

**Is the Hofstadter energy spectrum
observable in
far-infrared absorption?**

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für Festkörperforschung

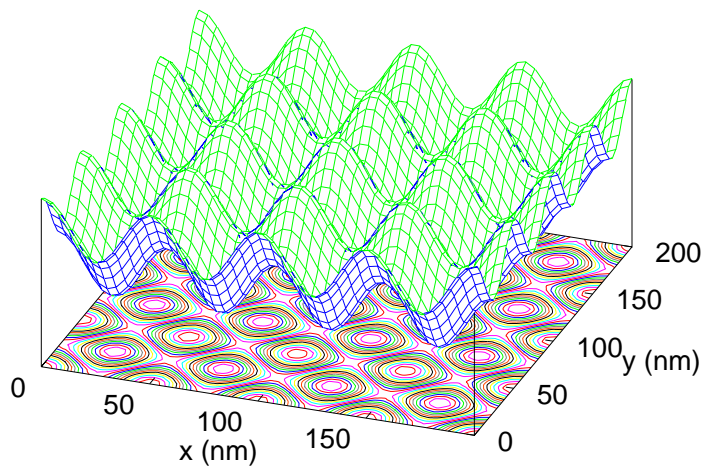
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Goals

- Bernstein modes?
- Effects of modulation
- Is the Hofstadter butterfly visible?



Ground state

2DEG in a periodic potential

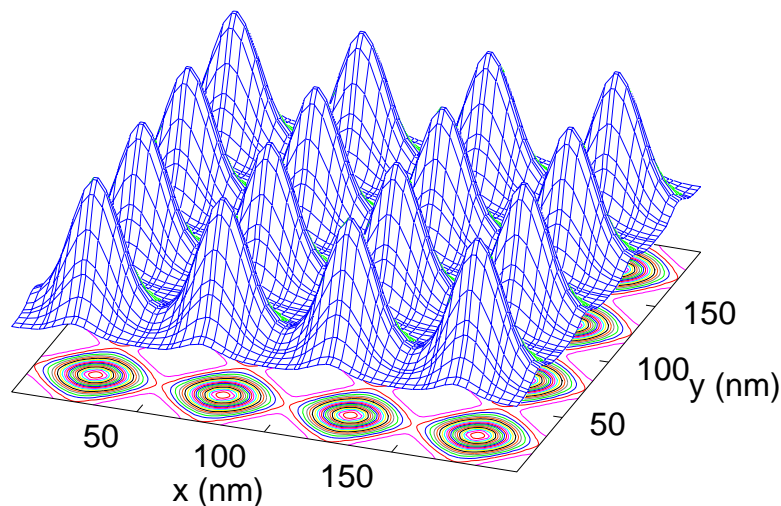
$$V(x, y) = V \{ \cos(gx) + \cos(gy) \}$$

$g = 2\pi/L$, with the periodic length L

Perpendicular magnetic field $\vec{B} = B\hat{z}$

Integer number pq of flux units $\Phi_0 = hc/e$
flows through a lattice unit cell with area

$$A = L^2 \longrightarrow B = pq\Phi_0/A$$



Magnetic length: $l = \sqrt{c\hbar/eB}$

Cyclotron frequency: $\omega_c = eB/m^*c$

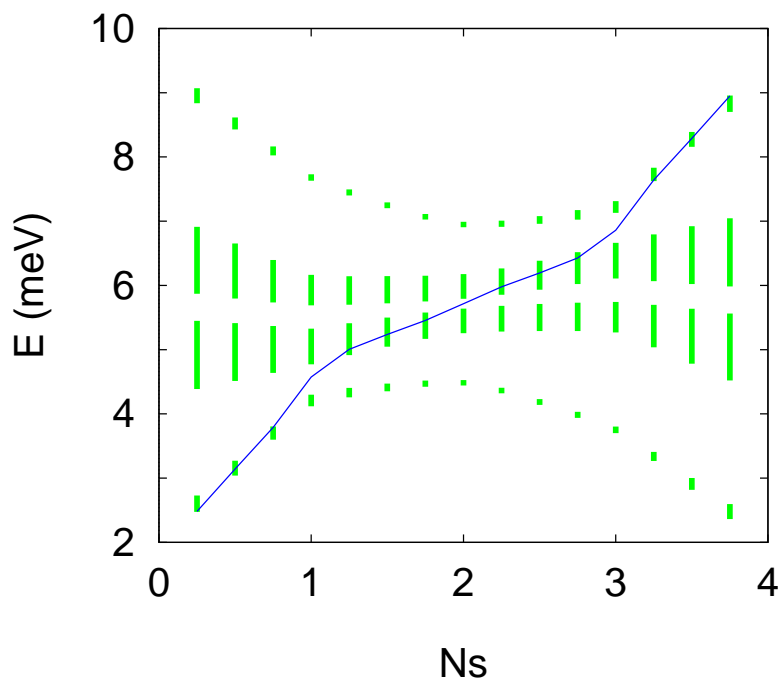
Commensurability between l and L splits each Landau level into pq subbands

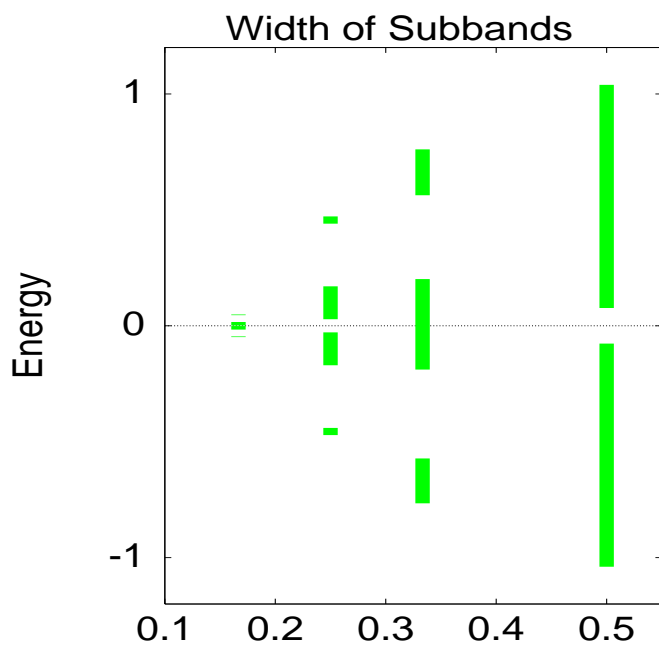
Hartree interacting electrons

Finite temperature T

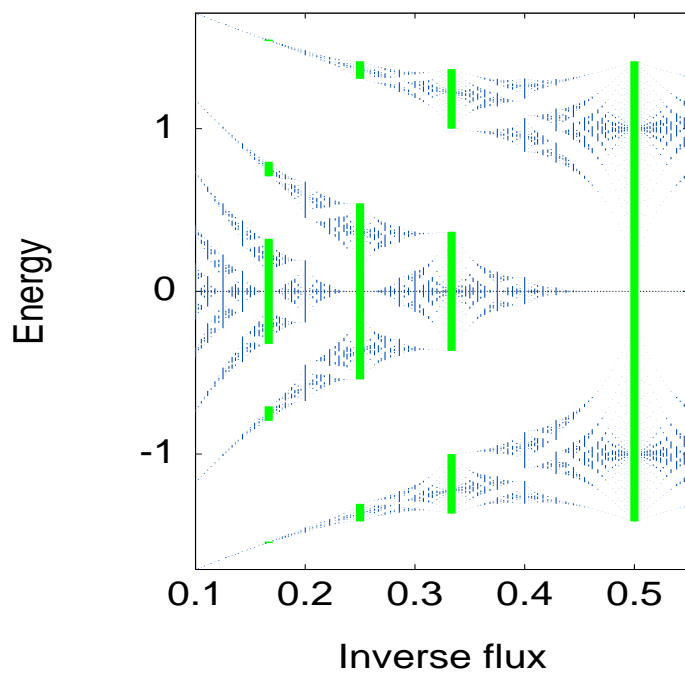
N_s electrons in a unit lattice cell

Filling factor of Landau levels is $\nu = N_s/pq$





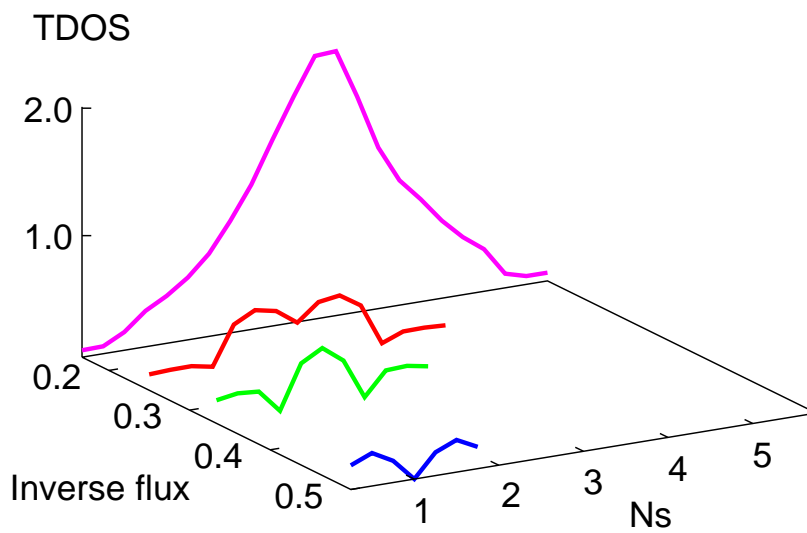
Interacting



Non interacting

$L=50\text{nm}$, $V=4\text{meV}$, $T=1\text{K}$, one Landau level

Subbands can be seen in TDOS!



Can they be seen in FIR?

Is strong modulation and short period needed?

What are the effects of ν ?

FIR-absorption

Self-consistent response to the in-field

$$\mathbf{E}_{ext}(\mathbf{r}, t) = -i\mathcal{E}_0 \frac{\mathbf{k} + \mathbf{G}}{|\mathbf{k} + \mathbf{G}|} \exp \{i(\mathbf{k} + \mathbf{G}) \cdot \mathbf{r} - i\omega t\}$$

System properties $\rightarrow \epsilon_{\mathbf{G}, \mathbf{G}'}(\mathbf{k}, \omega) \rightarrow$
self-consistent field $-\nabla \phi_{sc}$

$$\sum_{\mathbf{G}'} \epsilon_{\mathbf{G}, \mathbf{G}'}(\mathbf{k}, \omega) \phi_{sc}(\mathbf{k} + \mathbf{G}', \omega) = \phi_{ext}(\mathbf{k} + \mathbf{G}, \omega)$$

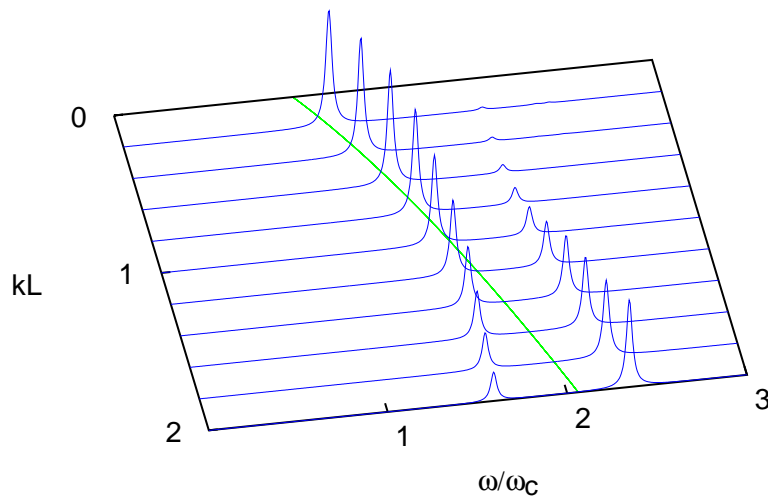
Joule heating \rightarrow power absorption

$$P(\mathbf{k} + \mathbf{G}, \omega) = -\frac{\omega}{4\pi} \Im \{ \mathcal{E}_0 \phi_{sc}(\mathbf{k} + \mathbf{G}, \omega) \}$$

No modulation, Bernstein modes

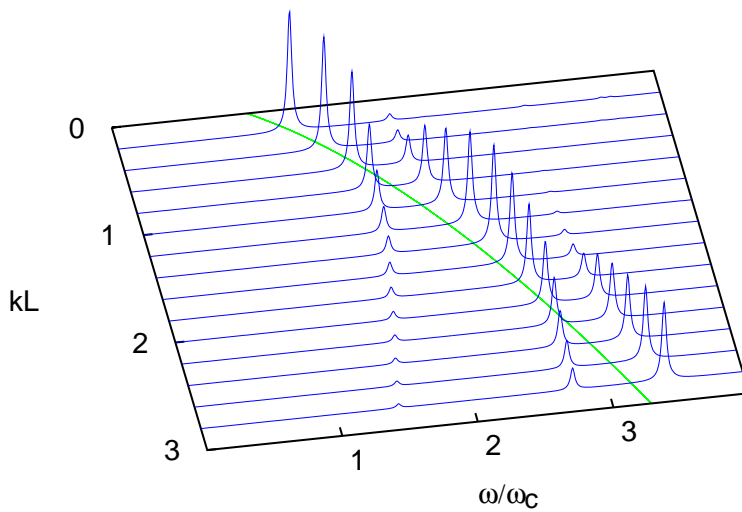
$L=100\text{nm}$, $pq=1$, $\hbar\omega_c=0.715\text{meV}$, $T=1\text{K}$, $V=0.0\text{meV}$, $N_s=0.25$

Absorption



$L=200\text{nm}$, $pq=1$, $\hbar\omega_c=0.1786\text{meV}$, $T=1\text{K}$, $V=0.0\text{meV}$, $N_s=1.00$

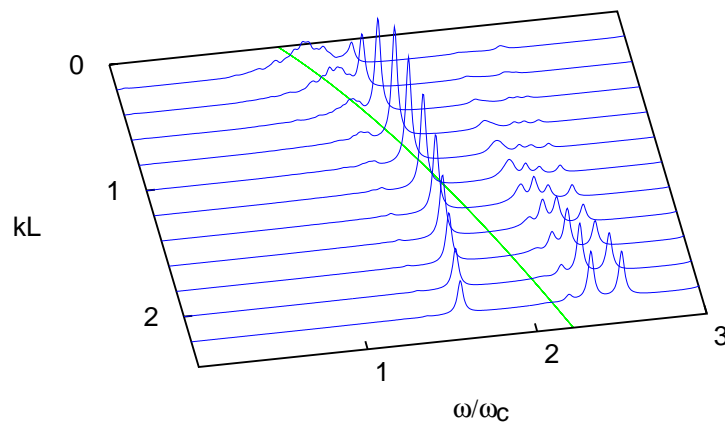
Absorption



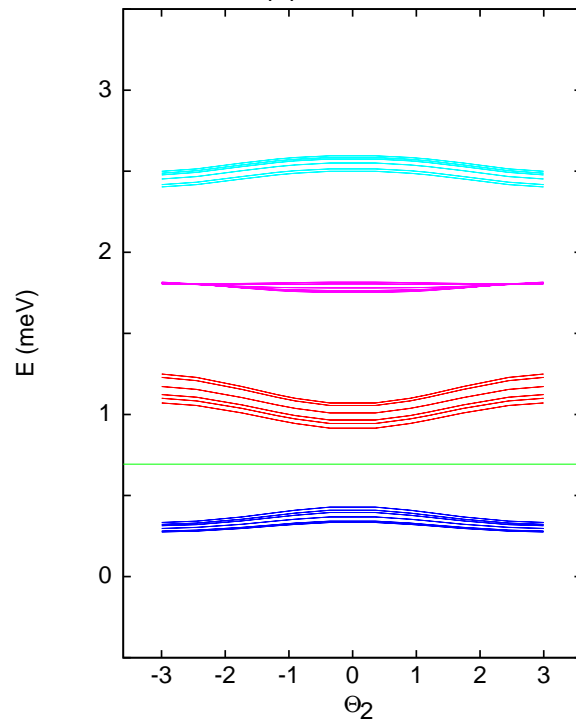
Modulation, $pq = 1, \nu = 1$

$L=100\text{nm}, pq=1, h\omega_c=0.715\text{meV}, T=1\text{K}, V=0.4\text{meV}, N_s=1.00$

Absorption



$L=100\text{nm}, pq=1, V=0.4\text{meV}, N_s=1.00$

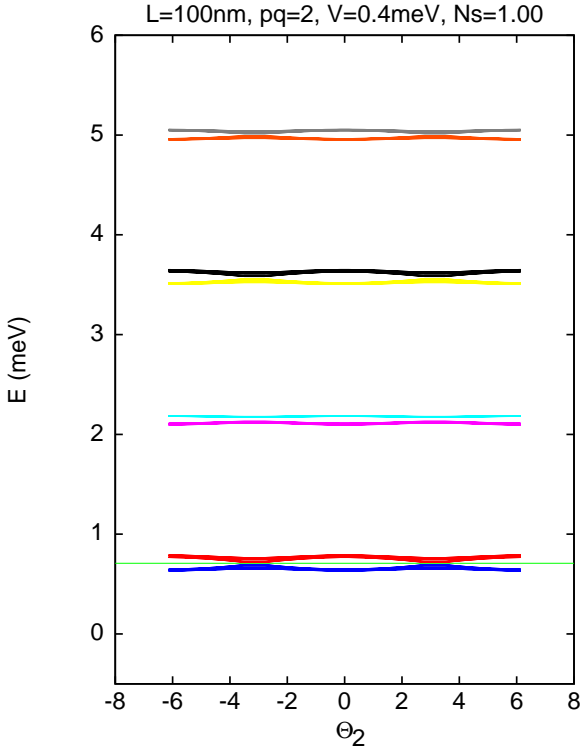
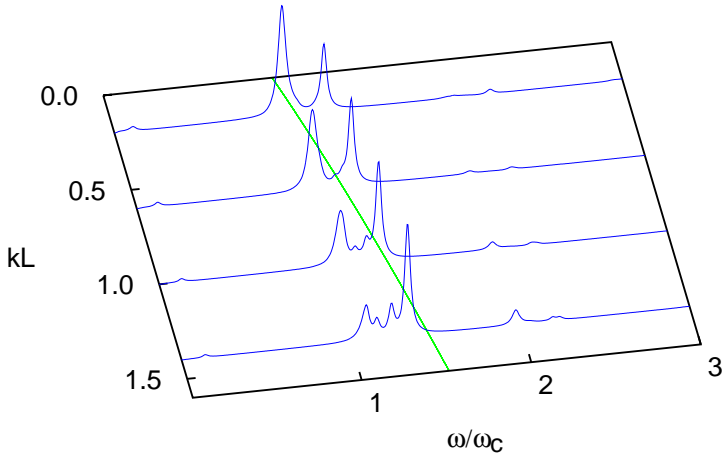


Modulation, $pq = 2$, $\nu = 1/2$

Inter and intra Landau band excitations

$L=100\text{nm}$, $pq=2$, $\hbar\omega_c=1.429\text{meV}$, $T=1\text{K}$, $V=0.4\text{meV}$, $N_s=1.00$

Absorption



Are we seeing the **two** Hofstadter subbands?

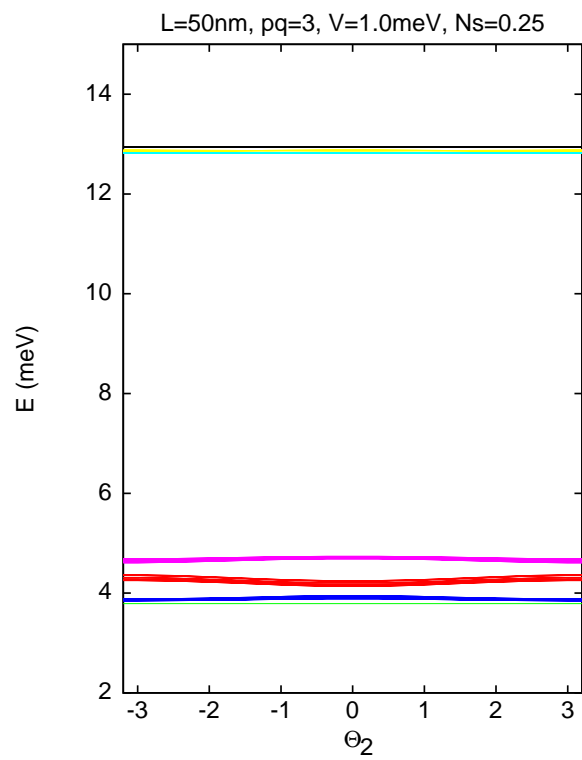
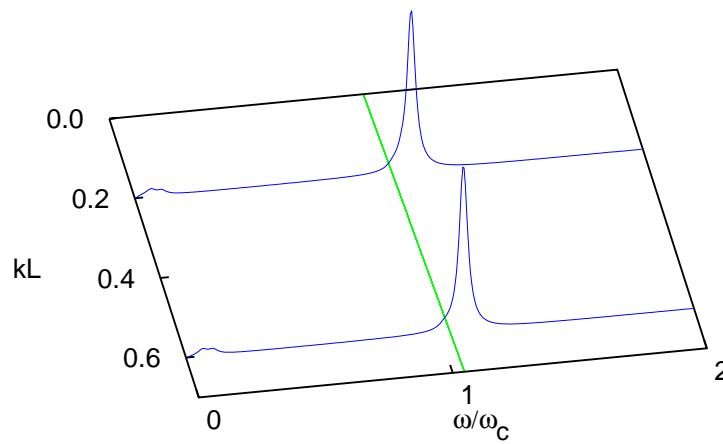
Two peaks or not, for $pq = 2$ are sensitive to modulation strength and filling factor ν

Can we see **three** peaks for $pq = 3$?

Modulation, $pq = 3$, $\nu = 1/12$

$L=50\text{nm}$, $pq=3$, $\hbar\omega_C=8.574\text{meV}$, $T=1\text{K}$, $V=1.0\text{meV}$, $N_s=0.25$

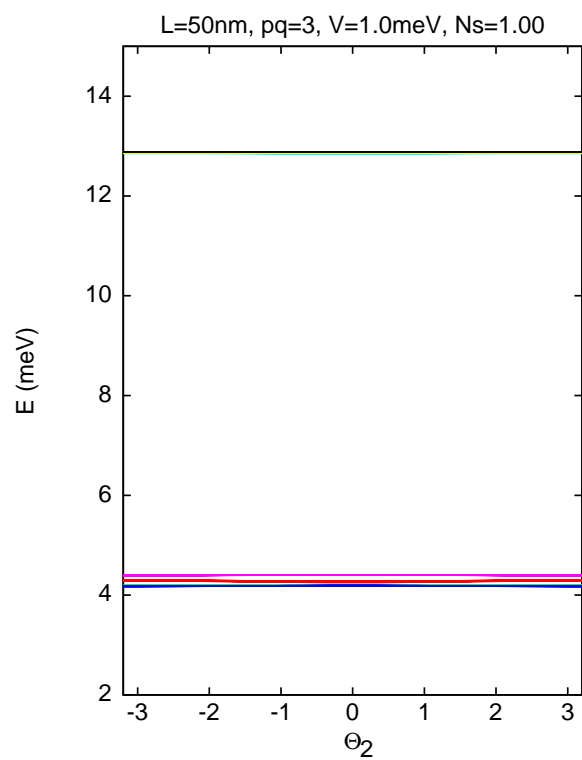
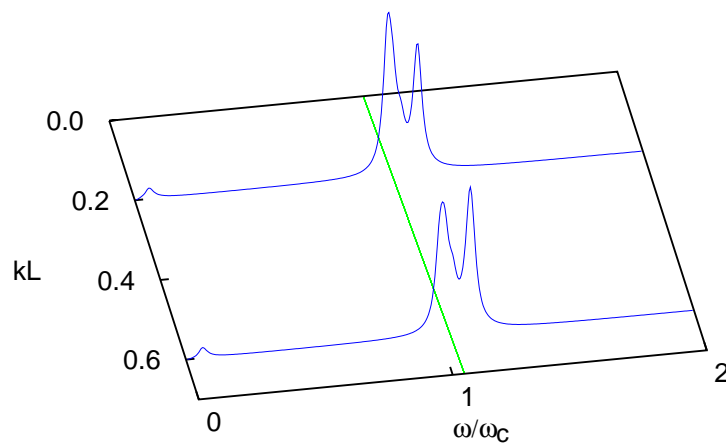
Absorption



Modulation, $pq = 3$, $\nu = 1/3$

$L=50\text{nm}$, $pq=3$, $\hbar\omega_c=8.574\text{meV}$, $T=1\text{K}$, $V=1.0\text{meV}$, $N_s=1.00$

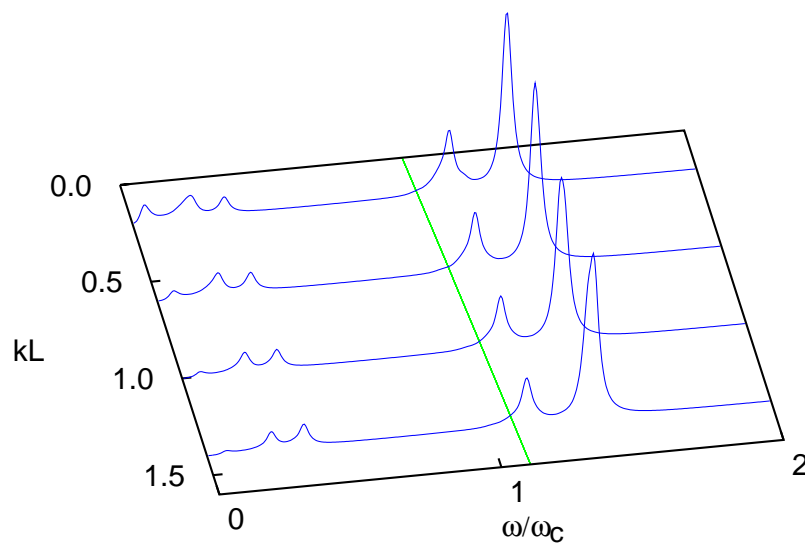
Absorption



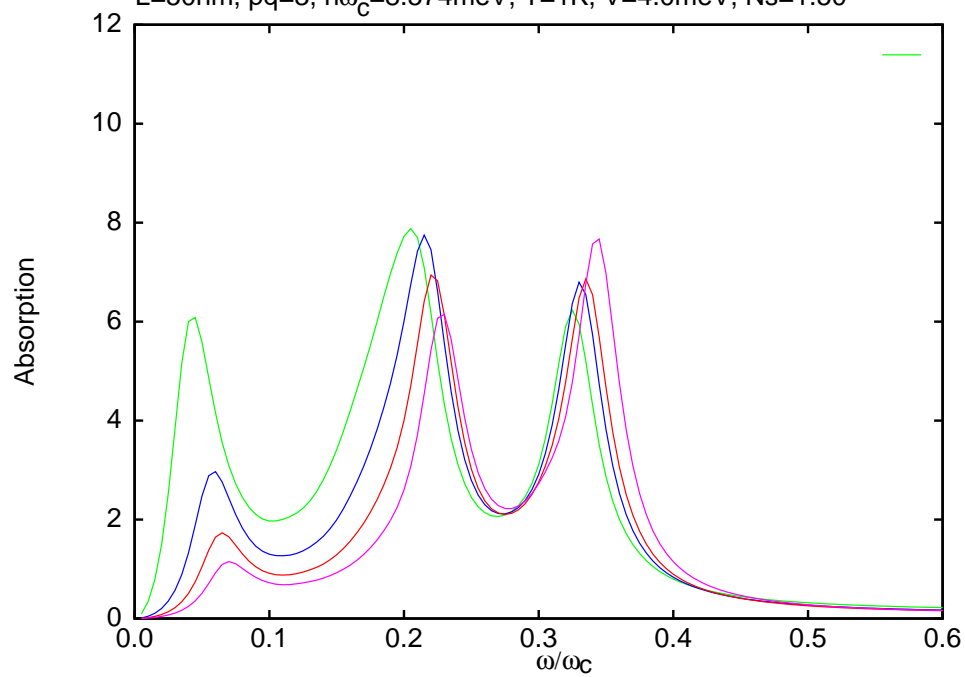
Modulation, $pq = 3$, $\nu = 1/2$

$L=50\text{nm}$, $pq=3$, $\hbar\omega_c=8.574\text{meV}$, $T=1\text{K}$, $V=4.0\text{meV}$, $N_s=1.50$

Absorption

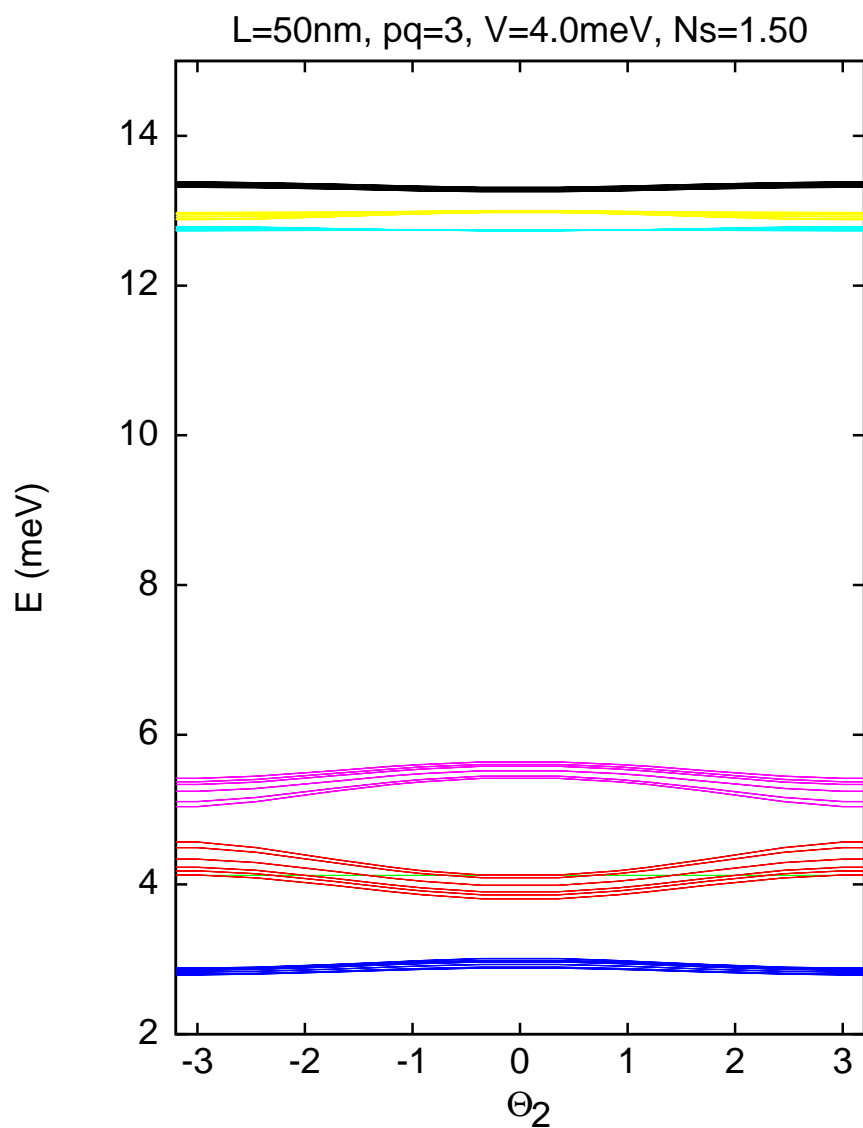


$L=50\text{nm}$, $pq=3$, $\hbar\omega_c=8.574\text{meV}$, $T=1\text{K}$, $V=4.0\text{meV}$, $N_s=1.50$



$L = 50 \text{ nm}$, $V = 4 \text{ meV}$, $pq = 3$, and $\nu = 1/2$

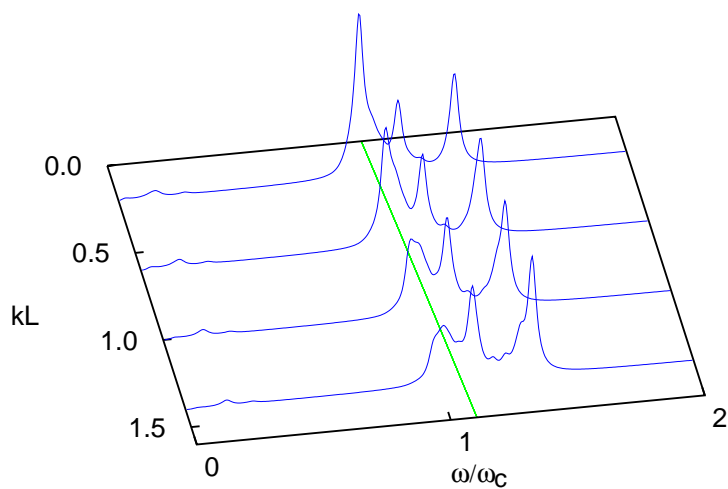
Three intraband excitations



Modulation, $pq = 3$, $\nu = 5/6$

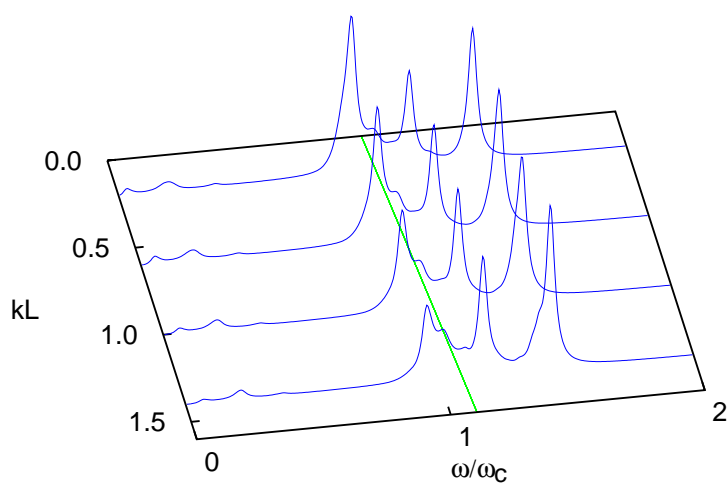
$L=50\text{nm}$, $pq=3$, $\hbar\omega_c=8.574\text{meV}$, $T=1\text{K}$, $V=3.0\text{meV}$, $N_s=2.50$

Absorption



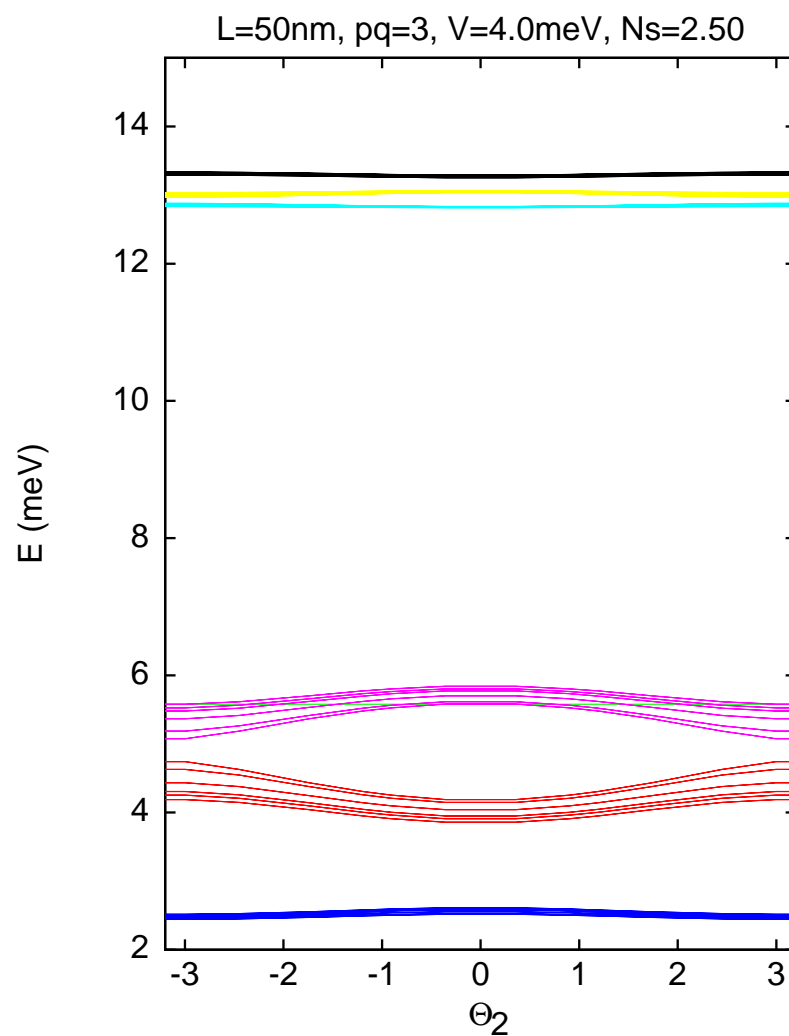
$L=50\text{nm}$, $pq=3$, $\hbar\omega_c=8.574\text{meV}$, $T=1\text{K}$, $V=4.0\text{meV}$, $N_s=2.50$

Absorption



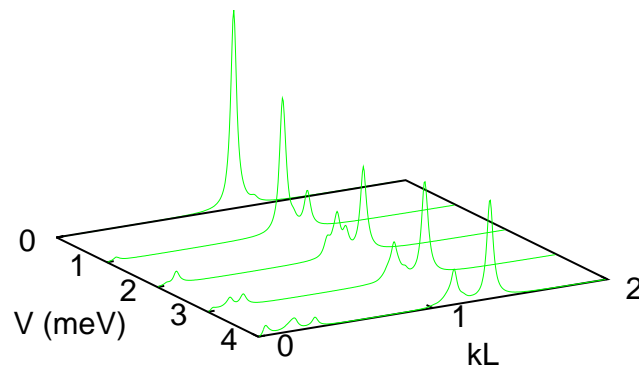
$L = 50 \text{ nm}$, $V = 4 \text{ meV}$, $pq = 3$, and $\nu = 5/6$

Three interband excitations

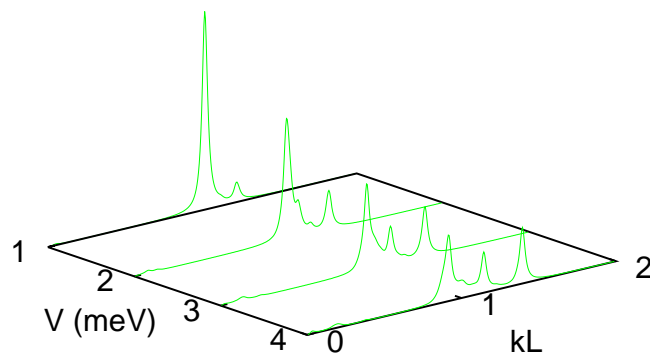


$pq = 3, L = 50\text{nm}$ Splitting as a function of V

$$\nu = 1/2$$



$$\nu = 5/6$$



Conclusions

- **Bernstein** modes
- **Hofstadter subband structure is found** in absorption due both to inter- and intra-Landau-band magnetoplasmon
- It **depends strongly** on filling factor ν and modulation strength V
- **FIR**-absorption has similar “sensitivity” as the **TDOS**
- Coulomb interaction weakens **selection rules** when $\nu \neq$ integer