Coherent electronic transport in multimode quantum channel

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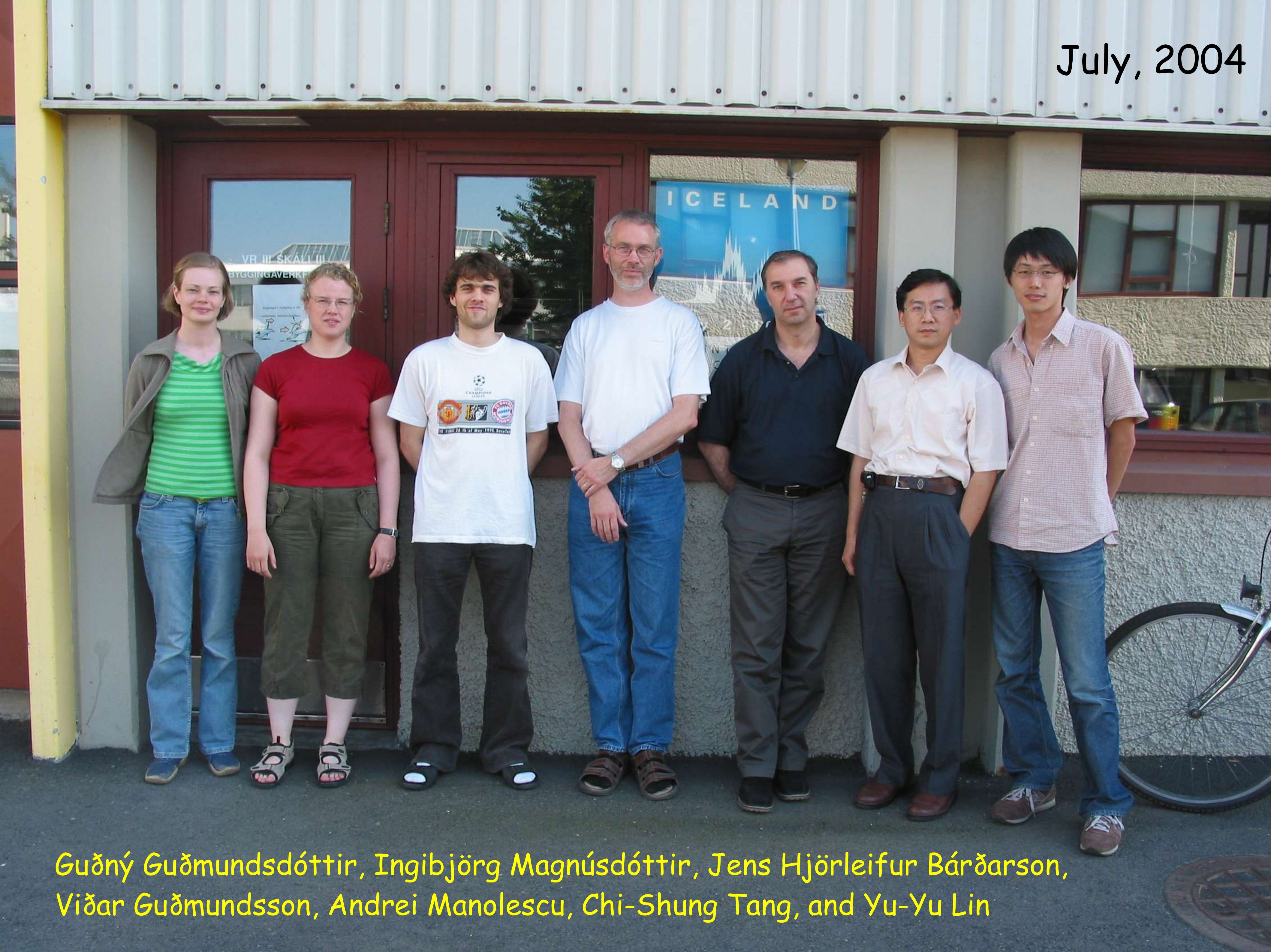

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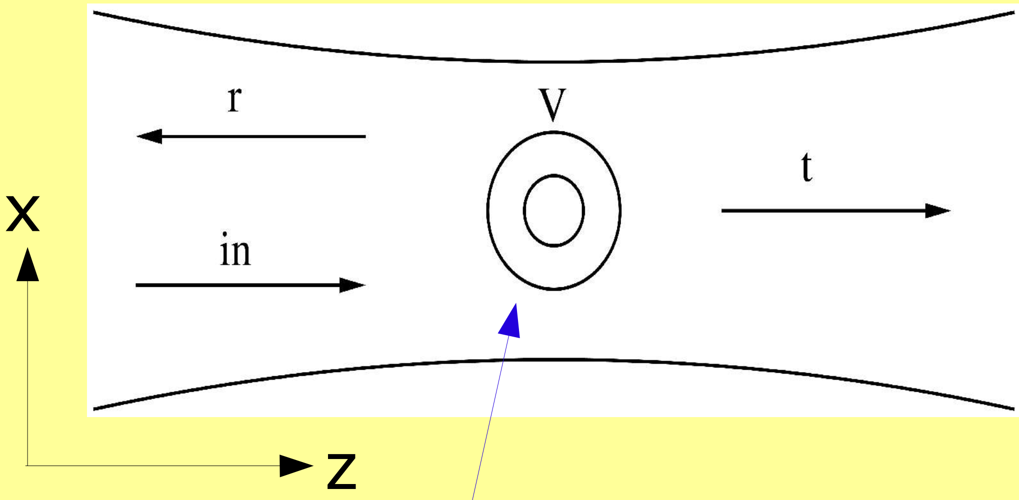
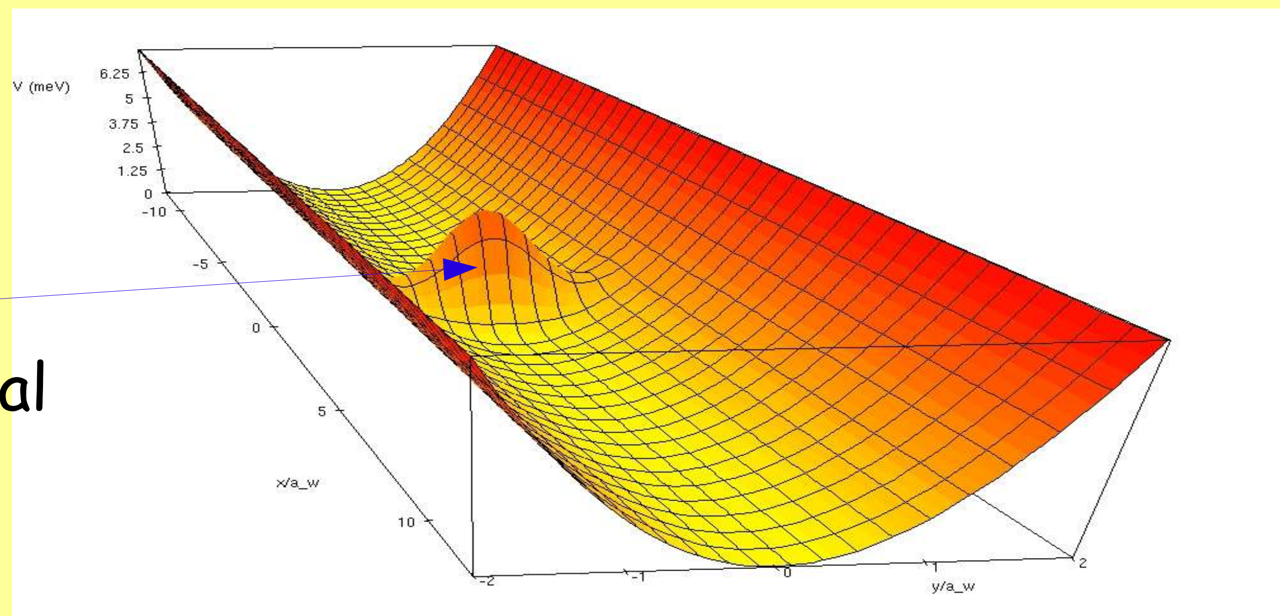
Guðný Guðmundsdóttir, Ingibjörg Magnúsdóttir, Jens Hjörleifur Bárðarson,
Viðar Guðmundsson, Andrei Manolescu, Chi-Shung Tang, and Yu-Yu Lin

System, quasi one-dimensional scattering



Top view of system



Scattering potential

Parabolic or hardwall confinement – (quantum wave guide)

Steady state equation of motion

Confinement

Scattering potential

$$\left(-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right]+V_c(x)+V(\mathbf{r})\right)\psi_E(\mathbf{r}) = E\psi_E(\mathbf{r})$$

In energy

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V_c(x)\right)\chi_n(x) = \varepsilon_n\chi_n(x)$$

Perpendicular mode

Possible solutions

$$\phi^{\pm}_{nE}(\mathbf{r}) = \frac{1}{\sqrt{k_n(E)}}e^{\pm ik_n(E)z}\chi_n(x)$$

Scattering boundary condition

(Asymmetric solution)

In-state

Reflected

$$\psi^+_{nE}(\mathbf{r}) = \begin{cases} \phi^+_{nE}(\mathbf{r}) + \sum_{m,\mathrm{prop}} r_{mn}\phi^-_{mE}(\mathbf{r}), & z \to -\infty \\ \sum_{m,\mathrm{prop}} t_{mn}\phi^+_{mE}(\mathbf{r}), & z \to \infty \end{cases}$$

$$G = \frac{2e^2}{h}\mathrm{Tr}[\mathbf{t}^\dagger \mathbf{t}]$$

Propagating channels, modes

Transmitted

Landauer-Buettiker conductance

Coupled channel equation

Scattering state

Channel mode

$$\psi^{+}_{nE}(\mathbf{r}) = \sum_m \varphi^n_{mE}(z)\chi_m(x)$$

$$\left(\frac{d^2}{dz^2} + k_m^2(E)\right)\varphi^n_{mE}(z) = \frac{2m}{\hbar^2}\sum_{m'} V_{mm'}(z)\varphi^n_{m'E}(z)$$

$$V_{mm'}(z) = \int dx\, \chi^*_m(x) V(\mathbf{r}) \chi_{m'}(x)$$

(Linear system)

Matrix element of scattering potential

Transformed into a linear system of integral equations

Why?

$$\varphi^{n}_{mE}(z) = \varphi^{n0}_{mE}(z) + \frac{2m}{\hbar^2}\sum_{m'}\int dz'\, \mathcal{G}^{0}_{mE}(z,z')V_{mm'}(z')\varphi^{n}_{m'E}(z')$$

1D scattering Greens function

$$\mathcal{G}^{0}_{nE}(z,z') = -\frac{i}{2k_n(E)}e^{ik_n(E)|z-z'|}$$

Structure
Global
Multiple scattering
Convenience

$$t_{mn} = \delta_{mn} + \frac{m}{i\hbar^2}\sum_{m'}\int dz'\, \frac{1}{\sqrt{k_m(E)}}e^{-ik_m(E)z'}V_{mm'}(z')\varphi^{n}_{m'}(z')$$

Transmission amplitudes, (n -> m)

Introduction of T-matrix

Self-consistency

$$\varphi = \varphi^0 + GV\varphi$$

Expansion

$$\varphi = \varphi^0 + GV\varphi^0 + GVGV\varphi^0 + \cdots = (1 + GT)\varphi^0$$

$$T = V + VGT$$

Exact solution

$$T_{mn}(k, k_n) = V_{mn}(k, k_n) + \frac{m}{\pi\hbar^2}\sum_l \int dq\,|q|\,\frac{V_{ml}(k, q)T_{ln}(q, k_n)}{k_l^2 - q^2 + i\eta}$$

$$t_{mn} = \delta_{mn} + \frac{m}{i\hbar^2}T_{mn}(k_m, k_n)$$

Transmission amplitude

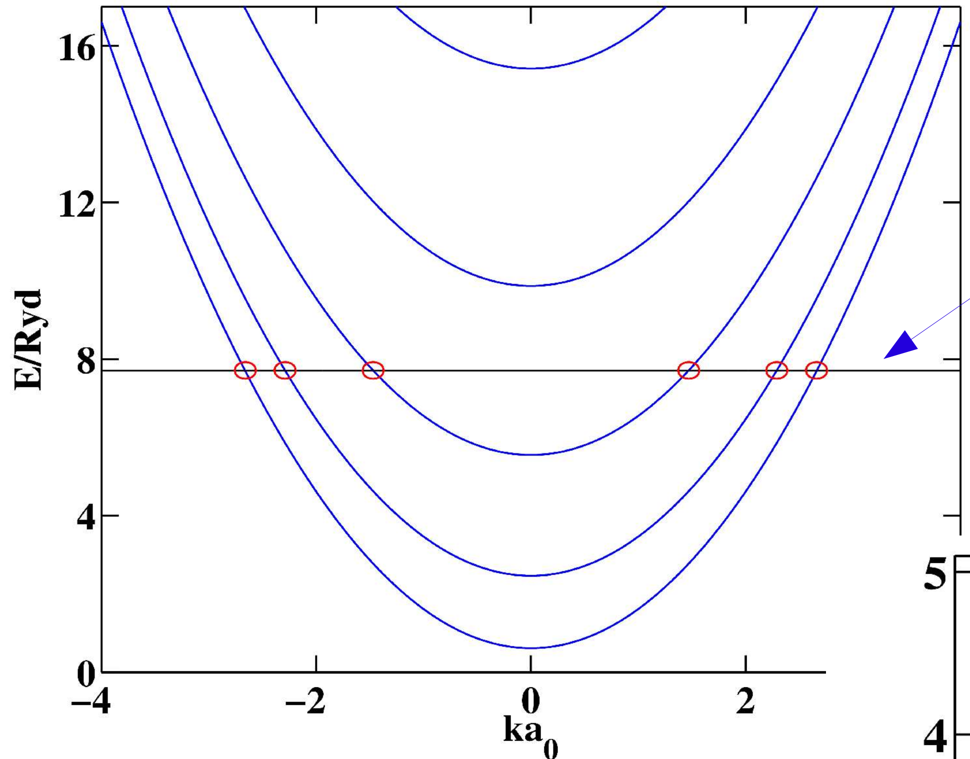
Size

Singularities

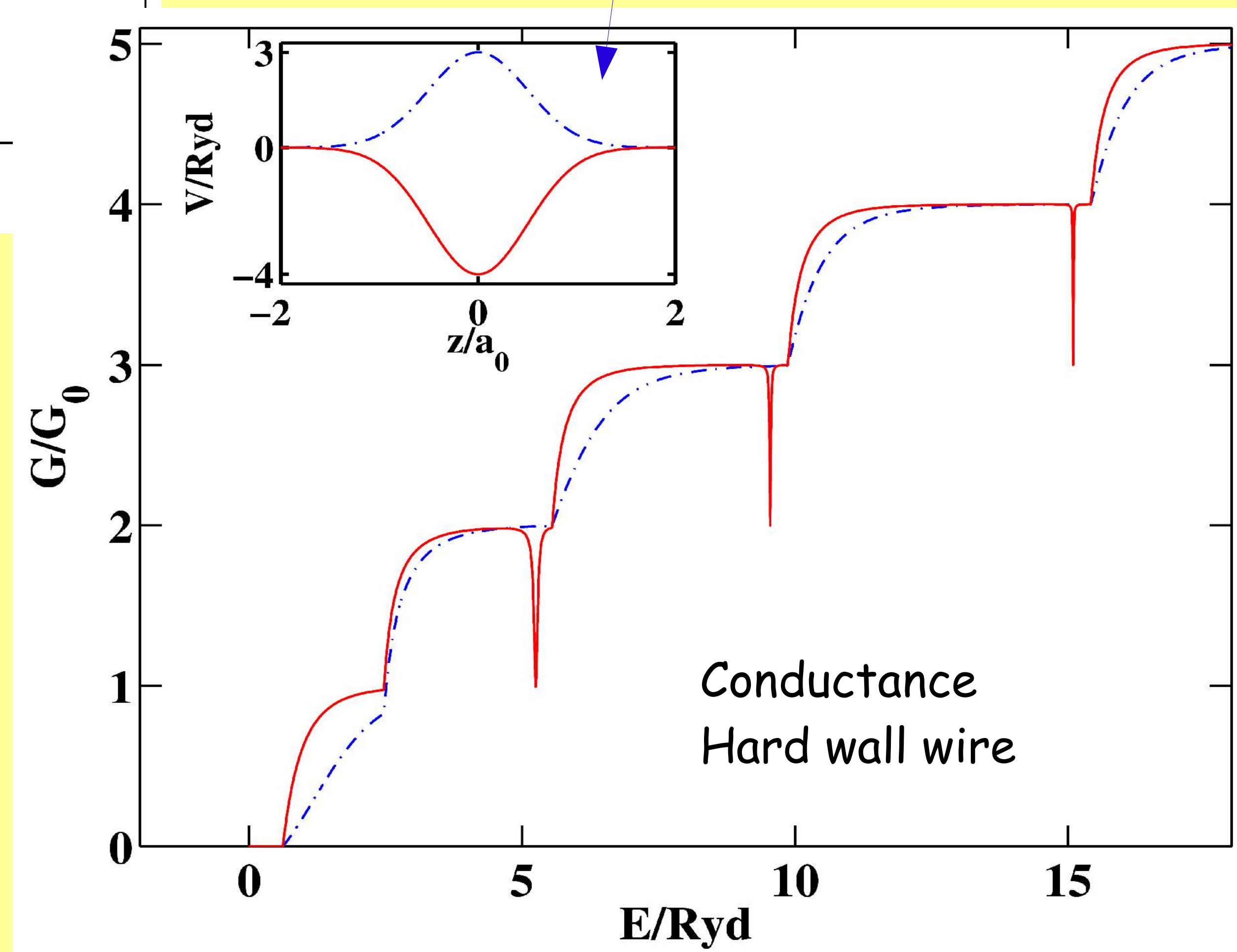
Energy spectrum

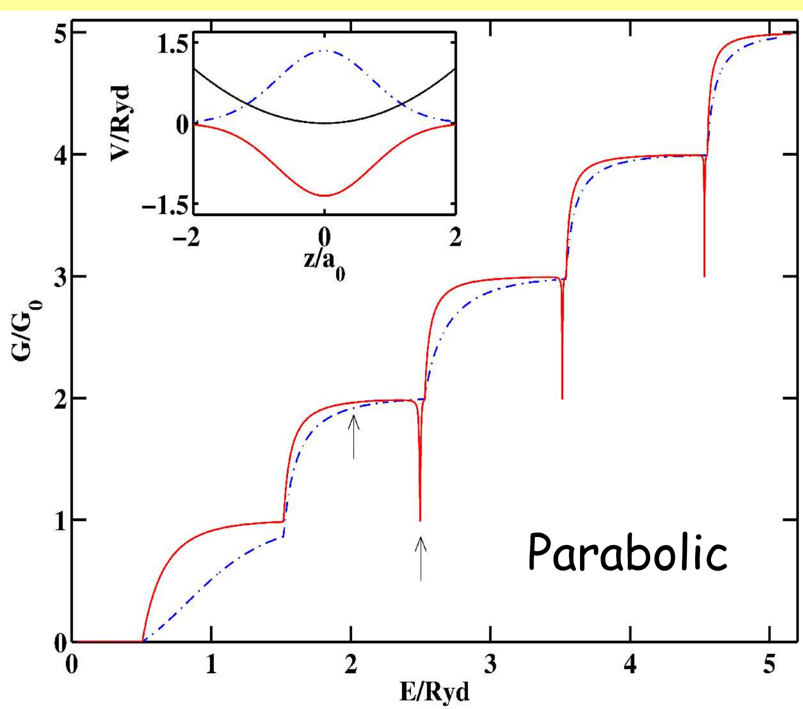
Propagating states- evanescent states

In-energy



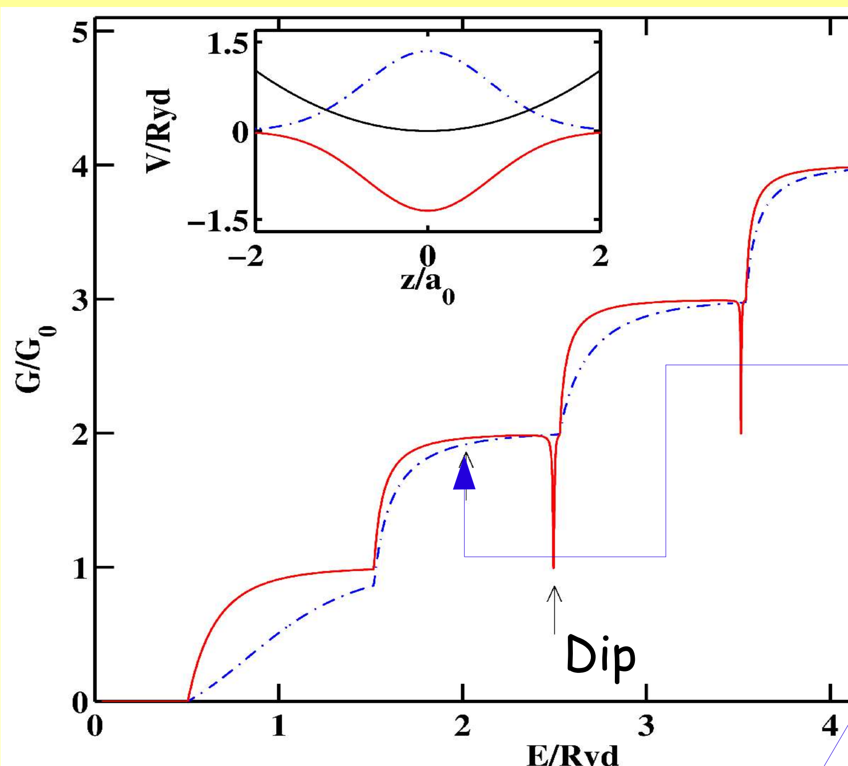
Scattering potential

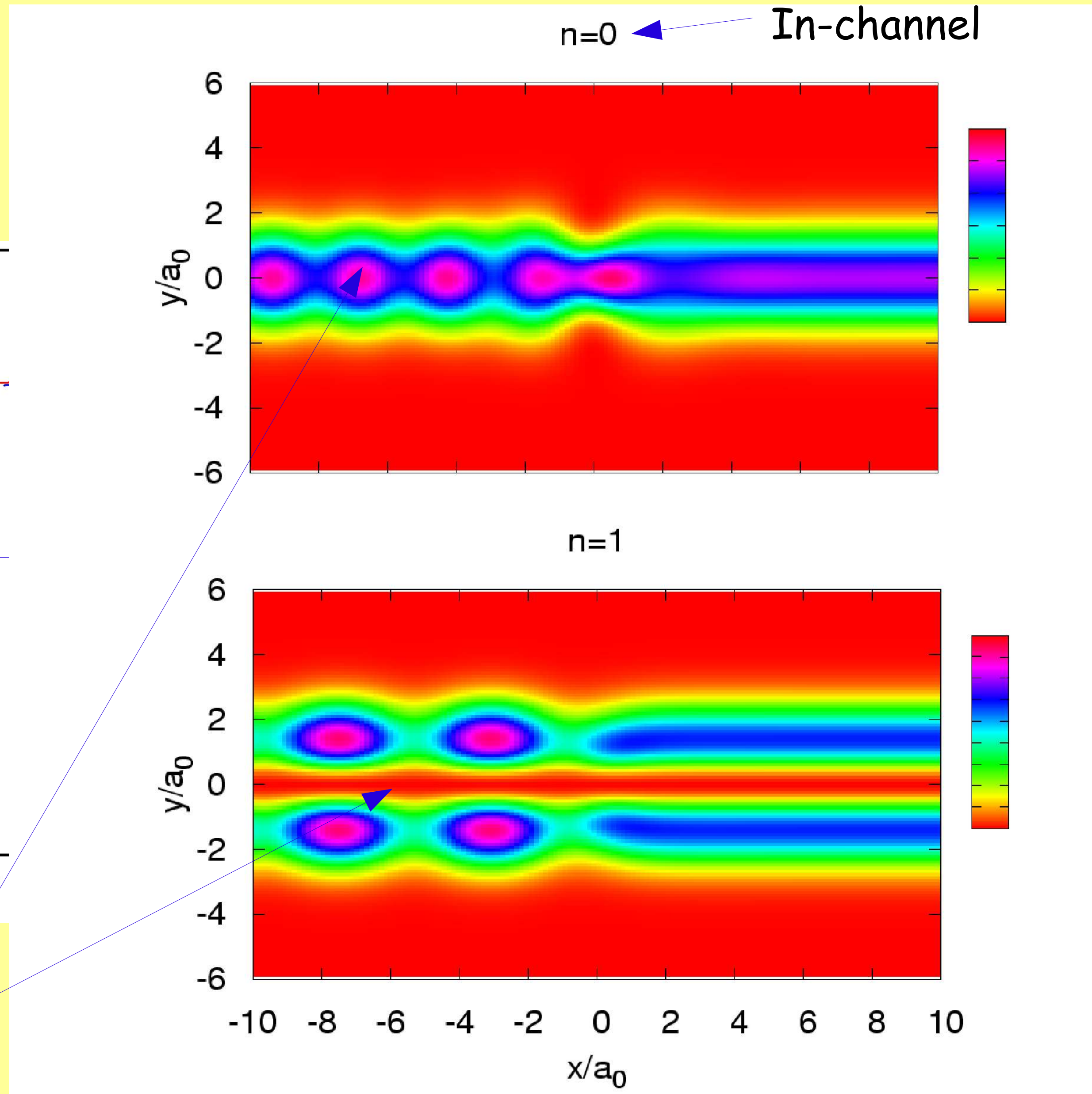




$$\phi^n_{mE}(z) = \phi^{n0}_{mE}(z) + \frac{m}{\pi\hbar^2}\int_{-\infty}^{+\infty} dp \frac{\sqrt{|p|}e^{ipz}}{k_m^2 - p^2} T_{mn}(p, k_n).$$

Scattering state



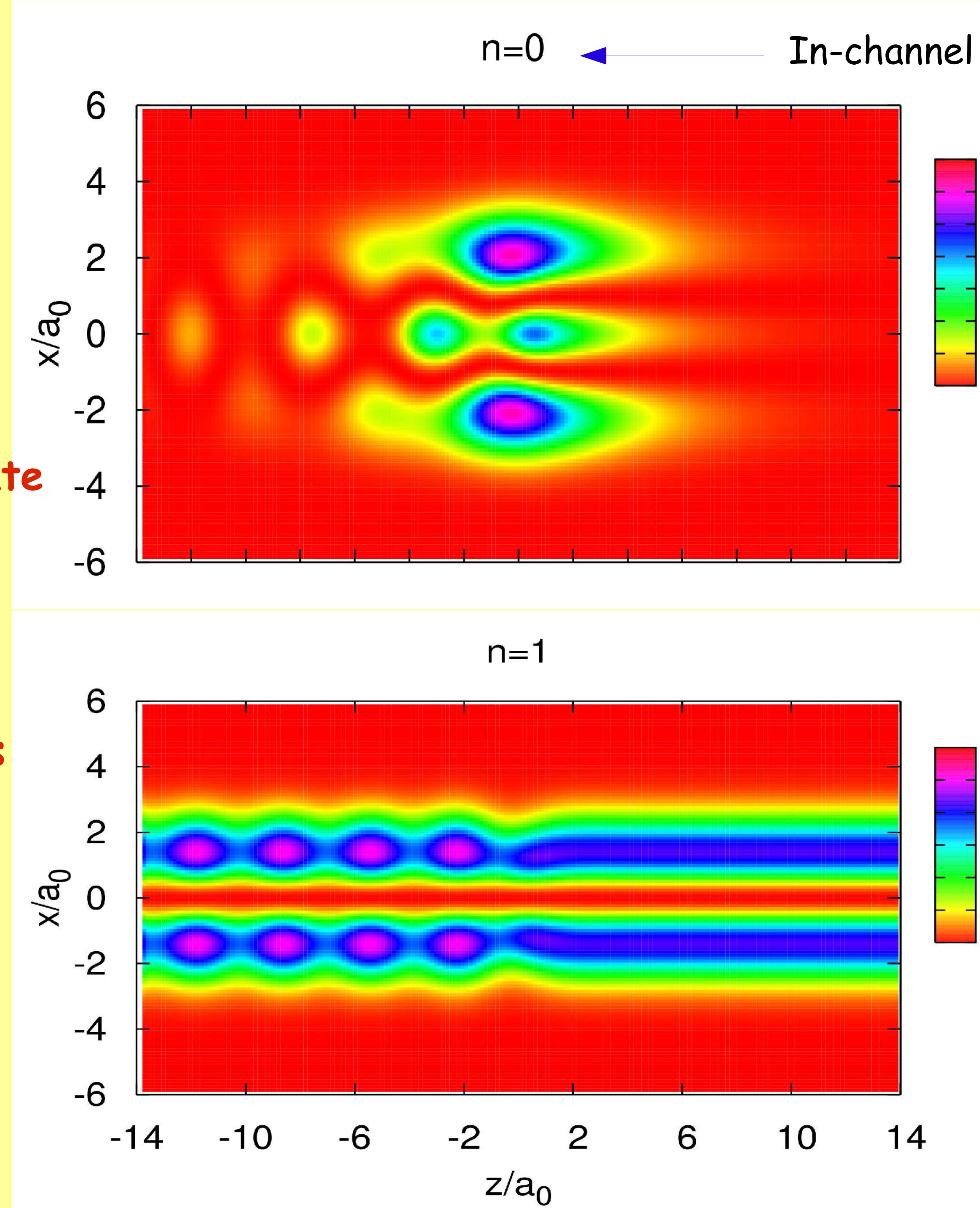


In – reflected interference

What happens in a dip?

Total reflectance of one channel through evanescent state

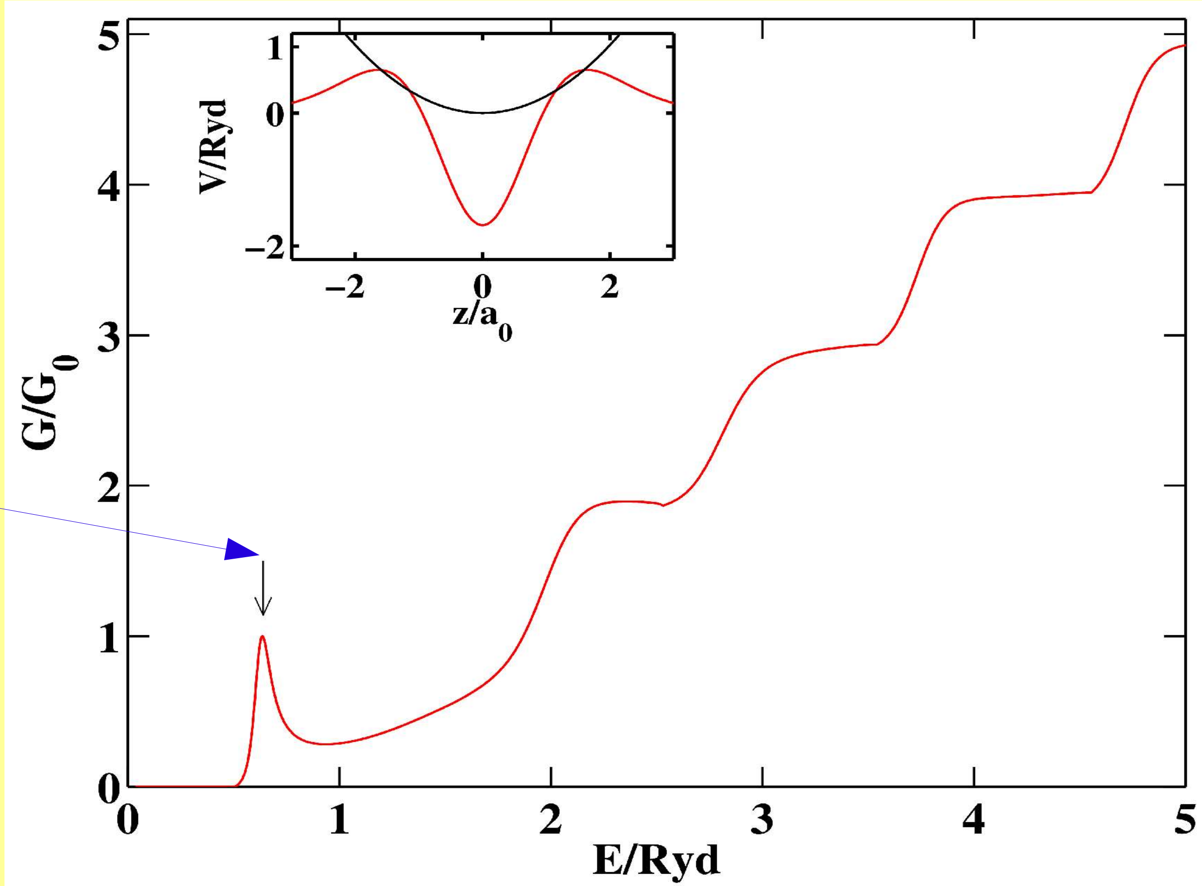
Symmetry - Selection rules



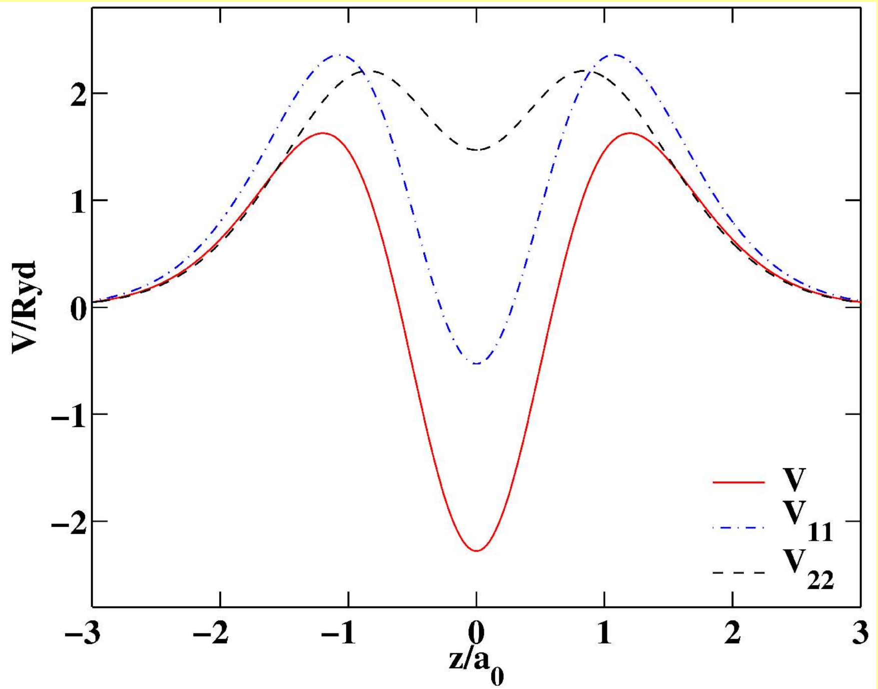
Embedded quantum dot

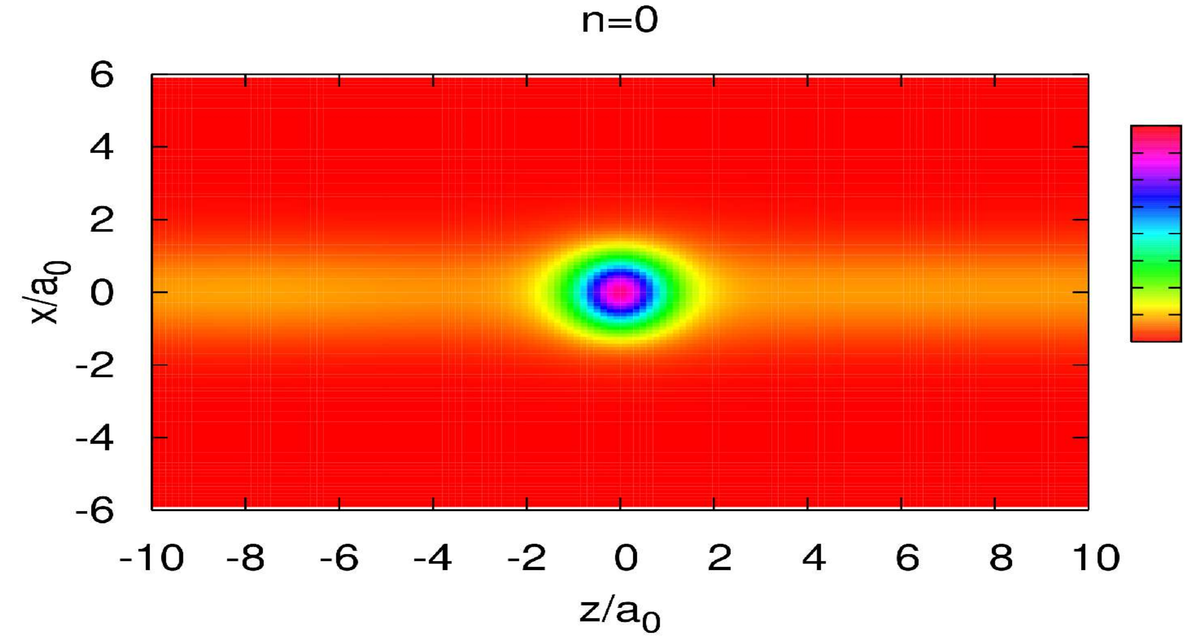
Total resonant transmission

(lifetime)



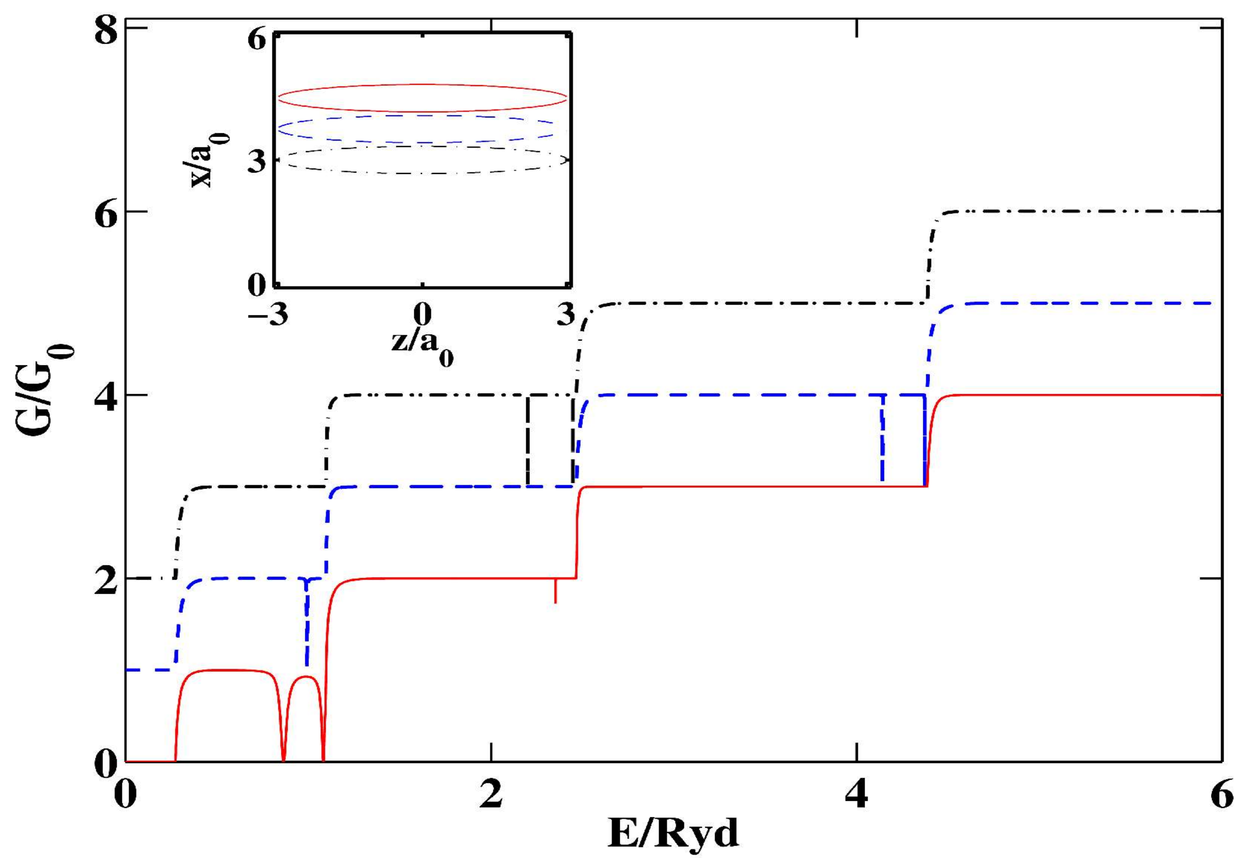
Effective scattering potential

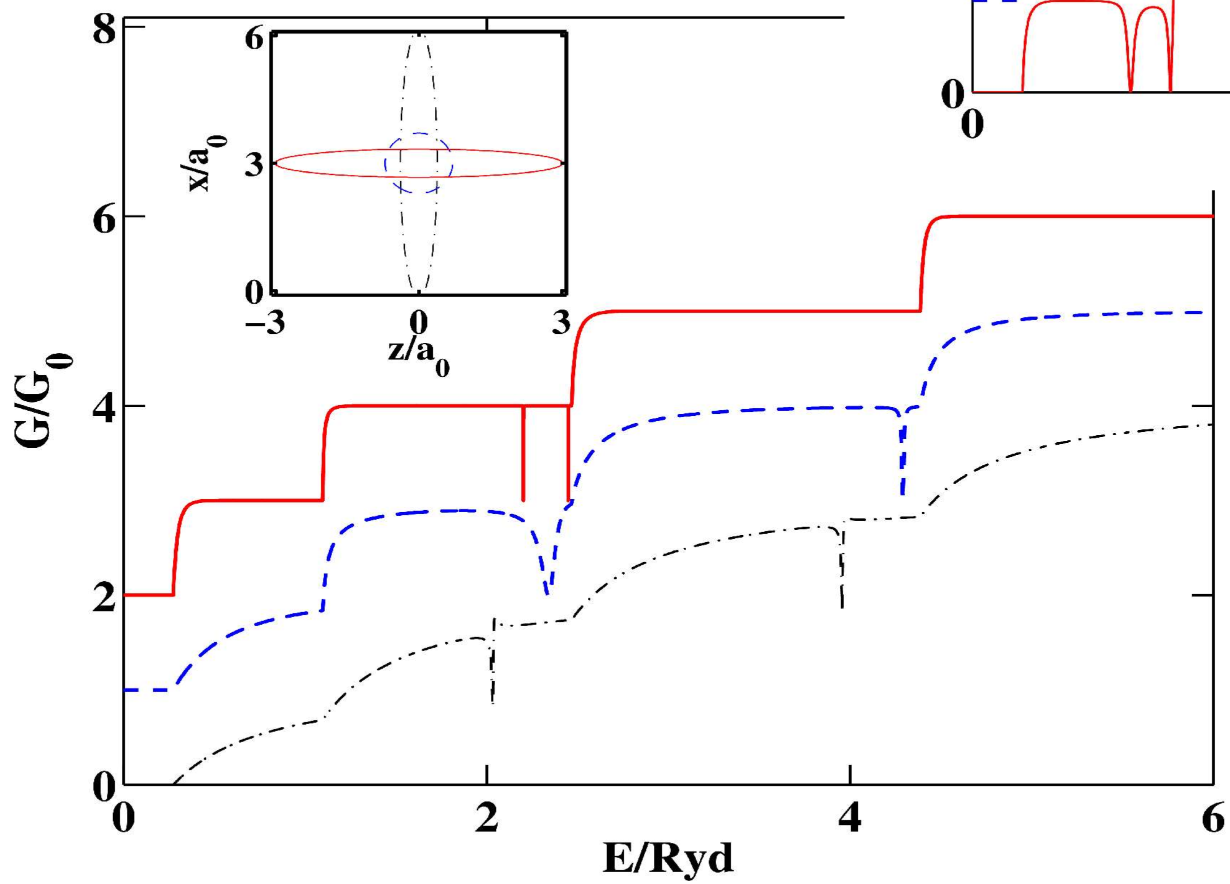


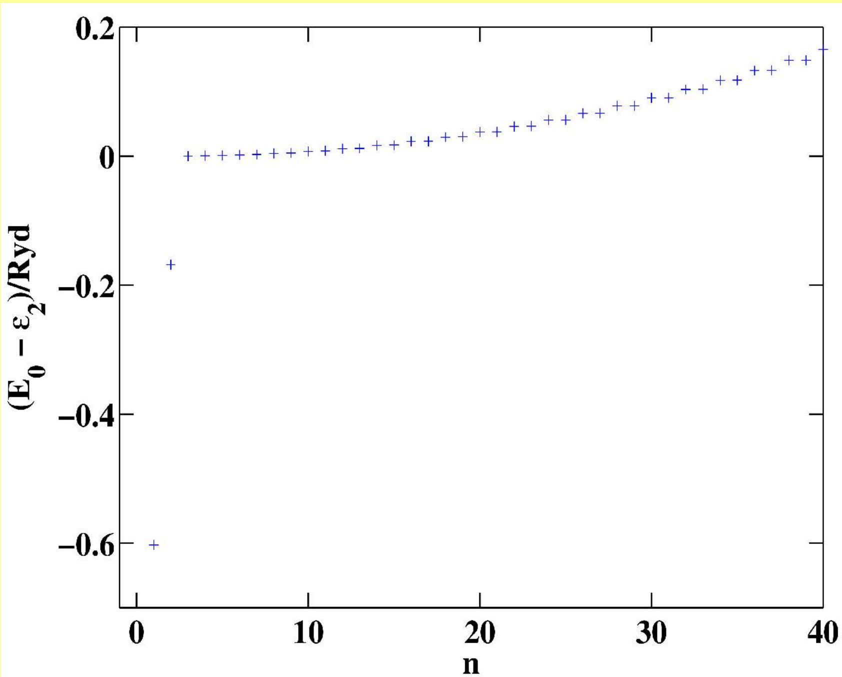


More complex scattering potentials

Symmetry breaking



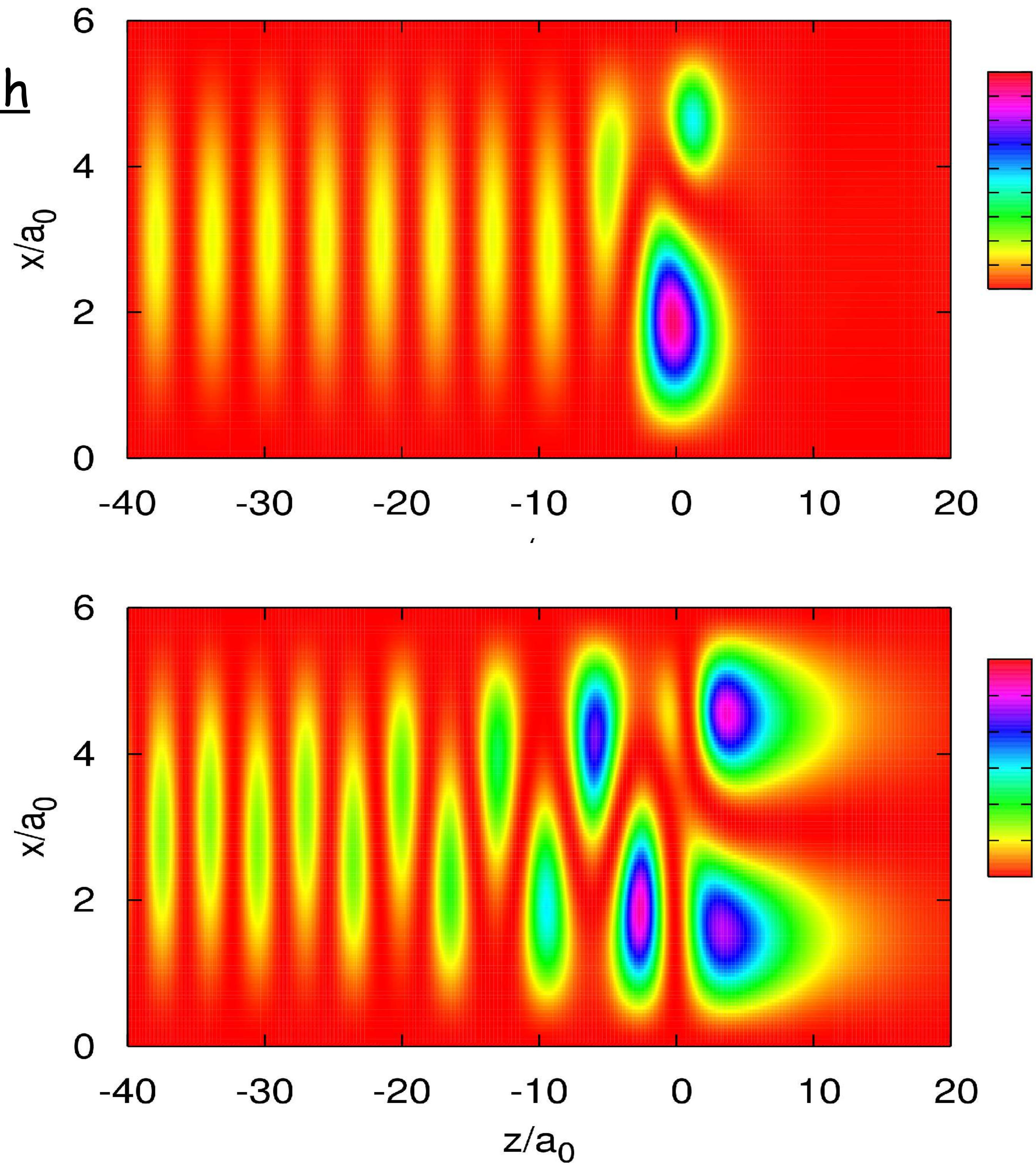




Total reflectance through evanescent states

Probability for the double dip structure

Where is the well?



Conclusion

Lippmann-Schwinger formalism -> far and near fields
-> we can see what happens

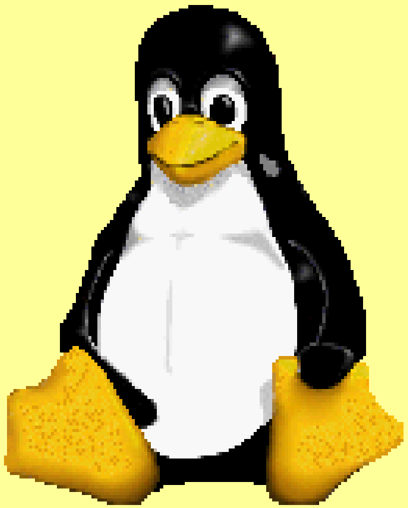
Lot of team work, complex analytical and numerical work, requires computational facilities

Magnetic field.....

Bias?

Time-dependence?

Interaction?



Anyway, a lot of fun.....