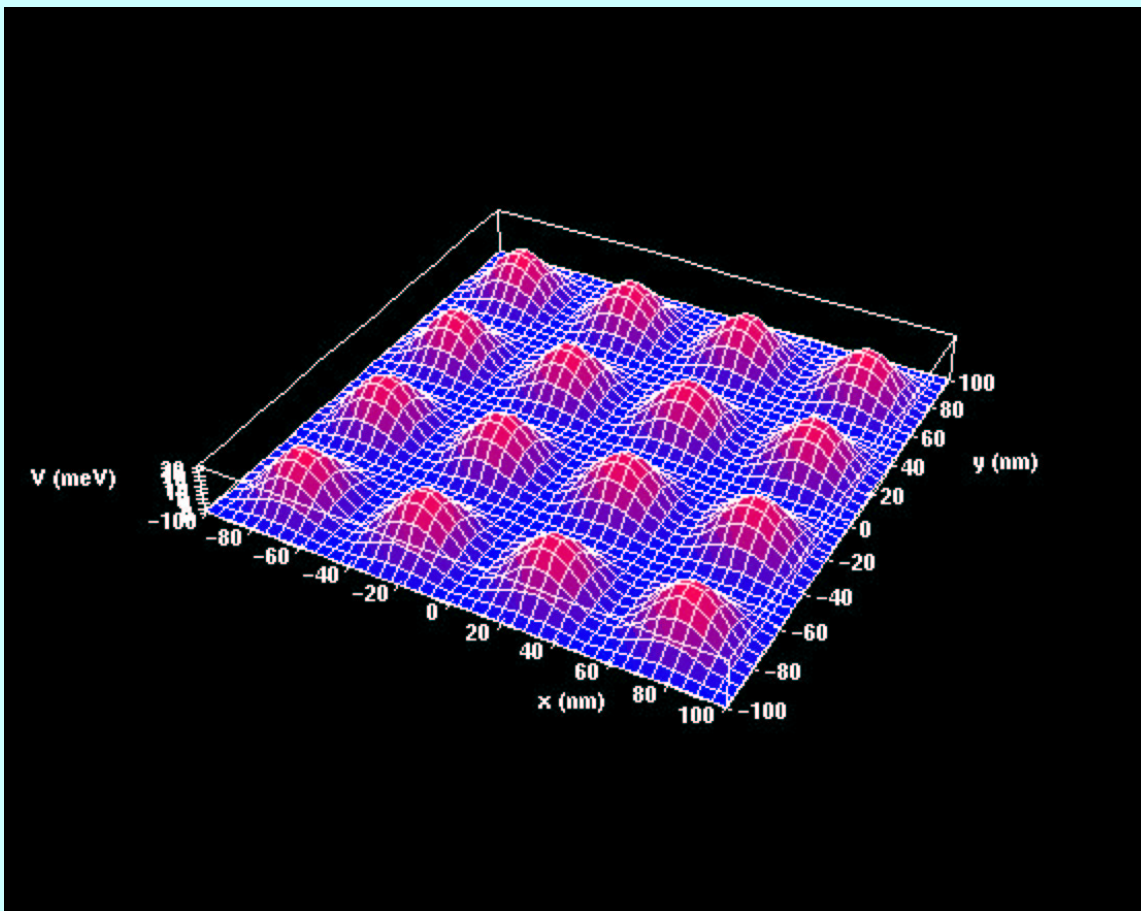
(a): **Homogeneous** 2DEG (HFA)

(b): **1D modulation**,  $V_0 = 1.5 \text{ meV}$ , (HFA)

(c): (b)+disorder:  $\Gamma = 2.6 \text{ meV}$

(d):  $V_0 = 5 \text{ meV}$

## Finite 2DEG, 2D modulation, (VG)



**2D modulation** → commensurability problem

→ integer  $\Phi_0$  through “unit cell”

→ Hofstadter energy spectrum

→ **technical difficulty** calculating  $M[E, T, S, B]$

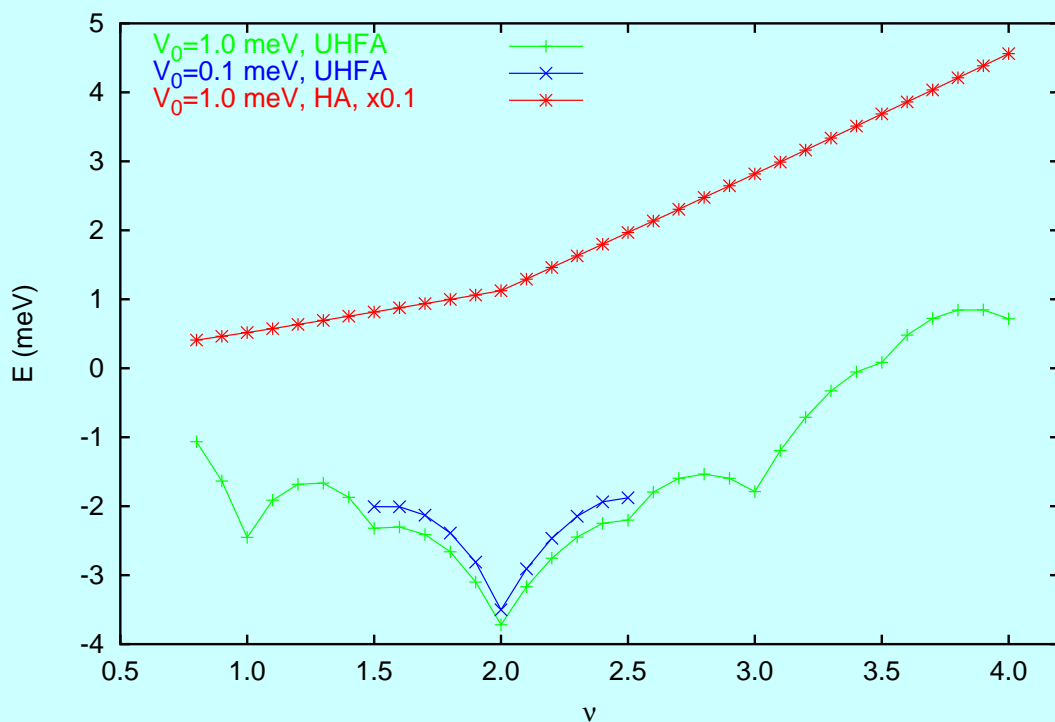
→ Vary electron density, **experiment**



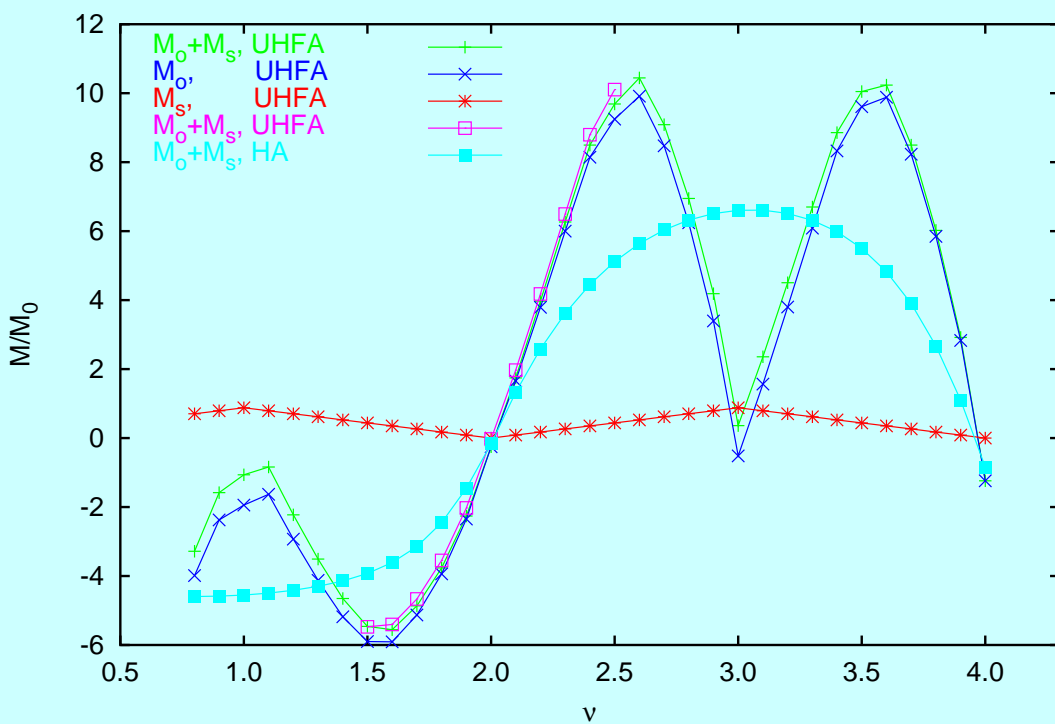
- In a **finite** system  $M[E, T, S, B]$  and  $M[J, \sigma, B]$  are **equivalent**
- $M[E, T, S, B]$  can be derived in a **homogeneous infinite** system from  $M[J, \sigma, B]$  for a finite system in the **proper limit** “size  $\rightarrow \infty$ ” w.r.t. the **edge**
- Without the proper limit  $M[J, \sigma, B] = 0$  for an infinite homogeneous system  $\leftarrow$  **no edge**
- In our 2D modulated 2DEG  $M[J, \sigma, B] \neq 0$   
 $\leftarrow$  modulation  $\neq 0$
- **Experiment: SQUID-loop inside system!**

Hartree-Fock approximation,  $pq \times \Phi_0$  in a unit cell

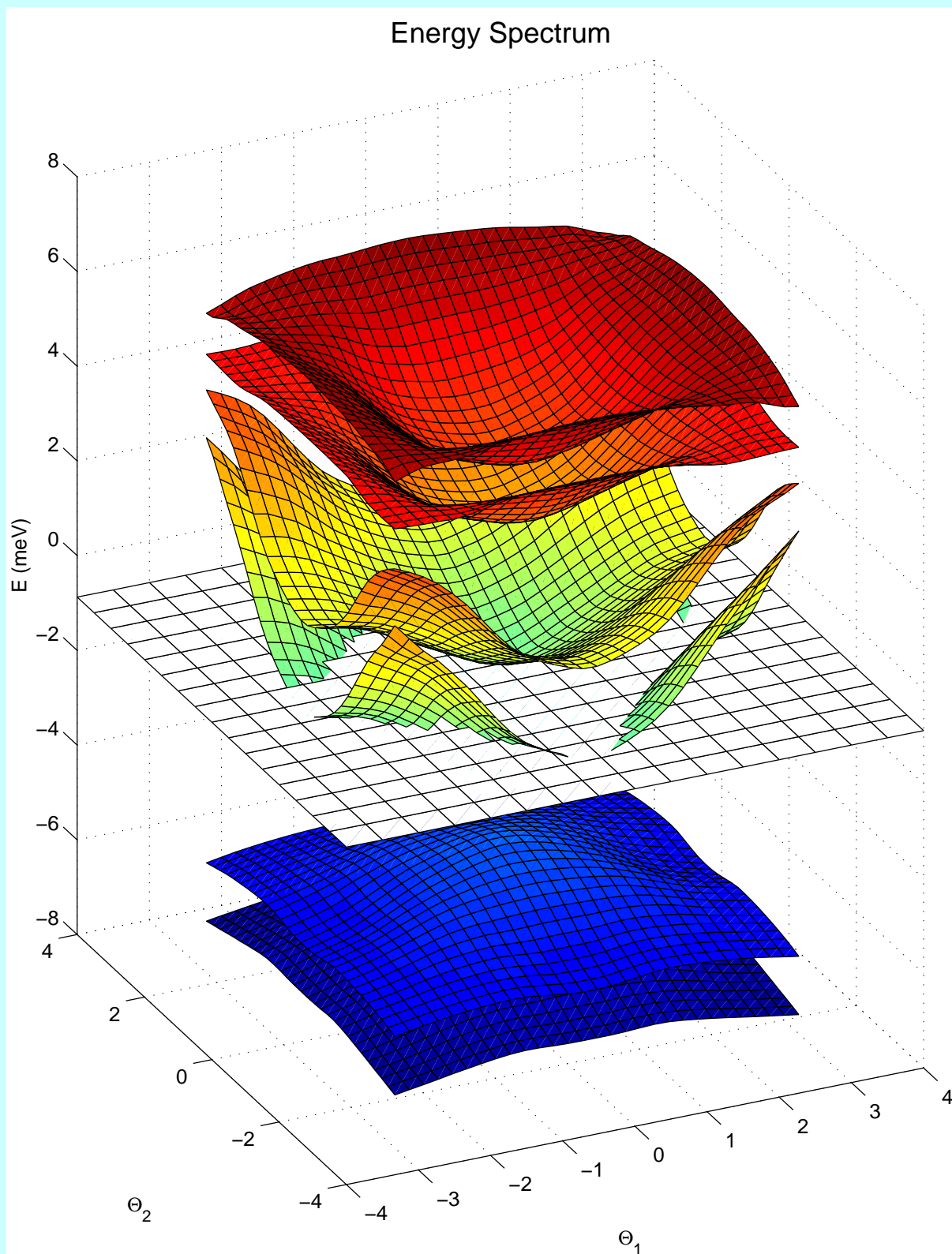
# Total energy, $pq=2$



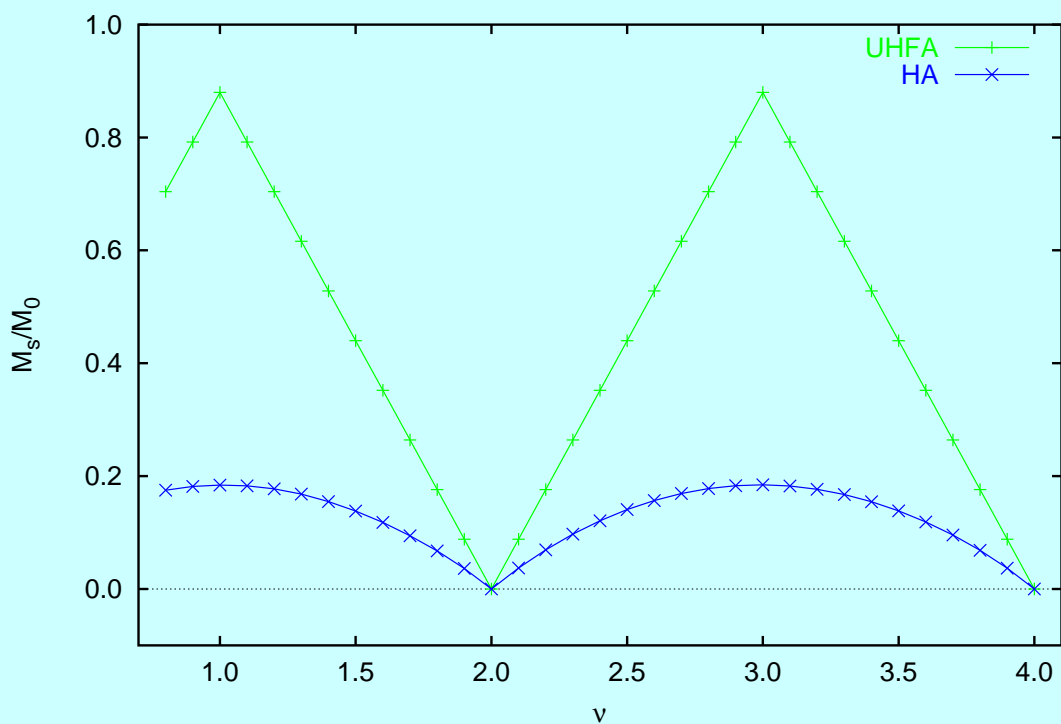
# Magnetization, (cosine modulation)



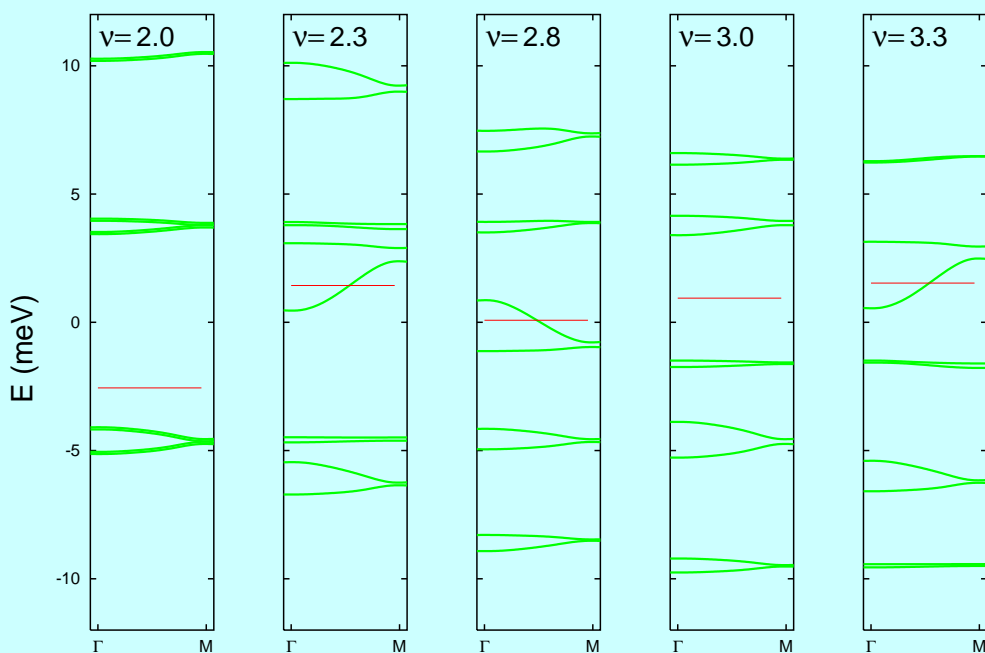
$pq = 1$  2 fold Landau-bands, (spin),  $\nu = 2.55$



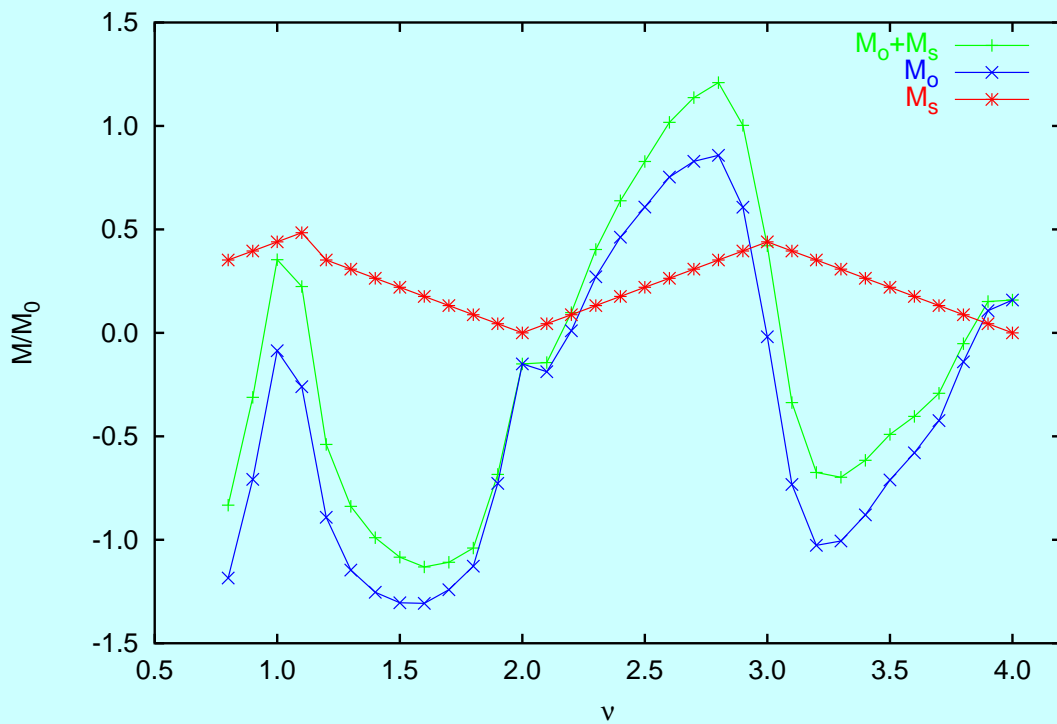
$M_s$ : Spin contribution HA or UHFA



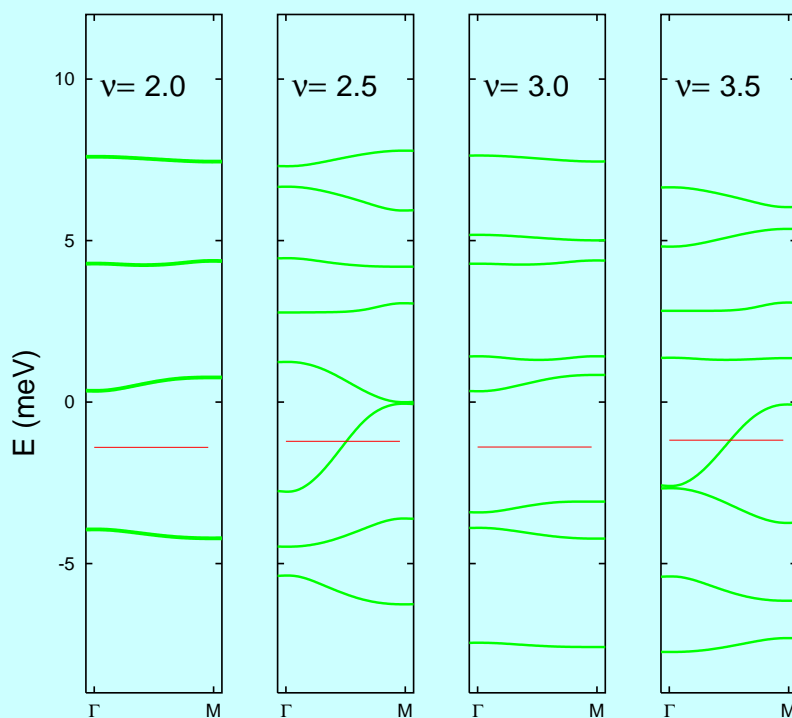
Bandstructure  $pq = 2$ , 4 fold Landau-bands, (2 spin, 2 Hofstadter)



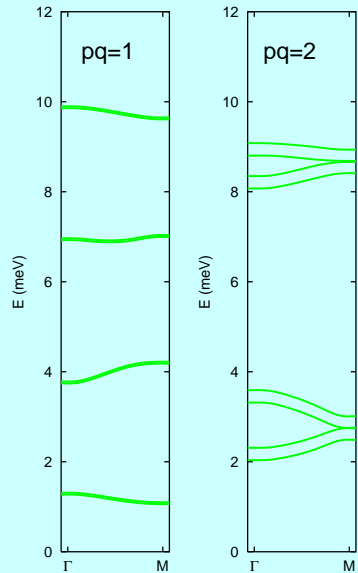
# Magnetization $pq = 1$



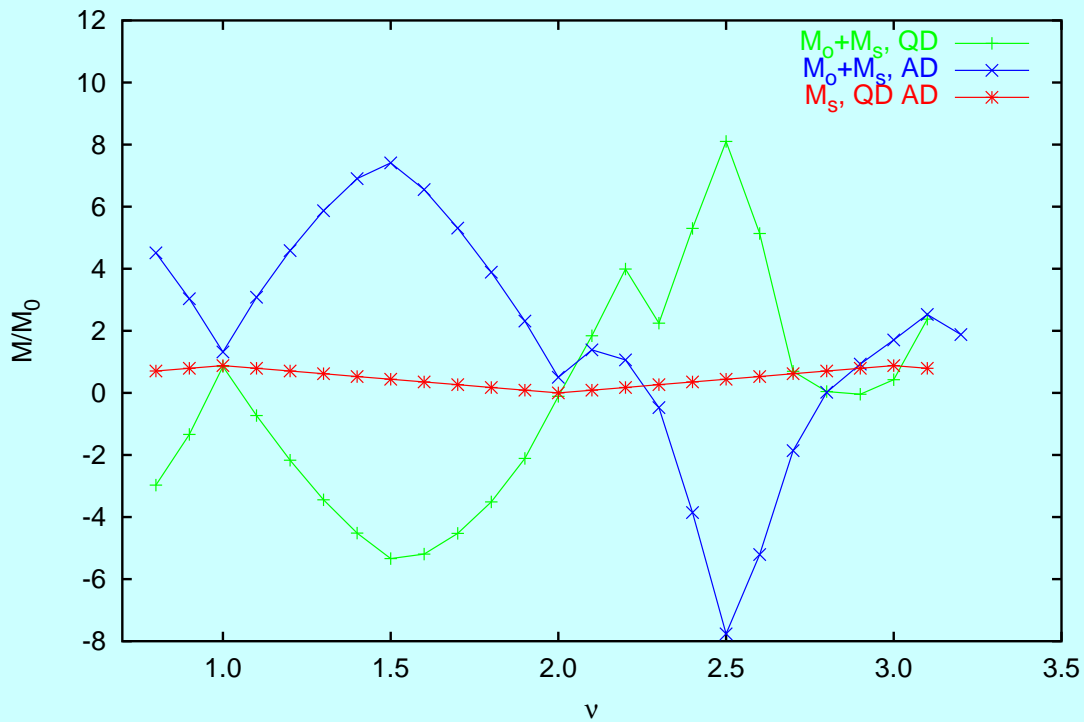
Bandstructure  $pq = 1$ , **2 fold Landau-bands**, (2 spin, 1 Hofstadter)



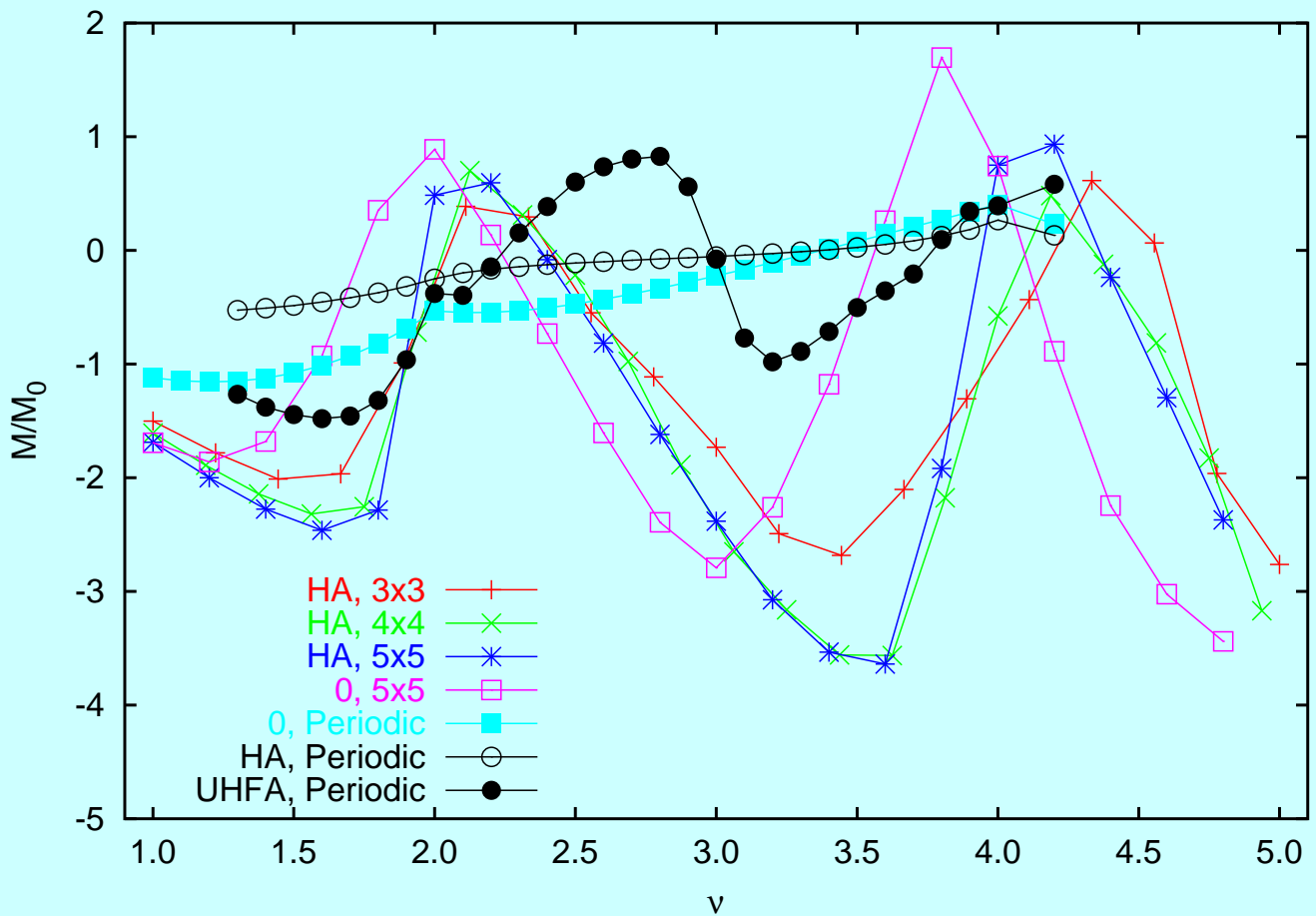
# Noninteracting Bandstructure, (static)



$pq = 2$ , Dot- antidot array,  $V_0 = \pm 5$  meV



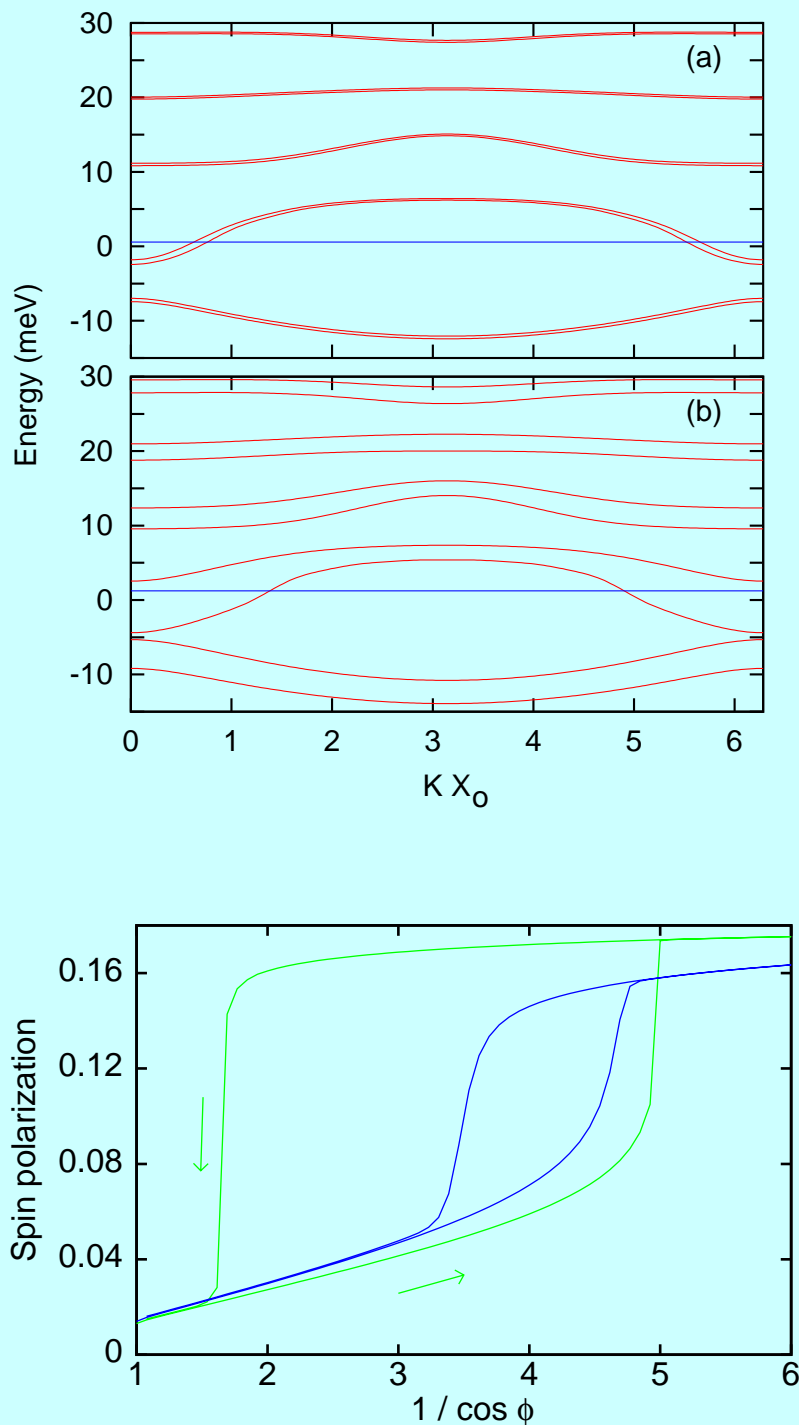
Symmetry ← finite  $M$  only due to modulation



- Infinite 2D modulated 2DEG has no boundary
- Effects of edge states is not simple
  - Direct effects on  $M_e$
  - Indirect effects on  $M_b$  through self-consistent shape of energy bands
  - Connected to motion of  $\mu$  through bands

## Hysteresis, (AM)

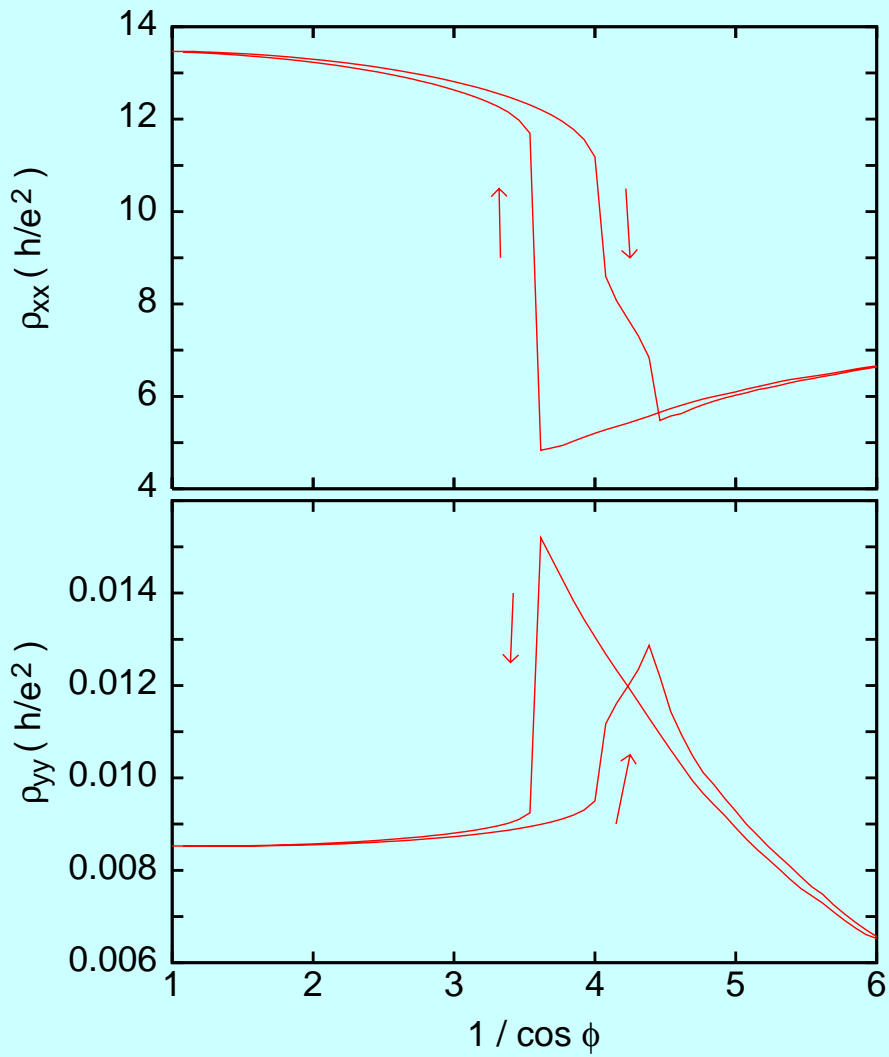
1D modulation, short period:  $a = 40$  nm,  
 $V_0 = 9$  meV, cosine modulation



Sample tilted  $\phi$ ,  $T = 5$  K, 7 K



## Visible in transport



Short period, strong modulation

→ weak screening of exchange force

## Results

- Magnetization is promising for investigation of the ground state
- Magnetization of a Hofstadter system?
- Essential to improve and study approximation of the Coulomb interaction
- Magnetization measurements of structured systems