# Transport through a nano-wire in a magnetic field

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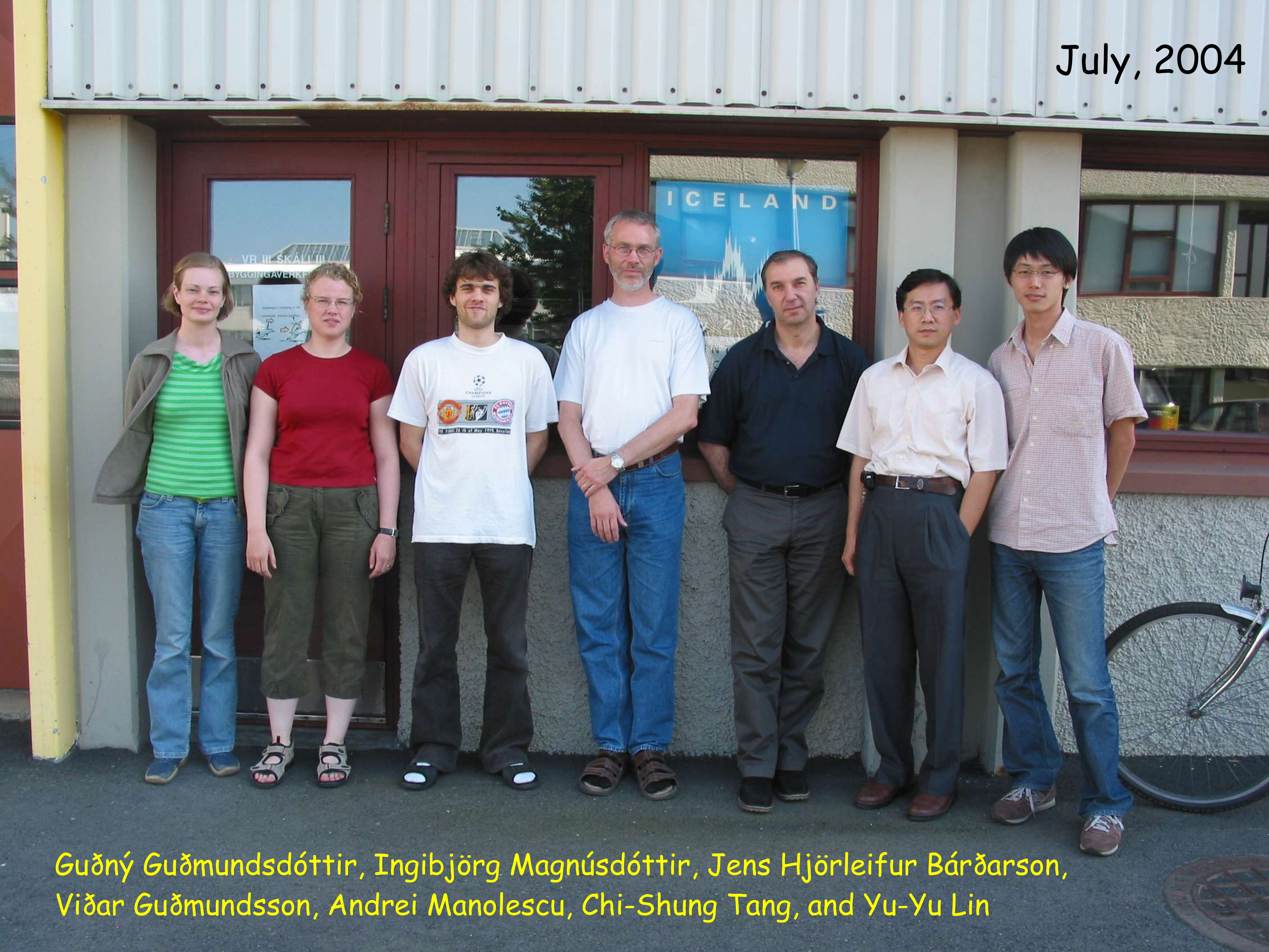


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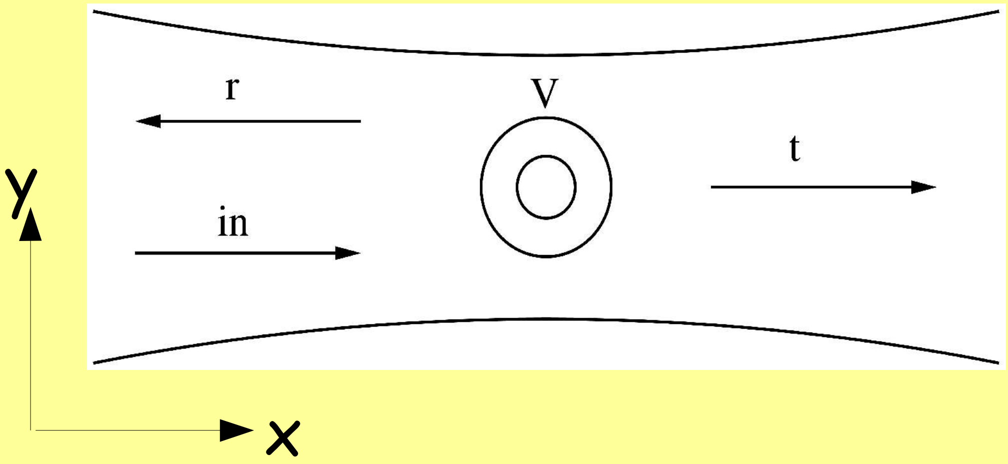
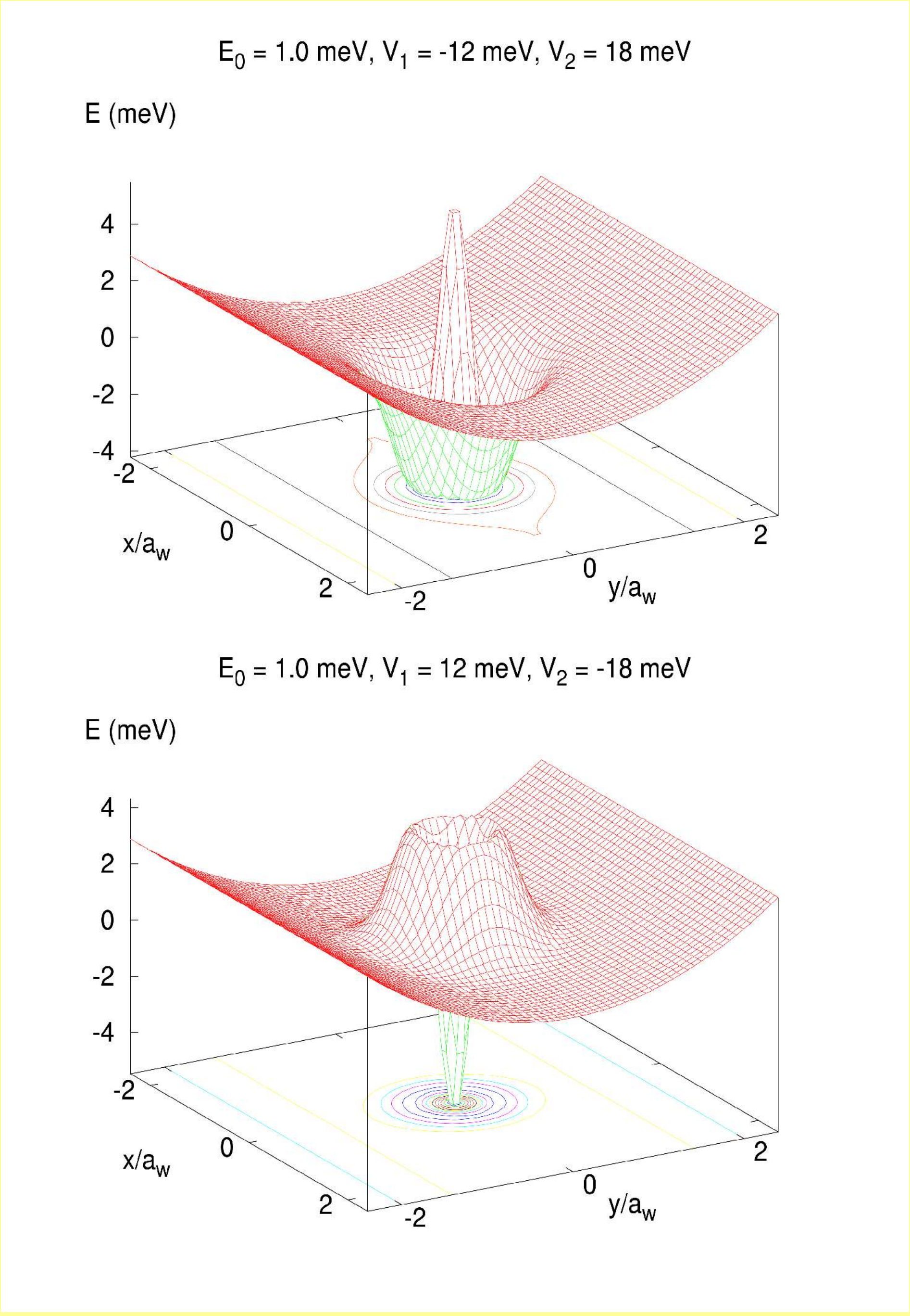
Guðný Guðmundsdóttir, Ingibjörg Magnúsdóttir, Jens Hjörleifur Bárðarson,
Viðar Guðmundsson, Andrei Manolescu, Chi-Shung Tang, and Yu-Yu Lin

# Quantum ring or dot embedded in wire

B=Bz,   magnetic field




B=0.2 T -> $a_w$ =32.8 nm

**Model**

$$H_0 = \frac{\hbar^2}{2m^*}\left[-i\boldsymbol{\nabla} - \frac{eB}{\hbar c} y\hat{\boldsymbol{x}}\right]^2 + V_{\rm c}(y)$$

Center coordinate

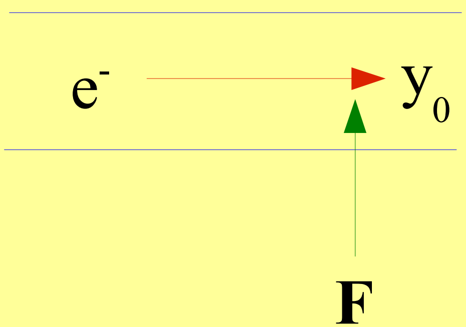
$$\psi^+(x,y,k_n) = e^{ik_n x}\chi_n(y-y_0)$$

$$E = \left(n+\frac{1}{2}\right)\hbar\Omega_w + \mathcal{K}(k_n)$$

Asymptotic region

$$\Omega_w = \sqrt{\omega_c^2+\Omega_0^2}, \quad y_0 = k_n a_w^2 \frac{\omega_c}{\Omega_w}, \quad \omega_c = \frac{eB}{m^* c}$$

$$\mathcal{K}(k_n) = \frac{(k_n a_w)^2}{2}\left(\frac{\hbar^2\Omega_0^2}{\hbar\Omega_w}\right), \quad a_w^2\Omega_w = \frac{\hbar}{m^*}$$




**B** → "Non-orthogonality"

## Solution

Mixed momentum-coordinate representation

$$\Psi_E(p,y) = \int dx\, \psi_E(x,y) e^{-ipx}$$

$$\Psi_E(p,y) = \sum_n \varphi_n(p)\phi_n(p,y)$$

Expansion in terms of eigenfunctions of the shifted harmonic oscillator -> transport mode "n"

...transforms the Schrödinger eq.

$$\kappa_n(q)\varphi_n(q) + \sum_{n'} \int \frac{dp}{2\pi} V_{nn'}(q,p)\varphi_{n'}(p) = (E - E_n)\varphi_n(q)$$

$$V_{nn'}(q,p) = \int dy\, \phi_n^*(q,y) V(q-p,y) \phi_{n'}(p,y)$$

$$V(q-p,y) = \int dx\, e^{-i(q-p)x} V_{sc}(x,y)$$

a set of coupled integral eq.'s

Scattering potential

...rewrite

Non-local

$$\left[-(qa_w)^2+(k_n(E)a_w)^2\right]\varphi_n(q)=\frac{2\hbar\Omega_w}{(\hbar\Omega_0)^2}\sum_{n'}\int\frac{dp}{2\pi}\,V_{nn'}(q,p)\varphi_{n'}(p)$$

$$(E-E_n)=\frac{[k_n(E)]^2}{2}\frac{(\hbar\Omega_0)^2}{\hbar\Omega_w}$$

Effective band momentum

$$\left[-(qa_w)^2+(k_n(E)a_w)^2\right]\varphi_n^0(q)=0$$

Free equation

Suggests an interpretation....

...a Green's function

$$\left[-(qa_w)^2 + (k_n(E)a_w)^2\right] G_E^n(q) = 1$$

Lippmann-Schwinger eq. in q-space

$$\varphi_n(q) = \varphi_n^0(q) + G_E^n(q) \sum_{n'} \int \frac{dpa_w}{2\pi} \tilde{V}_{nn'}(q,p)\varphi_{n'}(p)$$

$$\varphi = \varphi^0 + G\tilde{V}\varphi^0 + G\tilde{V}G\tilde{V}\varphi^0 + \cdots = (1 + G\tilde{T})\varphi^0$$

$$\tilde{T}_{nn'}(q,p) = \tilde{V}_{nn'}(q,p) + \sum_{m'} \int \frac{dka_w}{2\pi} \tilde{V}_{nm'}(q,k) G_E^{m'}(k) \tilde{T}_{nm'}(k,p).$$

transformed into eq.'s for T-matrix

## Supplies

Conductance

$$G(E) = \frac{2e^2}{h}\mathrm{Tr}[\mathbf{t}^\dagger(E)\mathbf{t}(E)]$$

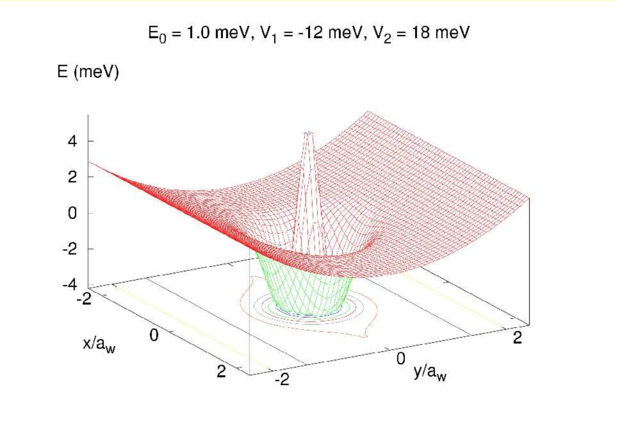
Transmission amplitudes

$$t_{nm}(E) = \delta_{nm} - \frac{i\sqrt{(k_m/k_n)}}{2(k_m a_w)}\left(\frac{\hbar\Omega_0}{\hbar\Omega_w}\right)^2 \tilde{T}_{nm}(k_n, k_m)$$

$$\psi_E(x,y) = e^{ik_n x}\phi_n(k_n, y) + \sum_m \int \frac{dq a_w}{2\pi} e^{iqx} G_E^m(q)\tilde{T}_{mn}(q, k_n)\phi_m(q, y)$$

Wave functions

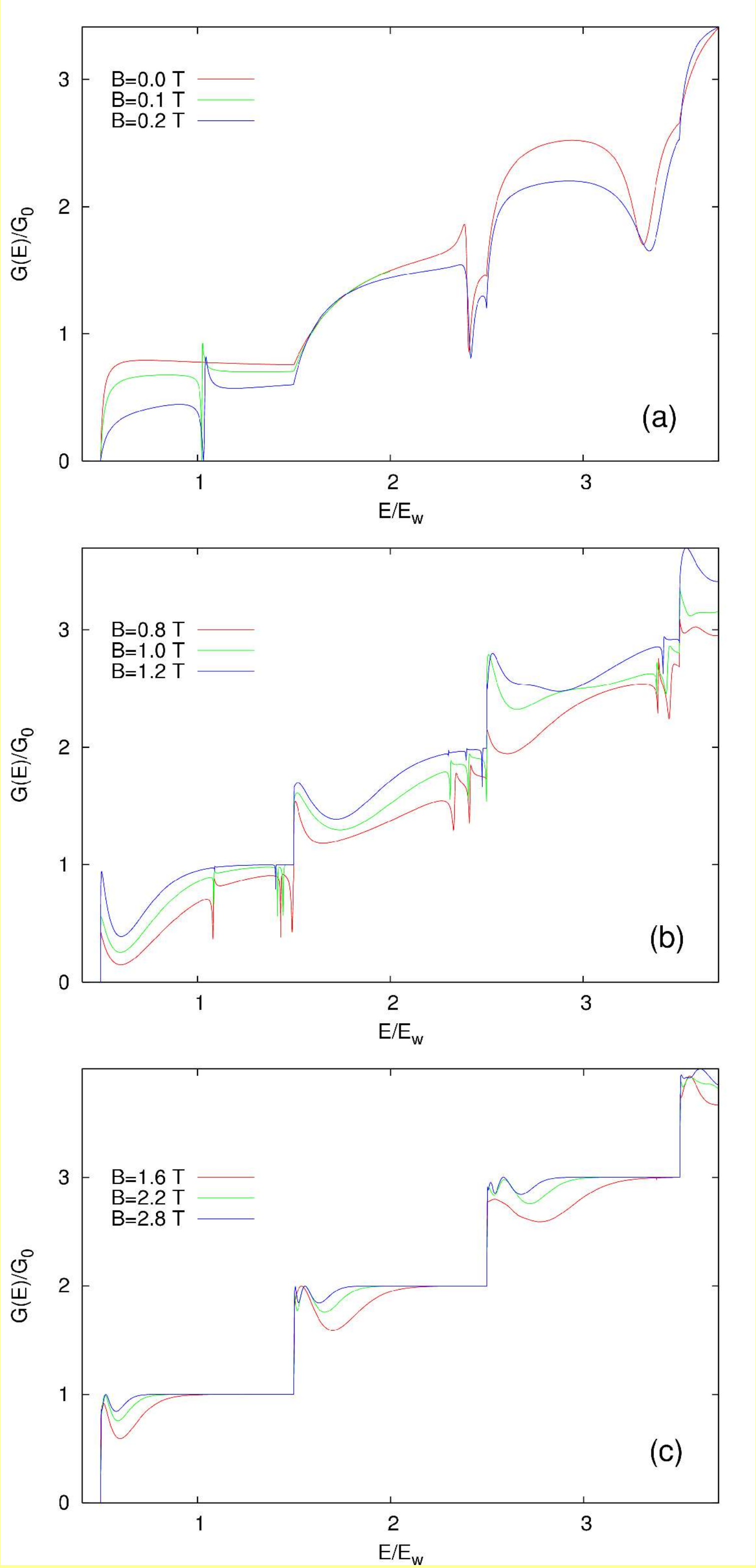
# Quantum ring




Low B

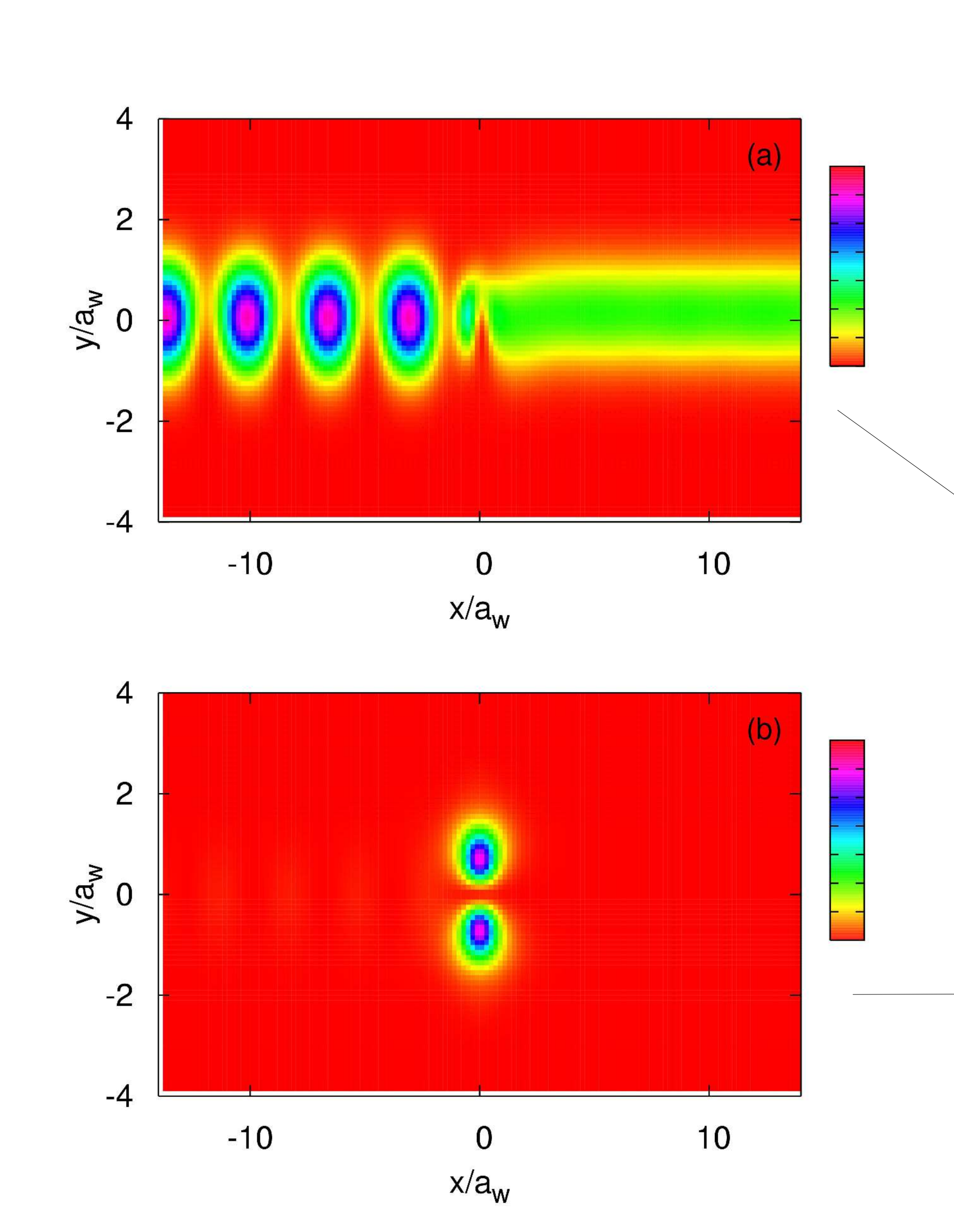
Intermediate B

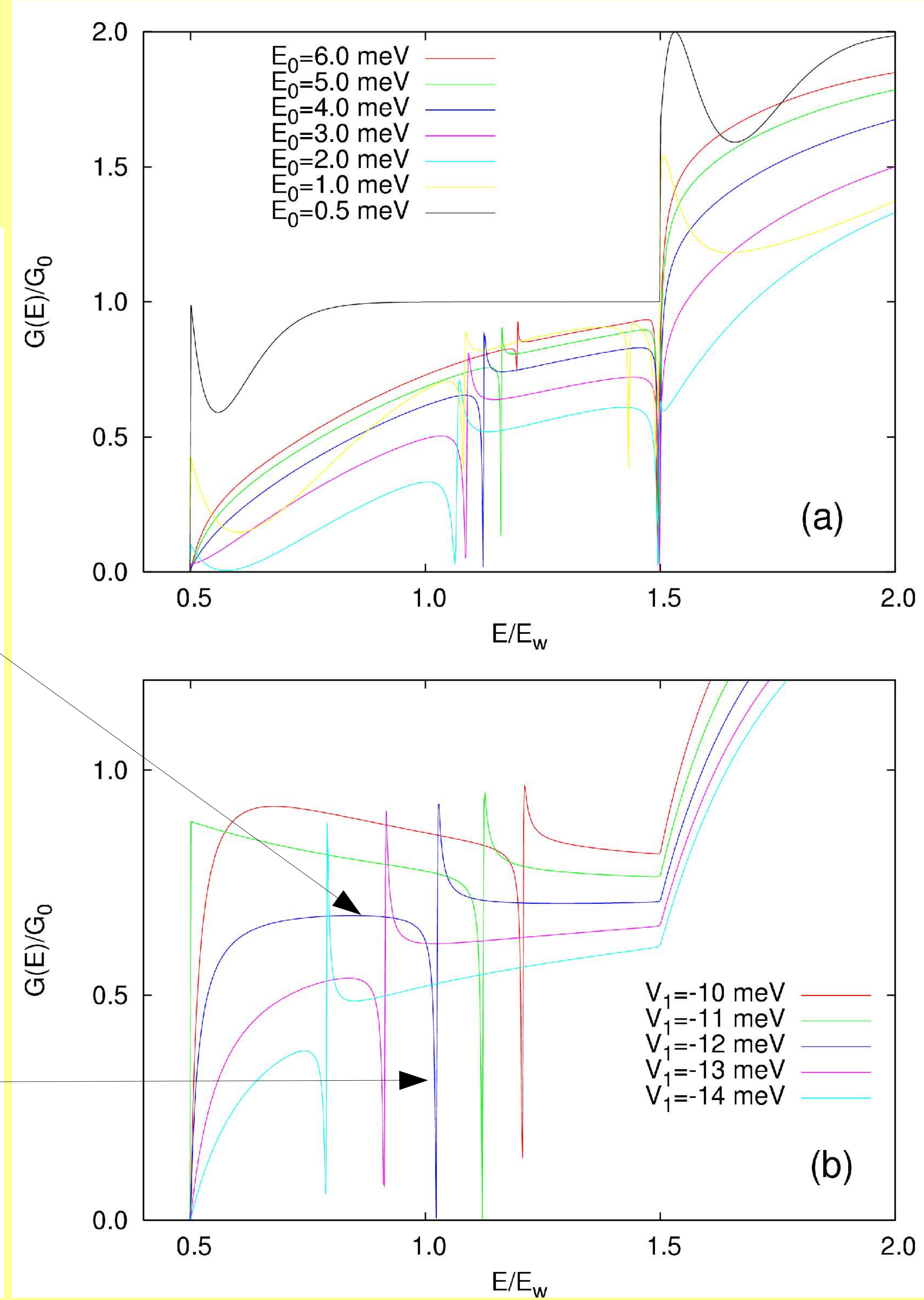
High B



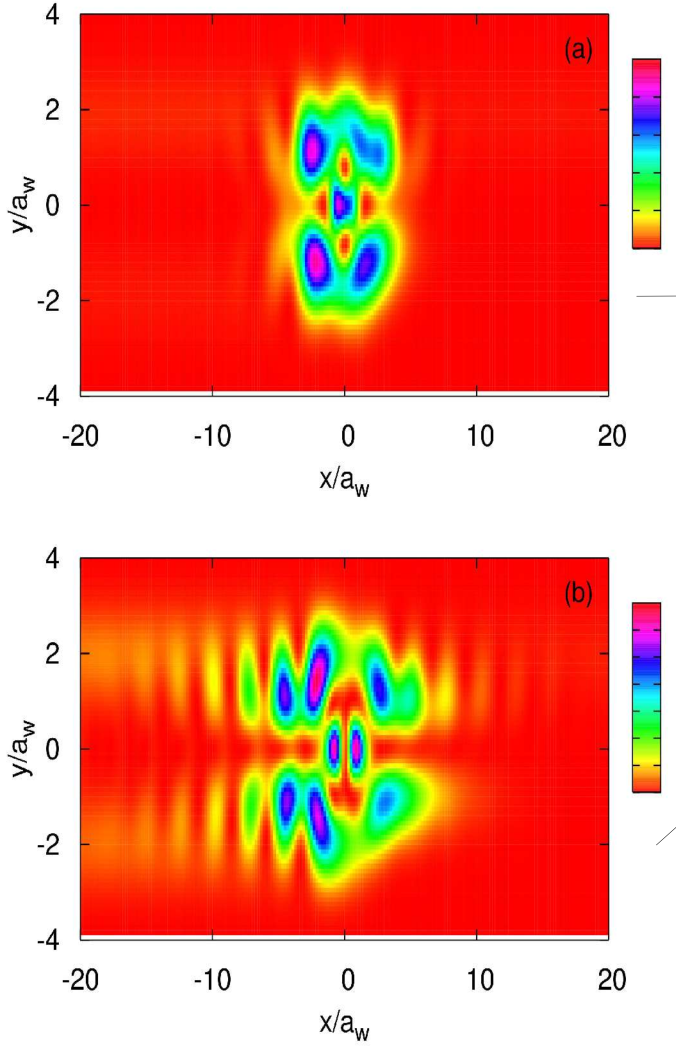
# Fano resonance

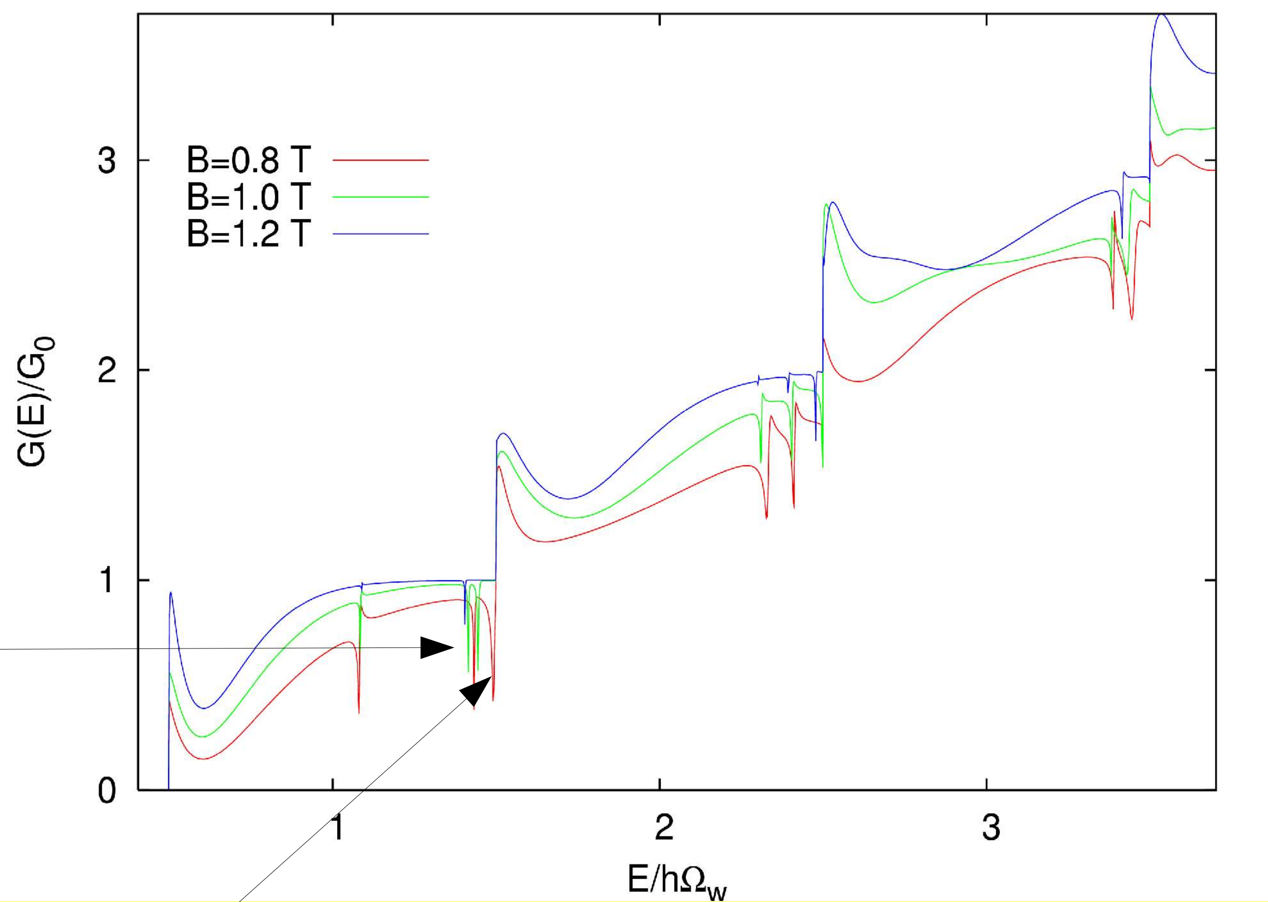
(Forbidden at B=0)



# Evanescent states



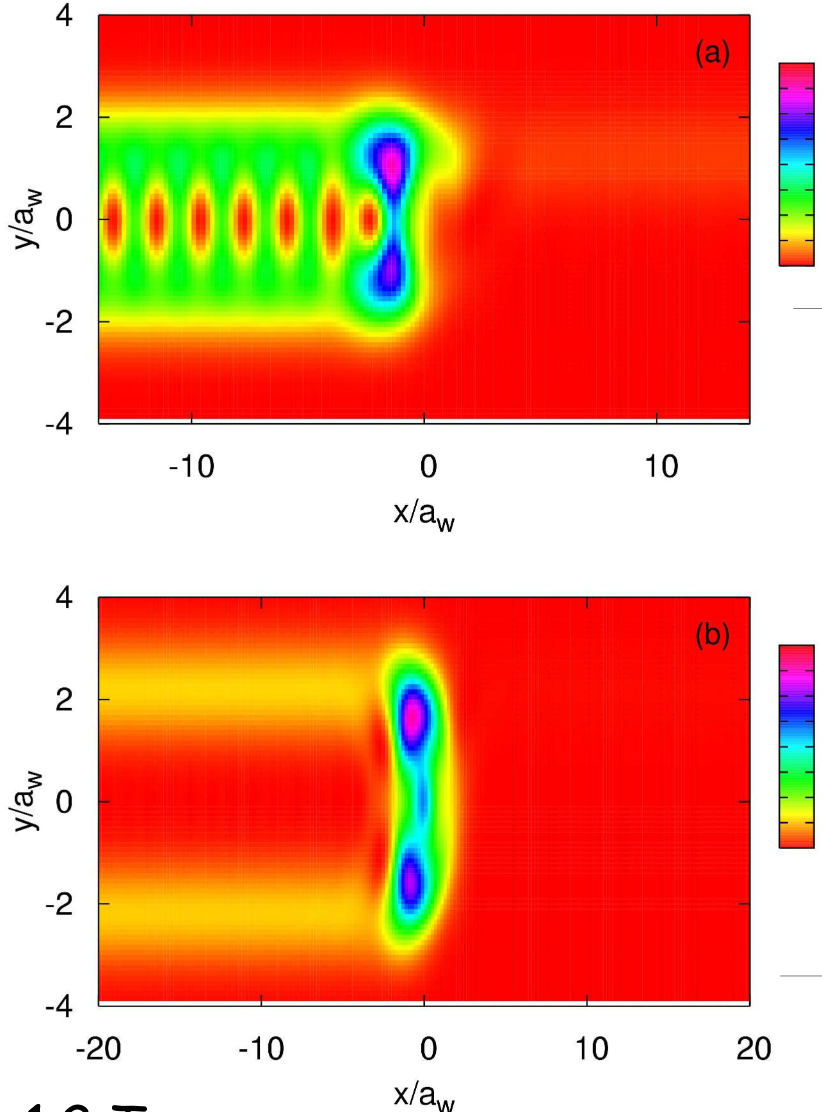




Partial separation of
in and reflected waves

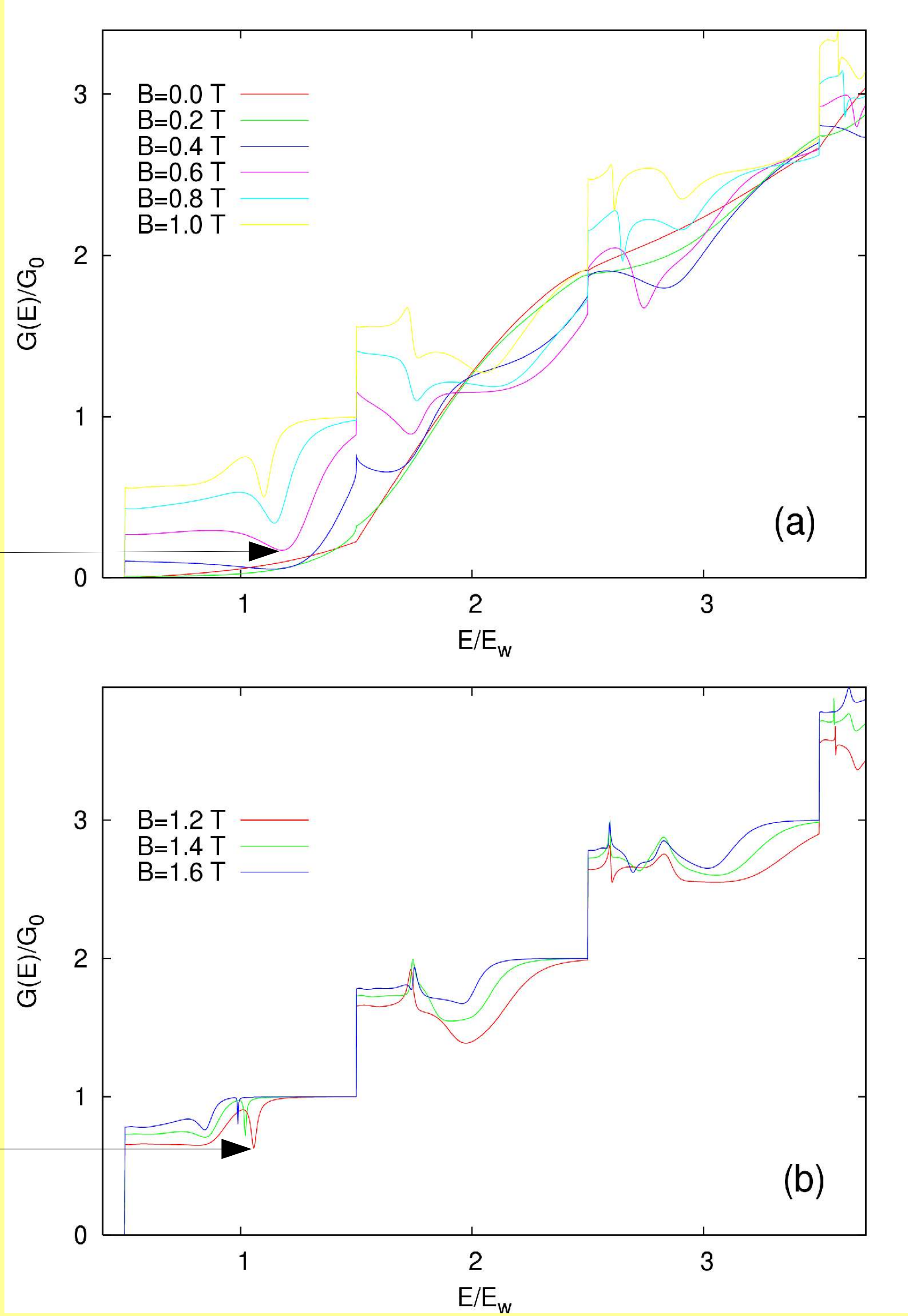
B=0.8 T

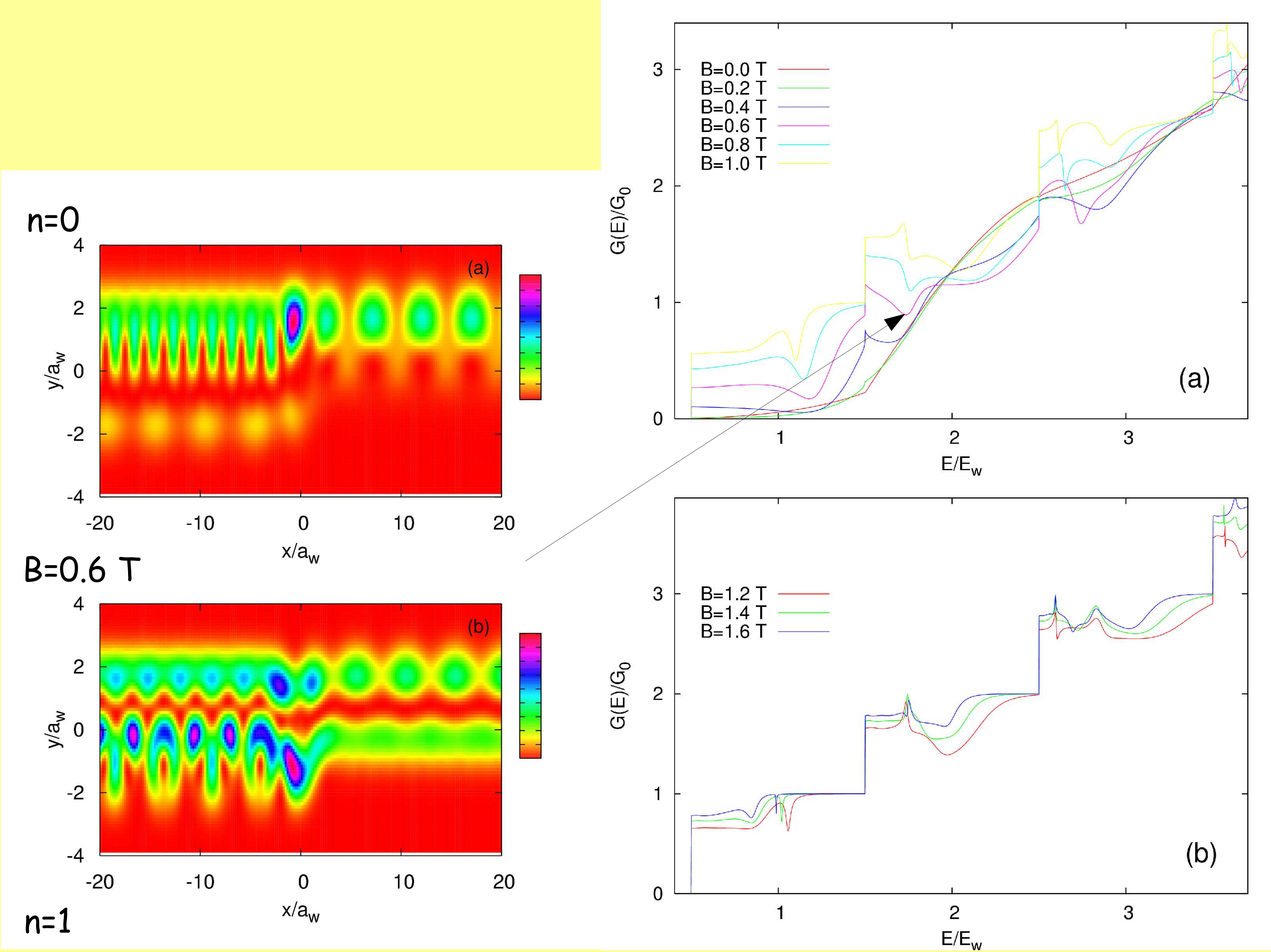
# Embedded dot

B=0.6 T

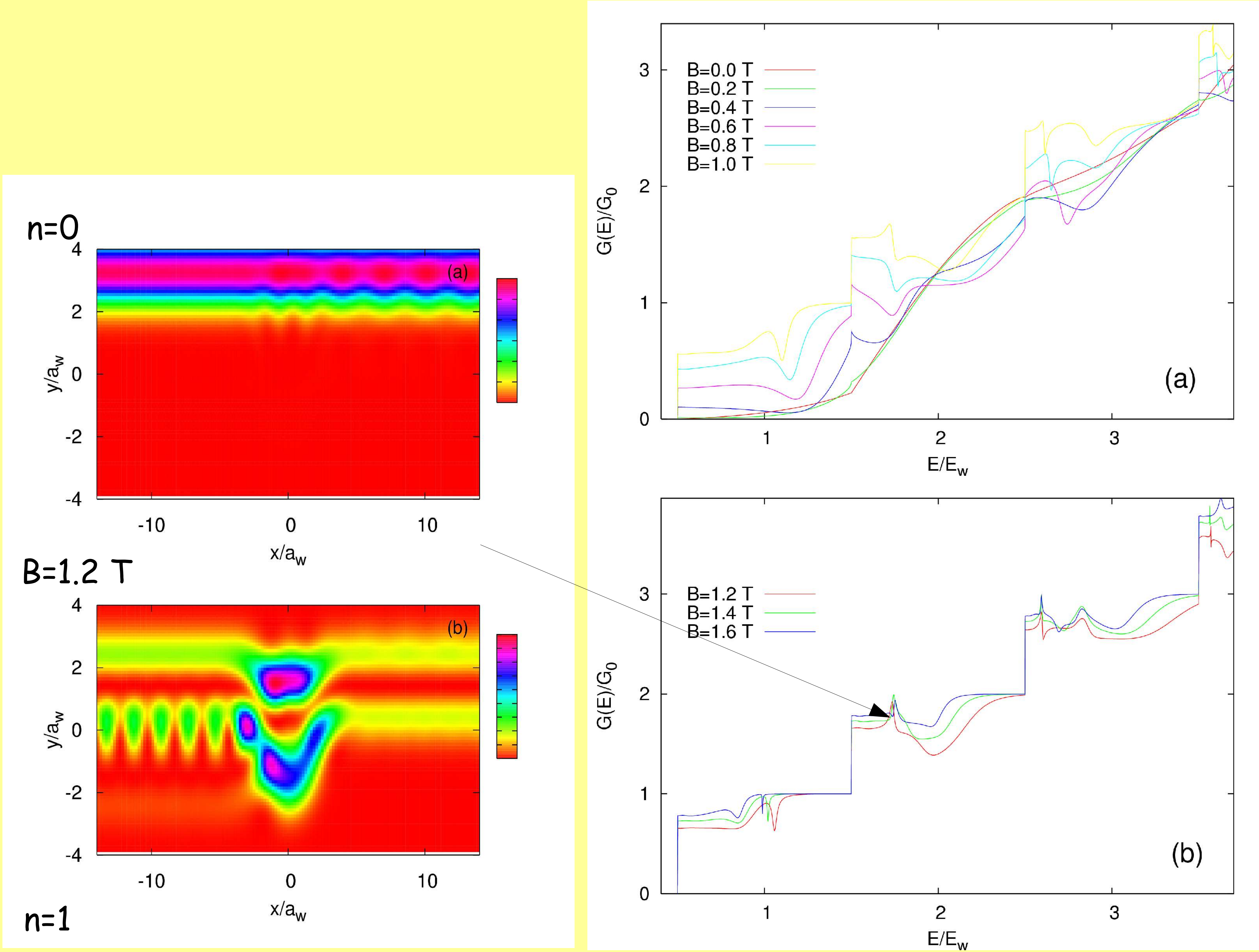



B=1.2 T





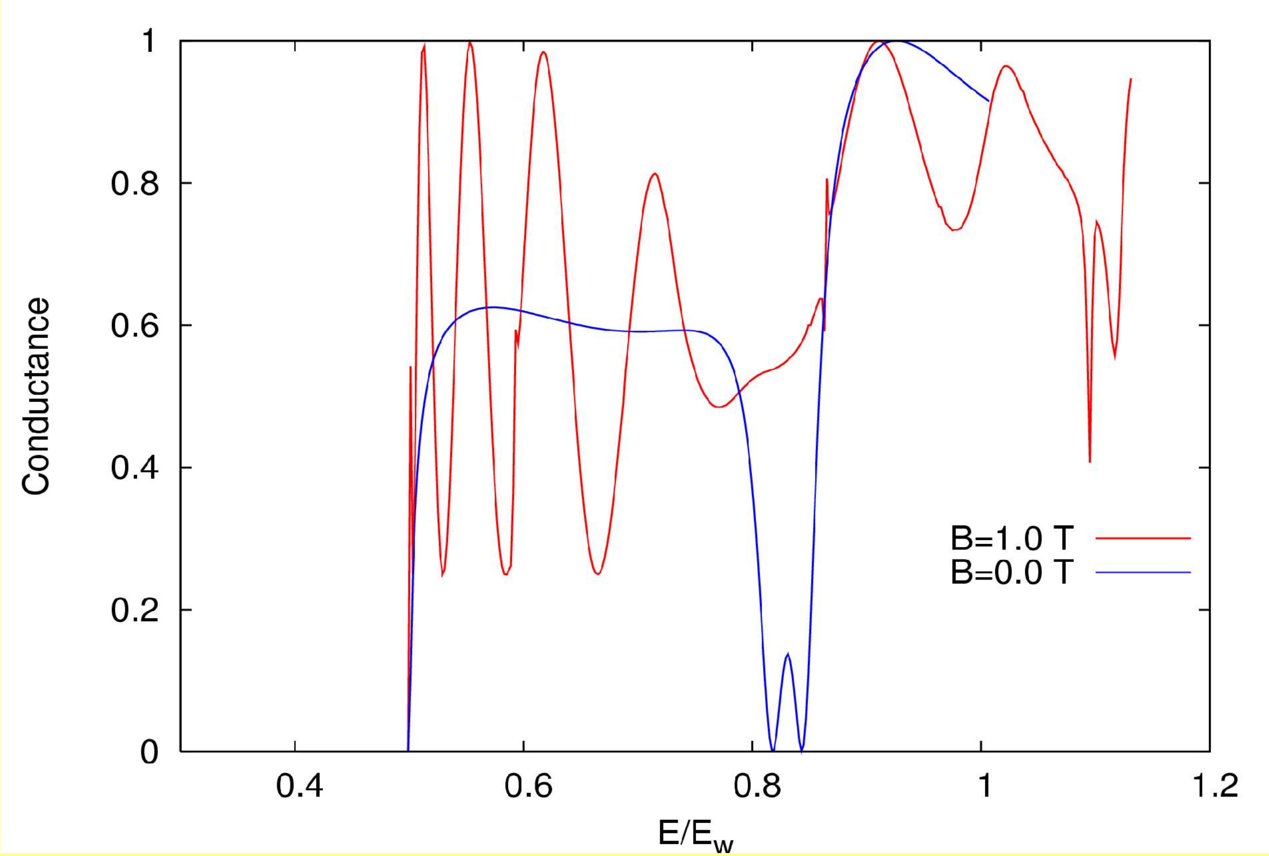
n=0
(a)
y/a_w
x/a_w
B=0.6 T
(b)
y/a_w
x/a_w
n=1
B=0.0 T
B=0.2 T
B=0.4 T
B=0.6 T
B=0.8 T
B=1.0 T
G(E)/G_0
E/E_w
(a)
B=1.2 T
B=1.4 T
B=1.6 T
G(E)/G_0
E/E_w
(b)

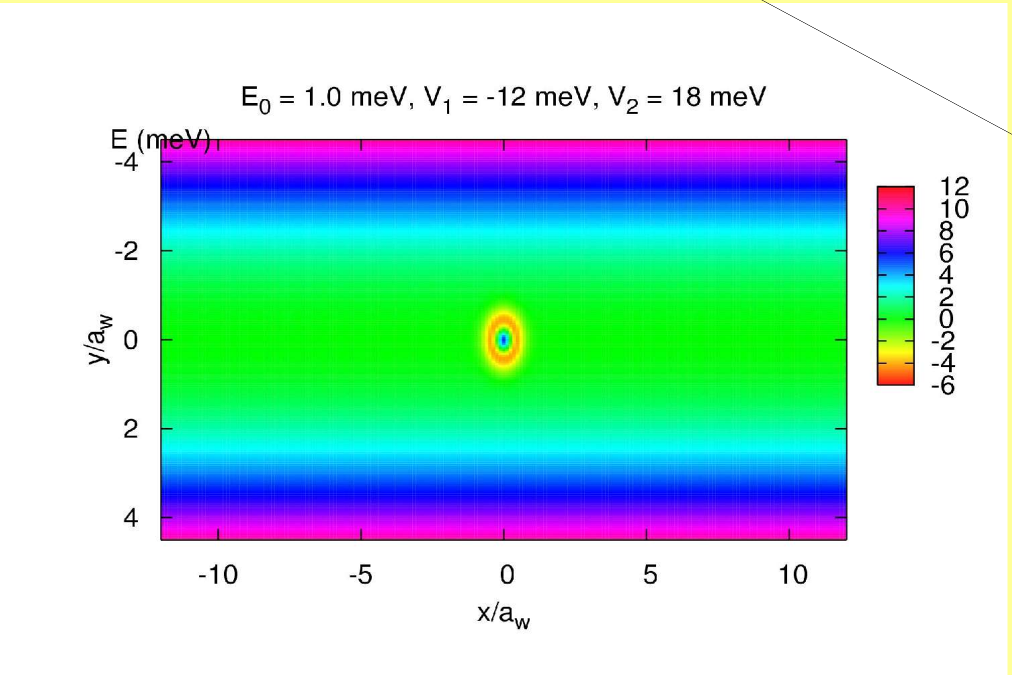


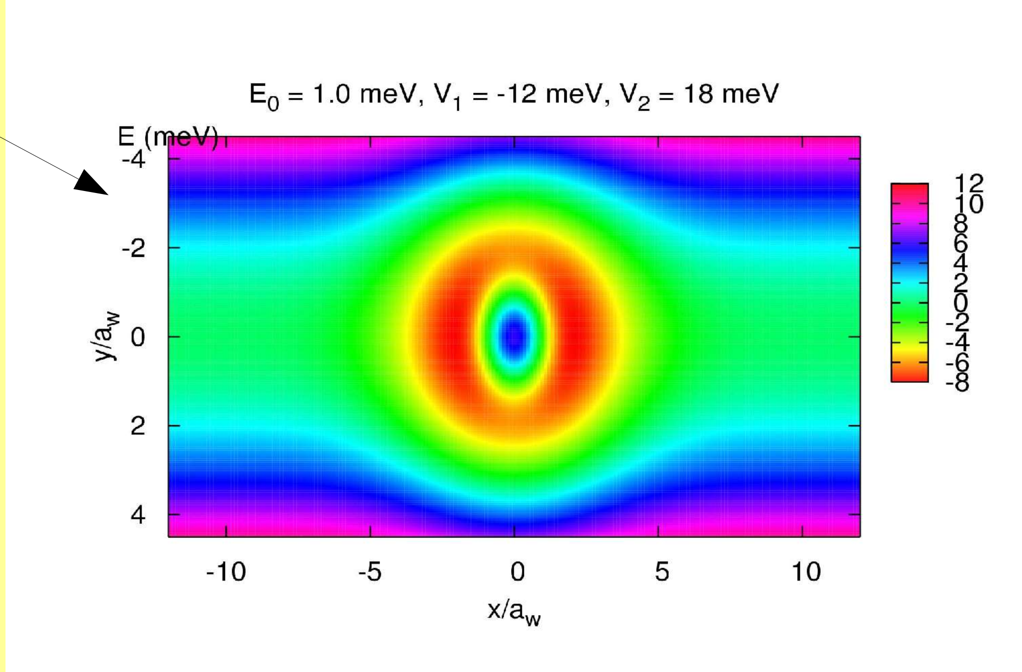
n=0
(a)
y/a_w
x/a_w
B=1.2 T
(b)
y/a_w
x/a_w
n=1
B=0.0 T
B=0.2 T
B=0.4 T
B=0.6 T
B=0.8 T
B=1.0 T
G(E)/G_0
E/E_w
(a)
B=1.2 T
B=1.4 T
B=1.6 T
G(E)/G_0
E/E_w
(b)

# Aharanov-Bohm

What happens in a **larger** ring



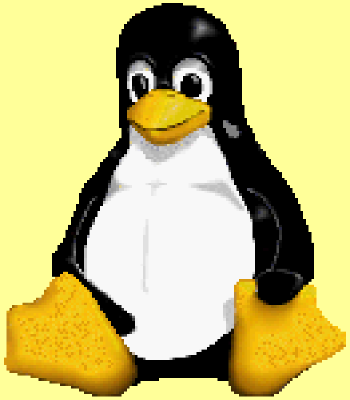



We can describe scattering in magnetic field

Relaxation of selection rules

Edge-modes, channels

Aharanov-Bohm oscillations



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