Time-dependent magnetotransport in a quantum wire

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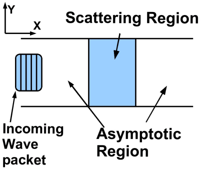
http://hartree.raunvis.hi.is/~vidar/Rann/Fyrirlestrar/Tainan_T.pdf

Tainan, September, 2007

Two cases of time-dependent magnetotransport

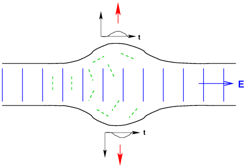
Wave packet transport

- Static potential
- Elastic scattering
- Life-time of quasi-bound states and resonances
- Delay times



Modulation of a current

- Plane in-wave $\rightarrow$ sharp in-energy E
- Time-dependent potential
- Potential flashed smoothly on and off, not periodic
- Inelastic scattering



Asymptotic regions, both cases

Landau gauge: $\mathbf{A} = -By\hat{\mathbf{x}} \quad \rightarrow \quad \mathbf{B} = B\hat{\mathbf{z}}, \quad l = \sqrt{\hbar c/(eB)}$

Equation of motion

$$i\hbar\partial_t\Psi(\mathbf{r},t) = \left\{ -\frac{\hbar^2}{2m^*}\left(\nabla^2 - \frac{2i}{l^2}y\partial_x - \frac{y^2}{l^4}\right) + \frac{1}{2}m^*\Omega_0^2 y^2 \right\}\Psi(\mathbf{r},t)$$

Fourier transform: $(x,y,t) \rightarrow (p,y,\omega)$

$$\Psi(\mathbf{r},t) = \int \frac{dp}{2\pi}\frac{d\omega'}{2\pi} e^{i(px-\omega' t)}\Psi(p,y,\omega')$$

Separation into modes in (p,y)-space, (S. A. Gurvitz, PRB **51**, 7123 (1995))

$$\Psi(q,y,t) = \sum_n \varphi_n(q,t)\phi_n(q,y)$$

H.O. eigenfunctions $\phi_n(q, y)$, (not orthogonal for different q's)

Frequency $\Omega_w = \sqrt{\omega_c^2 + \Omega_0^2}$, cyclotron frequency $\hbar\omega_c = eB/(m^* c)$

Shifted by $y_0 = q a_w^2 \omega_c / \Omega_w$, new length scale $a_w = \sqrt{\hbar/(m^* \Omega_w)}$

For a static system:

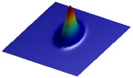
Energy bands

$$E_{nq} = \hbar\omega_{nq}^0 = E_n^0 + U_w \frac{(q a_w)^2}{2}, \qquad U_w = \frac{(\hbar\Omega_0)^2}{\hbar\Omega_w}$$

With band edges $E_n^0 = \hbar\Omega_w(n + 1/2)$ for $n = 0, 1, 2, \cdots$

Wave packet propagation

Wave packet



$$\varphi_n^0(p,\omega) = 2\pi g_n(p)\delta\left[\omega - \frac{E_{np}}{\hbar}\right]$$

$$g_n(p) = \delta_{nn'}\exp\left[-\gamma(p-p_0)^2\right]$$

and the Green function

$$G_n(p,\omega) = \frac{1}{(k_n(\omega)a_w)^2 - (pa_w)^2}$$

with

$$k_n(\omega)a_w = \sqrt{\frac{(\hbar\omega - E_n^0)}{U_w}}$$

gives a Lippmann-Schwinger equation

$$\varphi_n(p,\omega) = \varphi_n^0(p,\omega) + G_n(p,\omega)\int_{-\infty}^{\infty}\frac{dqa_w}{2\pi}U_{nn'}(p,q)\varphi_{n'}(q),$$

where

$$U_{nn'}(p,q)U_w = \int_{-\infty}^{\infty} dy\, \phi_{n'}^*(q,y)V_{\rm sc}(p-q,y)\phi_n(p,y)$$

With a T-matrix

$$T_{nn'}(p,q,\omega) = U_{nn'}(p,q) + \sum_m \int_{-\infty}^{\infty}\frac{dka_w}{2\pi}U_{nm}(p,k)G_m(k,\omega)T_{mn'}(k,q,\omega)$$

the wave function

$$\Psi(x,y,t) = \Psi_0(x,y,t) + \Psi_{\rm sc}(x,y,t)$$

with the in-wave

$$\Psi_0(x,y,t) = \sum_n \int_{-\infty}^{\infty} dp\; g_n(p)\phi_n(p,y)e^{i(px-\omega_{np}^0 t)}$$

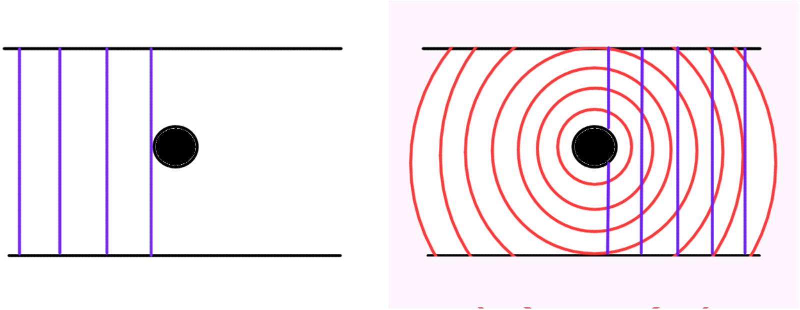
becomes

$$\begin{aligned}\Psi_{\rm sc}(x,y,t) \;&=\; \sum_{n'} \int_{E_{n'}^0/\hbar}^{\infty} d\omega\; e^{-i\omega t}\, \frac{\Omega_w g_{n'}[k_{n'}(\omega)]}{\Omega_0^2 |k_n'(\omega)a_w|} \\ &\times \sum_n \int_{-\infty}^{\infty} \frac{dpa_w}{2\pi} G_n(p,\omega)e^{ipx}\, T_{nn'}(p,k_{n'}(\omega))\phi_n(p,y)\end{aligned}$$

$\cdots$ or graphically

$$\Psi(x,y,t) = \Psi_0(x,y,t) + \Psi_{sc}(x,y,t).$$

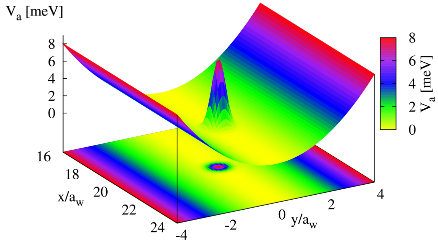
before and after scattering



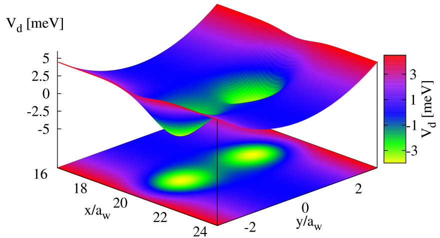
Propagation of a wave packet

Static potentials

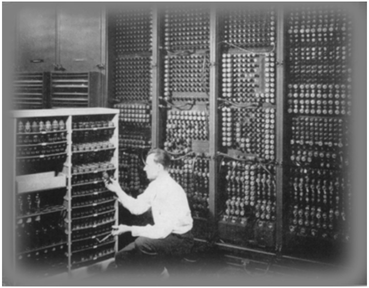
Antidot



Parallel double dot



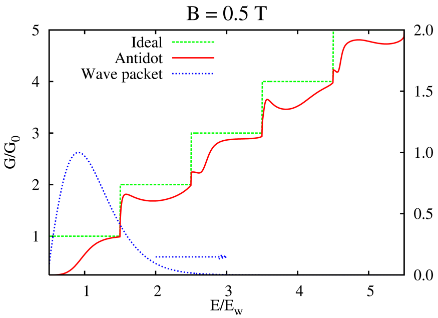
- Calculations implemented on the cluster jotunn.rhi.hi.is
- ω-integration programmed for parallel execution



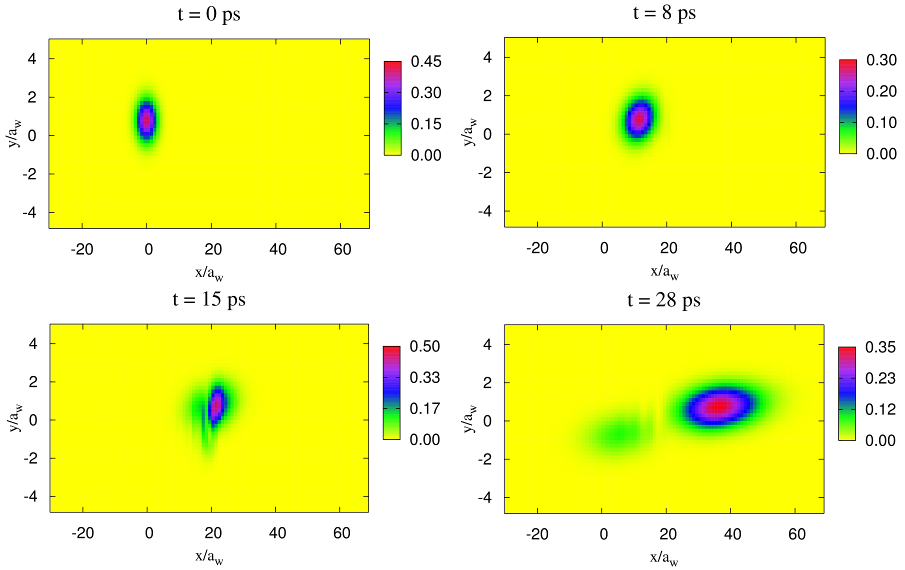
- Large calculations:
  - 20 nodes $\rightarrow\sim$ 40 hours
  - 3 GB of memory used on each node

Antidot

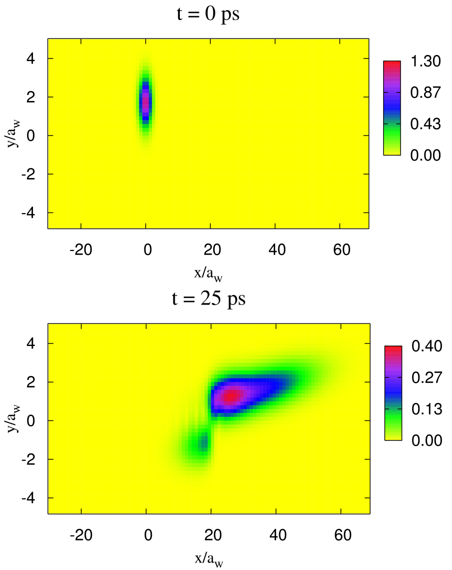
Static conductance – wave packet



Antidot, $B = 0.5$ T

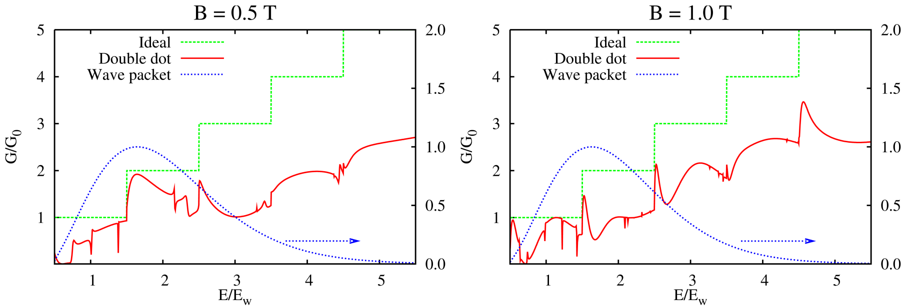


Antidot, $B = 1.0$ T

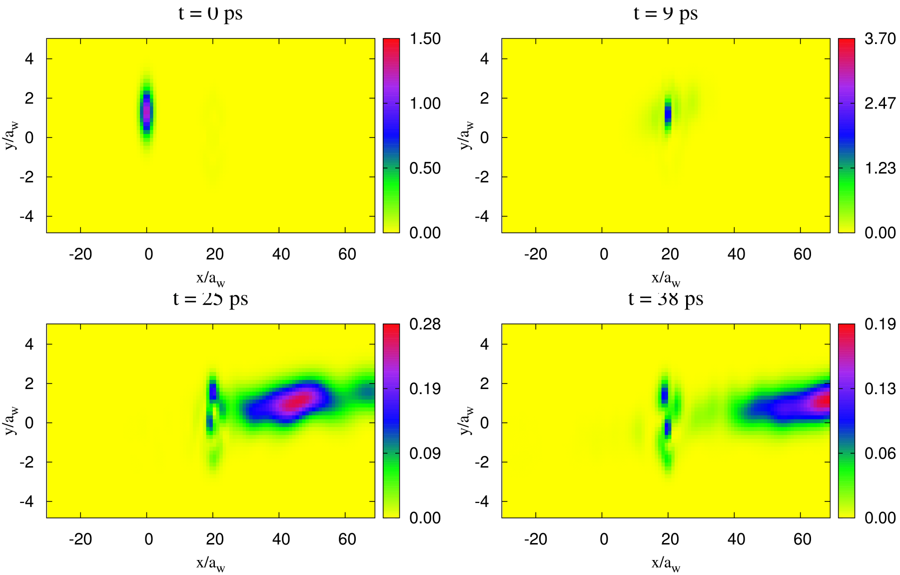


Parallel double dot

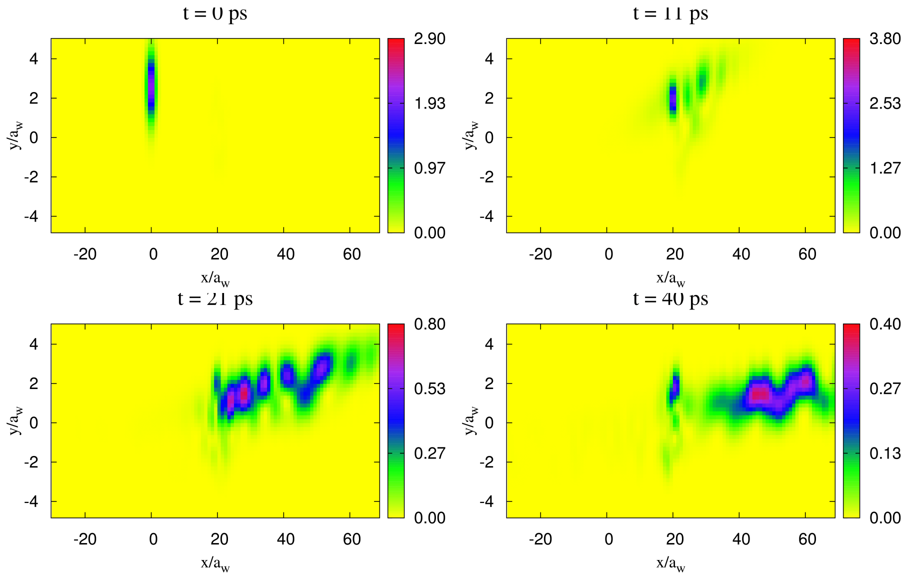
Static conductance – wave packet



Parallel double dot, $B = 0.5$ T

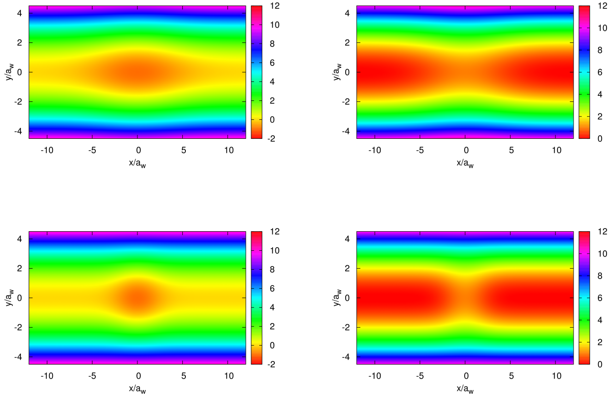


Parallel double dot, $B = 1.0$ T



Current modulation

$V_{\rm sc}(\mathbf{r},t) = V_0 e^{-\beta r^2} e^{-\gamma t} \cos(\Omega t)$, view at $t = 0$:



$V_0 = \pm 1.0$ meV, $\Omega = 0.2\Omega_w$, $\gamma = 1.0\Omega_w^2$, $\beta = 1$ or 4×10^{-4} nm^{-2}, $\longrightarrow$ one smooth flash

Plane in-wave

$$\varphi_m^0(q,\omega) = (2\pi)^2\delta(q-k_n)\delta(\omega-\omega_{nq}^0)\delta_{m,n}$$

Green function

$$\{\hbar\omega - \hbar\omega_{nq}^0\}G_0^n(q,\omega) = 1$$

T-matrix

$$T_{nn'}(q\omega,p\nu) = V_{nn'}^{\mathrm{sc}}(q\omega,p\nu) + \sum_{m'}\int\frac{dk}{2\pi}\frac{d\omega'}{2\pi}V_{nm'}^{\mathrm{sc}}(q\omega,k\omega')G_0^{m'}(k\omega')T_{m'n'}(k\omega',p\nu)$$

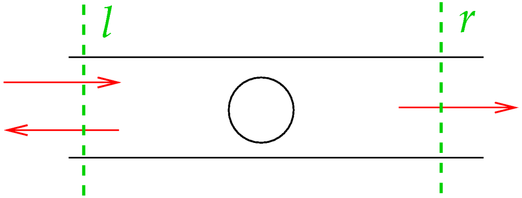
and full wave function

$$\mathbf{\Psi}(\mathbf{r},t) = e^{i(k_nx-\omega_{nk}^0t)}\phi_n(k_n,y) + \sum_m\int\frac{dq}{2\pi}\frac{d\omega}{2\pi}e^{i(qx-\omega t)}G_0^m(q\omega)T_{mn}(q\omega,k_n\omega_{nk}^0)\phi_m(q,y)$$

Left and right current of state α

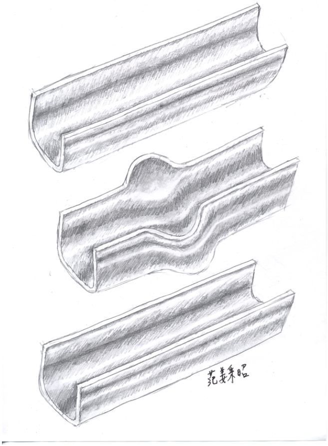
$$(I_\alpha^{r,l}(t))_x = \frac{\hbar}{m^*}\Re\left\{\int_{-\infty}^{\infty} dy\, (\Psi_\alpha^{r,l})^* D_x \Psi_\alpha^{r,l}\right\}$$

with $\hbar D_x = (p_x + (e/c)A_x) = \hbar(-i\partial_x - y/l^2)$



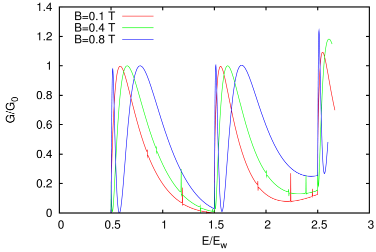
- Contributions from any point in sc-region for all earlier times
- Calculate for state α at Fermi energy
- Inelastic, any outstate possible, evanescent states explicitly in G

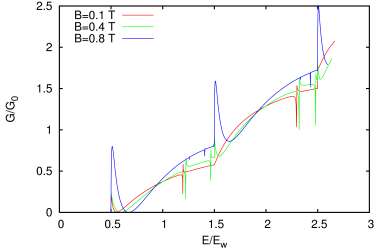
Smooth well-like pulse



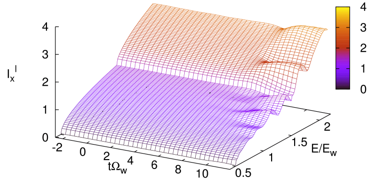
$\beta = 1 \times 10^{-4}\ \mathrm{nm}^{-2}$, $\beta = 4 \times 10^{-4}\ \mathrm{nm}^{-2}$

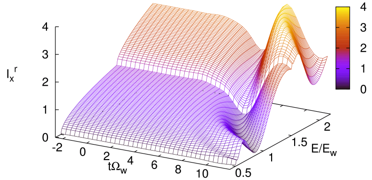
Static conductance



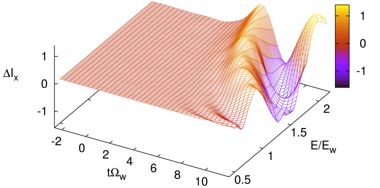


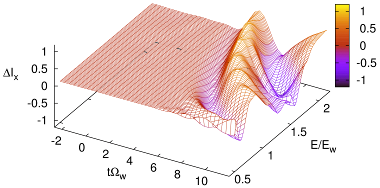
I_x^l and I_x^r, $B = 0.1$ T, $V_0 = -1$ meV



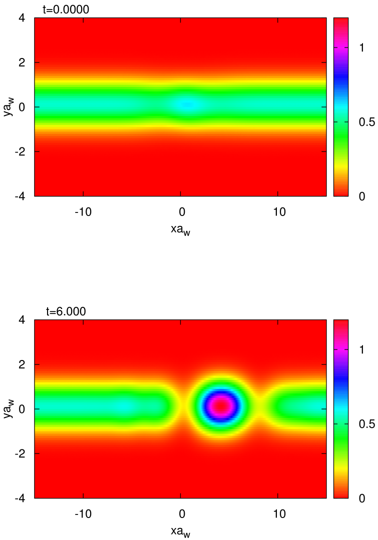


$I_x^l - I_x^r$, $\beta = 1 \times 10^{-4}$ nm^{-2}, $\beta = 4 \times 10^{-4}$ nm^{-2}

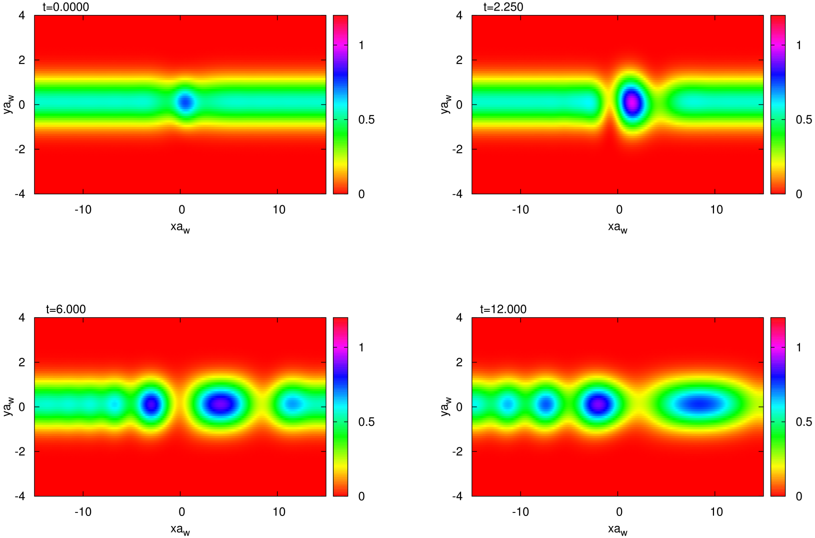




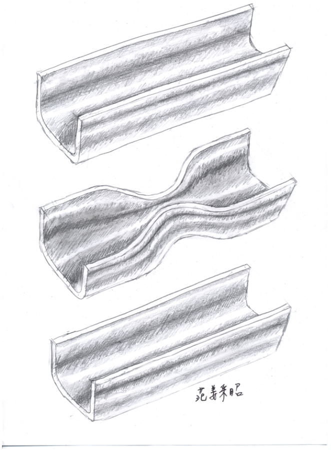
$|\Psi|^2$, $B = 0.1$ T, $V_0 = -1$ meV, $\beta = 1 \times 10^{-4}$ nm^{-2}, $E = 0.75E_w$



$|\Psi|^2$, $B = 0.1$ T, $V_0 = -1$ meV, $\beta = 4 \times 10^{-4}$ nm^{-2}, $E = 0.75E_w$

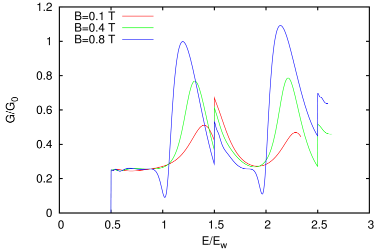


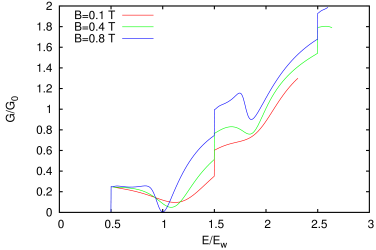
Smooth hill-like pulse



$\beta = 1 \times 10^{-4}$ nm^{-2}, $\beta = 4 \times 10^{-4}$ nm^{-2}

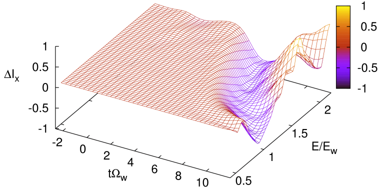
Static conductance

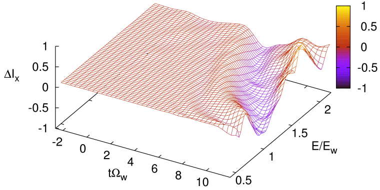




ΔI_x, $B = 0.4$ T, $V_0 = +1$ meV,

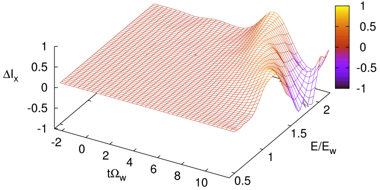
$\beta = 1 \times 10^{-4}$ nm^{-2}, $\beta = 4 \times 10^{-4}$ nm^{-2}

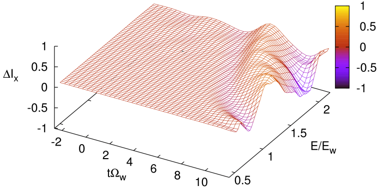




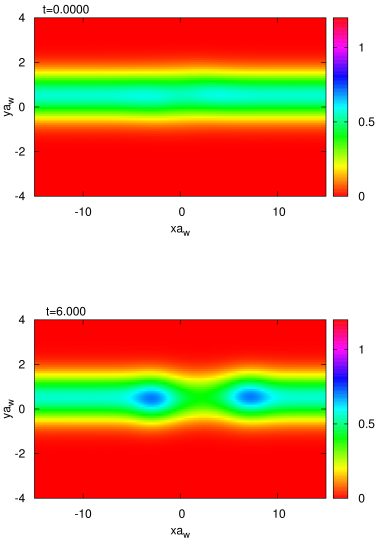
ΔI_x, $B = 0.4$ T, $V_0 = -1$ meV,

$\beta = 1 \times 10^{-4}$ nm^{-2}, $\beta = 4 \times 10^{-4}$ nm^{-2}





$|\Psi|^2$, $B = 0.6$ T, $V_0 = +1$ meV, $\beta = 1 \times 10^{-4}$ nm^{-2}, $E = 0.63 E_w$



Conclusions

Wave packet propagation

- Lifetimes of resonances
- Spreading of wave packets by resonances and magnetic field
- Mode-mixing $\rightarrow$ onset of skipping orbits

Current modulation

- Is modulation of current possible?
- Inelastic scattering
- Releasing of quasi-bound states?

- Magnetotransport in general smooth geometries
- Coulomb interaction, many-body effects?