Magnetotransport through systems embedded in a quantum wire

Viðar Guðmundsson

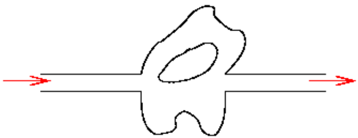
Science Institute, University of Iceland, Iceland

vidar@raunvis.hi.is

Berlin, Februar, 2008

Aim

- Model of magnetotransport in a 2D quantum wire
- Embedded subsystems

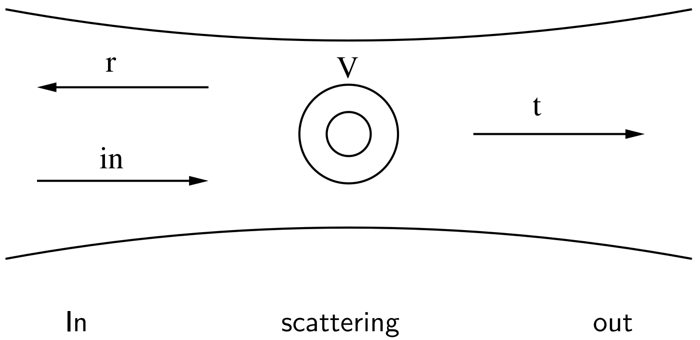


- Static and dynamic transport
- Effects $\leftarrow$ geometry $+$ magnetic field

Methods

- Scattering formalism, built on Lippmann-Schwinger approach
- Basis expansion – multimode transport – enhanced parallelization
- Cooperation and comparison with groups working on alternative approaches
    - Chi-Shung Tang: Wave function matching
    - Valeriu Moldoveanu: NEGF on a lattice
- Analytical $\rightarrow$ heavy numerical work

Asymptotic regions



External magnetic field, $\mathbf{B} \neq 0$

Asymptotic regions, free parabolic wire, perpendicular magnetic field

$$H_0 = \frac{\hbar^2}{2m^*}\left[-i\boldsymbol{\nabla} - \frac{eB}{\hbar c}y\hat{\boldsymbol{x}}\right]^2 + V_{\rm c}(y)$$

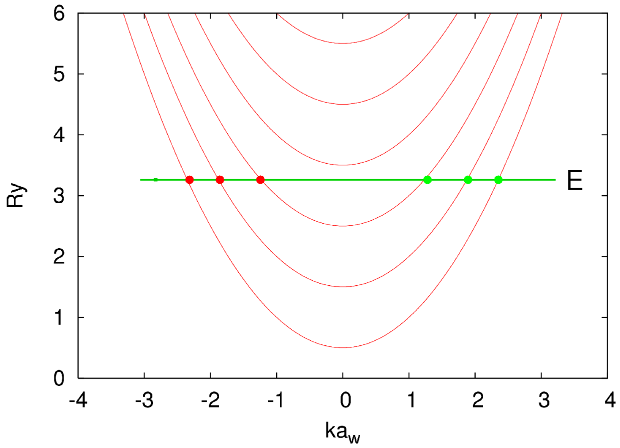
$$\psi^+(x,y,k_n) = e^{ik_nx}\phi_n(y-y_0)$$

$$E = \left(n+\frac{1}{2}\right)\hbar\Omega_w + \mathcal{K}_n(k_n)$$

$$\Omega_w = \sqrt{\omega_c^2+\Omega_0^2}, \quad y_0 = k_n a_w^2\frac{\omega_c}{\Omega_w}, \quad \omega_c = \frac{eB}{m^*c}$$

$$\mathcal{K}_n(k_n) = \frac{(k_n a_w)^2}{2}\left(\frac{\hbar^2\Omega_0^2}{\hbar\Omega_w}\right), \quad a_w^2\Omega_w = \frac{\hbar}{m^*}$$

Asymptotic energy spectrum



In-, out- states, energy is conserved

Consequences of $B \neq 0$

- Lorentz force couples the motion in x- and y-direction
- $\phi_n(y - y_0)$ with different y_0's and n's are not orthogonal
- No simple separation in modes, (k_n and y_0 are related)

Mixed momentum-coordinate representation, S. A. Gurvitz, PRB **51**, 7123 (1995)

$$\Psi_E(p, y) = \int dx\, \psi_E(x, y) e^{-ipx}$$

$$\Psi_E(p, y) = \sum_n \varphi_n(p) \phi_n(p, y)$$

Separation in (p,y)-space, p is Fourier variable!

Expansion in terms of eigenfunctions of the shifted harmonic oscillator $\rightarrow$ transport mode "n"

. . . transforms the Schrödinger equation (in q-space)

$$\mathcal{K}_n(q)\varphi_n(q) + \sum_{n'} \int \frac{dp}{2\pi}\, V_{nn'}(q,p)\varphi_{n'}(p) = (E - E_n)\varphi_n(q)$$

$$V_{nn'}(q,p) = \int dy\, \phi_n^*(q,y)\, V(q-p,y)\phi_{n'}(p,y)$$

$$V(q-p,y) = \int dx\, e^{-i(q-p)x}\, V_{\mathrm{sc}}(x,y)$$

into a set of coupled integral equations,

$V_{\mathrm{sc}}(x,y)$ is the scattering potential (nonlocal),
(analytic matrix elements)

. . . rewrite

Nonlocal potential

$$\left[-(qa_w)^2 + (k_n(E)a_w)^2\right]\varphi_n(q) = \frac{2\hbar\Omega_w}{(\hbar\Omega_0)^2}\sum_{n'}\int\frac{dp}{2\pi}\, V_{nn'}(q,p)\varphi_{n'}(p)$$

Effective band momentum $\quad (E - E_n) = \frac{[k_n(E)]^2}{2}\frac{(\hbar\Omega_0)^2}{\hbar\Omega_w}$

Free equation $\quad \left[-(qa_w)^2 + (k_n(E)a_w)^2\right]\varphi_n^0(q) = 0$

Suggests an interpretation. . .

. . . a Green function

$$\left[-(qa_w)^2 + (k_n(E)a_w)^2\right] G_E^n(q) = 1$$

Lippmann-Schwinger eq.'s in q-space

$$\varphi_n(q) = \varphi_n^0(q) + G_E^n(q)\sum_{n'}\int \frac{dpa_w}{2\pi}\tilde{V}_{nn'}(q,p)\varphi_{n'}(p)$$

$$\varphi = \varphi^0 + G\tilde{V}\varphi^0 + G\tilde{V}G\tilde{V}\varphi^0 + \cdots = (1 + G\tilde{T})\varphi^0$$

$$\tilde{T}_{nn'}(q,p) = \tilde{V}_{nn'}(q,p) + \sum_{m'}\int \frac{dka_w}{2\pi}\tilde{V}_{nm'}(q,k)G_E^{m'}(k)\tilde{T}_{nm'}(k,p).$$

Transformed into eq's for the T-matrix
(convenient for numerical calculations)

Supplies

Wavefunctions

$$\psi_E(x,y) = e^{ik_nx}\phi_n(k_n,y) + \sum_m \int \frac{dqa_w}{2\pi} e^{iqx} G_E^m(q)\,\tilde{T}_{mn}(q,k_n)\phi_m(q,y)$$

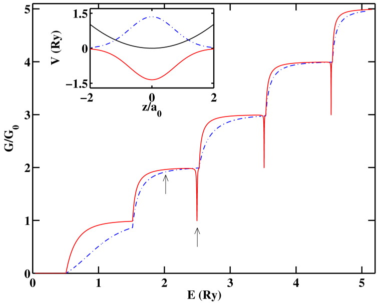
Transmission amplitudes

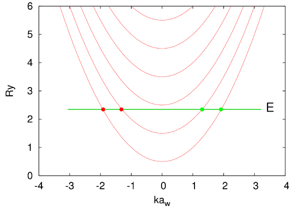
$$t_{nm}(E) = \delta_{nm} - \frac{i\sqrt{(k_m/k_n)}}{4(k_m a_w)}\tilde{T}_{nm}(k_n,k_m)$$

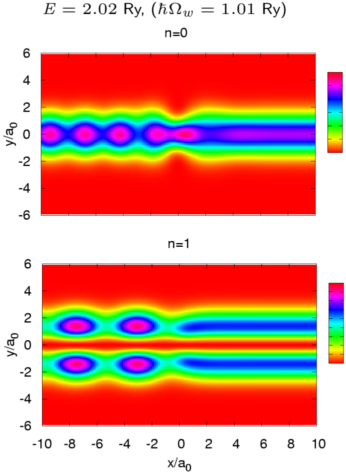
Conductance

$$G(E) = \frac{2e^2}{h}\mathrm{Tr}[\mathbf{t}^\dagger(E)\mathbf{t}(E)]$$

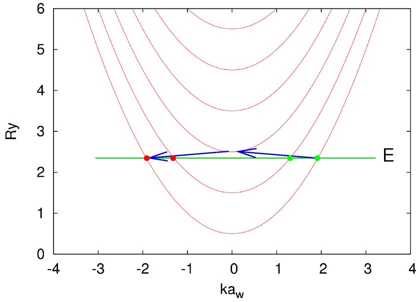
B=0, simple Gaussian hill or well

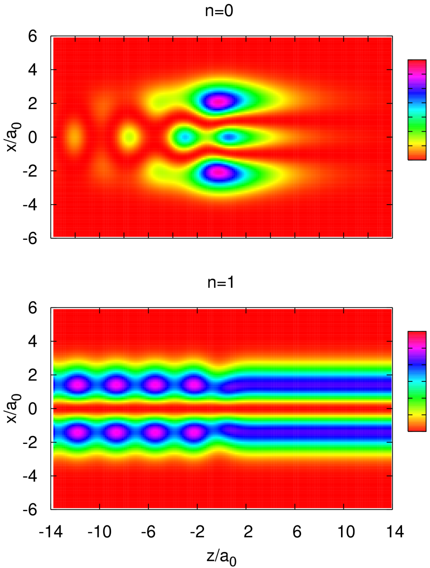






What causes a dip?

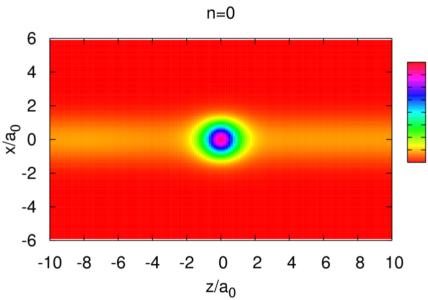


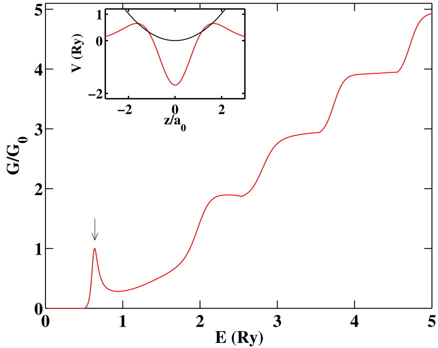


- Total reflection by an evanescent state
- Symmetry → selection rules
- All orders of scattering

Embedded dot, $B = 0$

- Total resonant transmission
- Finite lifetime



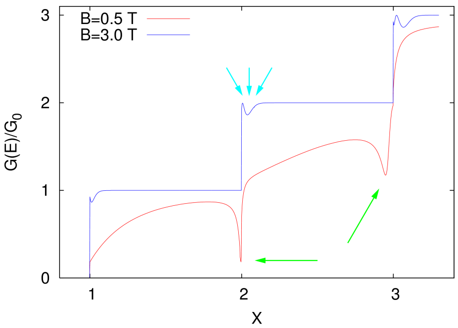


Phys. Rev. B **70** 245308, (2004)

Magnetic field, $B \neq 0$

Small open quantum dot, (Phys. Rev. B **71**, 235302 (2005))

(Further systems in: PRB **70** 245308, (2004), Euro. Phys. J. B **45**, 339 (2005), and PRB **72**, 195331 (2005))

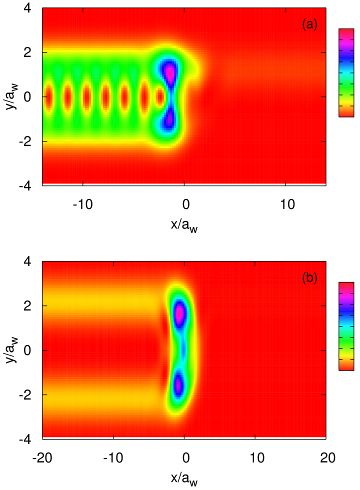


- $\hbar\Omega_0 = 1.0$ meV, broad wire
- $V_0 = -0.8$ meV, shallow dot
- $G_0 = \frac{2e^2}{h}$
- $\Omega_w = \sqrt{\omega_c^2 + \Omega_0^2}$
- $X = \frac{E}{\hbar\Omega_w} + \frac{1}{2}$

- Quantization, with or without B, symmetry breaking
- Lorentz force $\rightarrow$ electrons bypass dot at high B

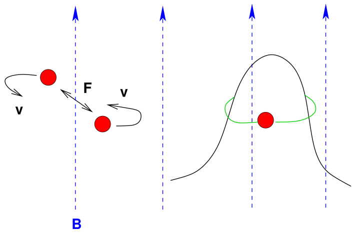
Lorentz force

- $B = 0.6$ T, or $B = 1.2$ T
- Separation of in- and out-states
- Interference
- Telltale symmetry



Negative binding energy

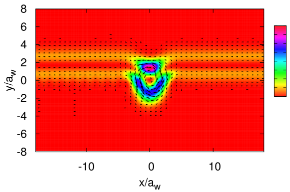
Small Gauss hill

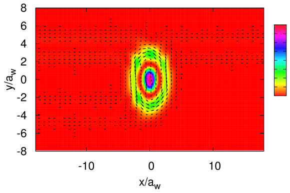


Probability density →

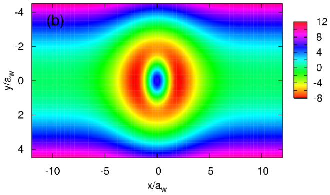
(Phys. Rev. B **72**, 153306 (2005))

Quasi-bound states

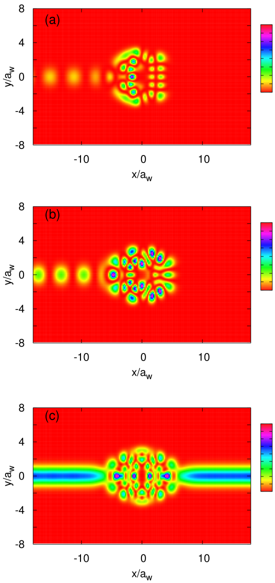




Quantum ring, $B = 0$



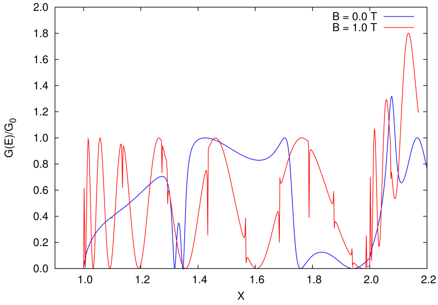
- Scarring of a wave function
  Persistence of eigenstates



$B \neq 0$

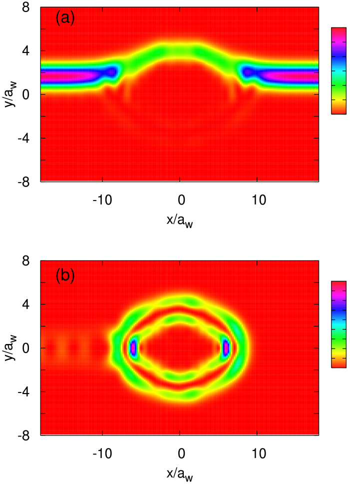
- Ahranov-Bohm oscillations
- Superimposed resonances

Conductance



(Phys. Rev. B **71**, 235302 (2005))

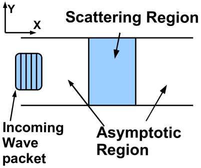
Probability density



Two cases of time-dependent magnetotransport

Wave packet transport

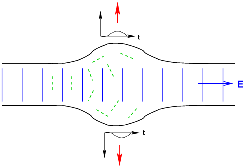
- Static potential
- Elastic scattering
- Life-time of quasi-bound states and resonances
- Delay times



(Phys. Rev. **B** 76, 195314 (2007))

Modulation of a current

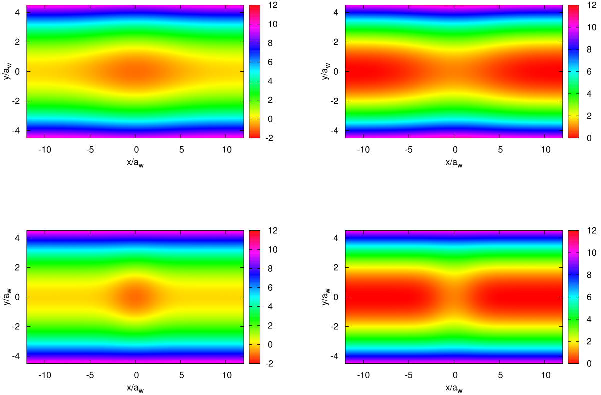
- Plane in-wave $\longrightarrow$ sharp in-energy E
- Time-dependent potential
- Potential flashed smoothly on and off, not periodic
- Inelastic scattering



(Phys. Rev. **B** 77, 035329 (2008))

Current modulation

$$V_{\rm sc}(\mathbf{r},t) = V_0 e^{-\beta r^2} e^{-\gamma t}\cos(\Omega t),$$ view at $t=0$:



$V_0 = \pm 1.0$ meV, $\Omega = 0.2\Omega_w$, $\gamma = 1.0\Omega_w^2$, $\beta = 1$ or 4×10^{-4} nm^{-2}, $\longrightarrow$ one smooth flash

Plane in-wave

$$\varphi_m^0(q,\omega) = (2\pi)^2\delta(q-k_n)\delta(\omega-\omega_{nq}^0)\delta_{m,n}$$

Green function

$$\{\hbar\omega - \hbar\omega_{nq}^0\}\, G_0^n(q,\omega) = 1$$

T-matrix

$$T_{nn'}(q\omega,p\nu) = V_{nn'}^{\rm sc}(q\omega,p\nu) + \sum_{m'}\int \frac{dk}{2\pi}\frac{d\omega'}{2\pi}\, V_{nm'}^{\rm sc}(q\omega,k\omega')\, G_0^{m'}(k\omega')\, T_{m'n'}(k\omega',p\nu)$$

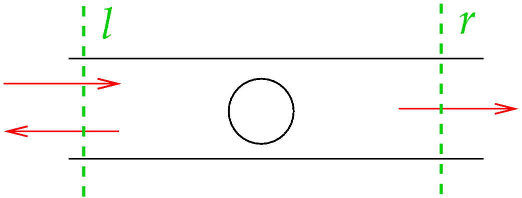
and full wave function

$$\Psi(\mathbf{r},t) = e^{i(k_n x - \omega_{nk}^0 t)}\phi_n(k_n,y) + \sum_m \int \frac{dq}{2\pi}\frac{d\omega}{2\pi}\, e^{i(qx-\omega t)}\, G_0^m(q\omega)\, T_{mn}(q\omega,k_n\omega_{nk}^0)\phi_m(q,y)$$

Left and right current of state α

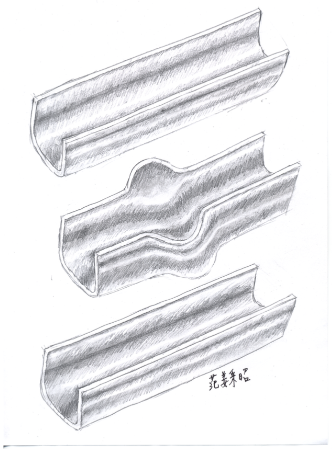
$$(I_\alpha^{r,l}(t))_x = \frac{\hbar}{m^*}\Re\left\{\int_{-\infty}^{\infty} dy\, (\Psi_\alpha^{r,l})^* D_x \Psi_\alpha^{r,l}\right\}$$

with $\hbar D_x = (p_x + (e/c)A_x) = \hbar(-i\partial_x - y/l^2)$



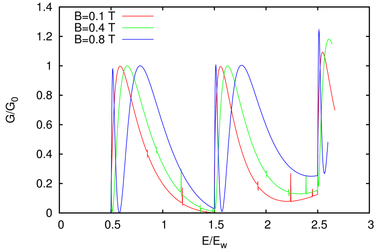
- Contributions from any point in sc-region for all earlier times
- Calculate for state α at Fermi energy
- Inelastic, any outstate possible, evanescent states explicitly in G

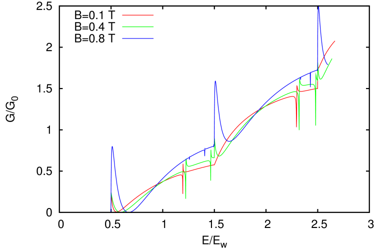
Smooth well-like pulse



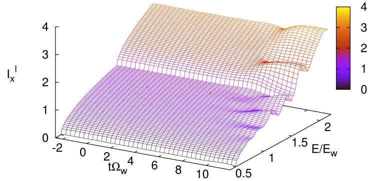
$\beta = 1 \times 10^{-4}$ nm^{-2}, $\beta = 4 \times 10^{-4}$ nm^{-2}

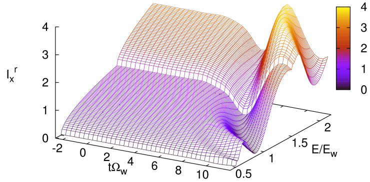
Static conductance



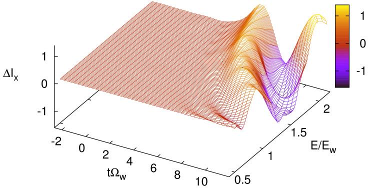


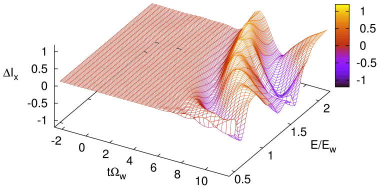
I_x^l and I_x^r, $B = 0.1$ T, $V_0 = -1$ meV



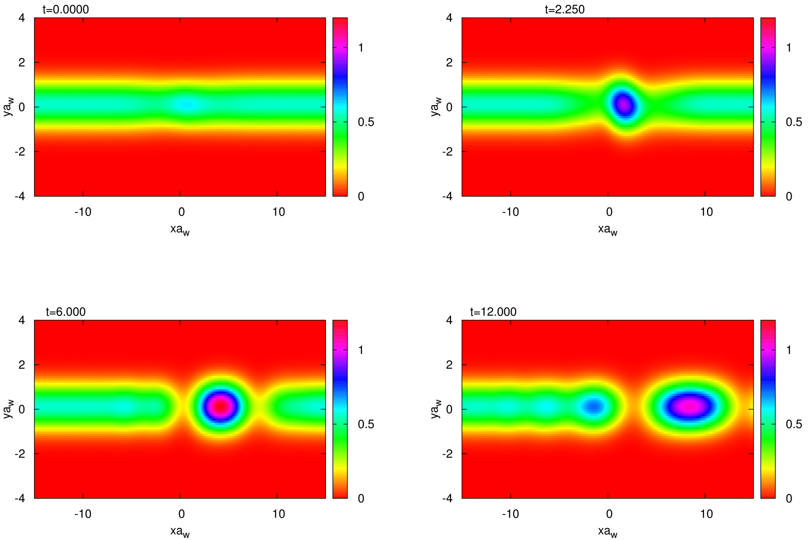


$I_x^l - I_x^r$, $\beta = 1 \times 10^{-4}$ nm^{-2}, $\beta = 4 \times 10^{-4}$ nm^{-2}

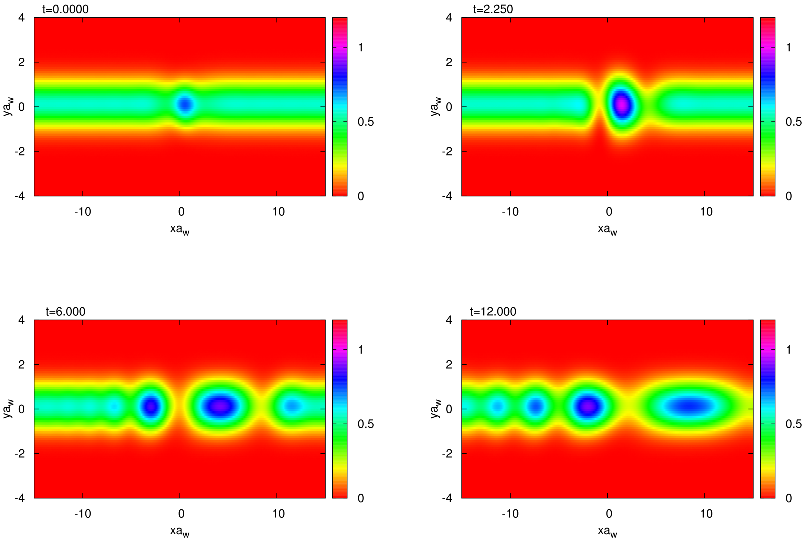




$|\Psi|^2$, $B = 0.1$ T, $V_0 = -1$ meV, $\beta = 1 \times 10^{-4}$ nm^{-2}, $E = 0.75E_w$



$|\Psi|^2$, $B = 0.1$ T, $V_0 = -1$ meV, $\beta = 4 \times 10^{-4}$ nm^{-2}, $E = 0.75 E_w$

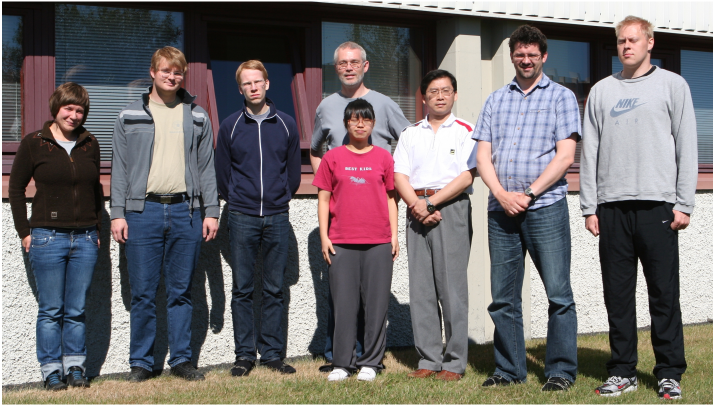


Summary

- General scattering potentials – embedded systems
- Magnetic field
- General confinement
- Heavy numerical – analytical calculations
- Single-electron formalism

- Interplay of geometry and magnetic field $\rightarrow$ interference
- Scattering to all orders
- Resonances, open systems
- Current modulation
- Releasing of quasi-bound states
- Time-dependence $\rightarrow$ inelastic processes

Cooperation



Ingibjörg Magnúsdóttir	Guðný Guðmundsdóttir	Jens H. Bárðarson
Gunnar Þorgilsson	Kristinn Torfason	Ómar Valsson
Yu-Yu Lin	Chi-Shung Tang	Cai-Jhao Fan-Jiang
Wing Wa Yu	Andrei Manolescu	Valeriu Moldoveanu