# QEDFT = QED + DFT applied to an array of quantum dots in a photon cavity

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# Experimental impetus. . .

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LETTERS

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## Collective non-perturbative coupling of 2D electrons with high-quality-factor terahertz cavity photons

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**The collective interaction of electrons with light in a high-quality-factor cavity is expected to reveal new quantum phenomena[1–7] and find applications in quantum-enabled technologies[8,9]. However, combining a long electronic coherence time, a large dipole moment, and a high quality-factor has proved difficult[10–13]. Here, we achieved these conditions simultaneously in a two-dimensional electron gas in a high-quality-factor terahertz cavity in a magnetic field. The vacuum Rabi splitting of cyclotron resonance exhibited a square-root dependence on the electron density, evidencing collective interaction. This splitting extended even where the detuning is larger than the resonance frequency. Furthermore, we observed a peak shift due to the normally negligible diamagnetic term in the Hamiltonian. Finally, the high-quality-factor cavity suppressed superradiant cyclotron resonance decay, revealing a narrow intrinsic linewidth of 5.6 GHz. High-quality-factor terahertz cavities will enable new experiments bridging the traditional disciplines of condensed-matter physics and cavity-based quantum optics.**

nonresonant matter decay rate, which is usually the decoherence rate in the case of solids. Strong coupling is achieved when the splitting, $2g$, is much larger than the linewidth, $(\kappa+\gamma)/2$, and ultrastrong coupling is achieved when $g$ becomes a considerable fraction of $\omega_0$. The two standard figures of merit to measure the coupling strength are $C \equiv 4g^2/(\kappa\gamma)$ and $g/\omega_0$; here, $C$ is called the cooperativity parameter[18], which is also the determining factor for the onset of optical bistability through resonant absorption saturation[20]. To maximize $C$ and $g/\omega_0$, one should construct a cavity QED set-up that combines a large dipole moment (that is, large $g$), a small decoherence rate (that is, small $\gamma$), a large cavity $Q$ factor (that is, small $\kappa$), and a small resonance frequency $\omega_0$.

Group III–V semiconductor quantum wells (QWs) provide one of the cleanest and most tunable solid-state environments with quantum-designable optical properties. Microcavity QW-exciton-polaritons represent a landmark realization of a strongly coupled light–condensed-matter system that exhibits a rich variety of coherent many-body phenomena[21]. However, the large values of $\omega_0$ and relatively small dipole moments for interband transitions make it

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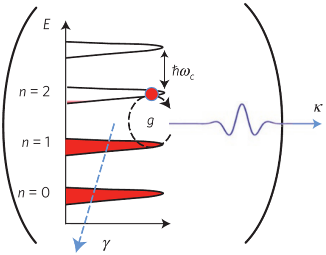
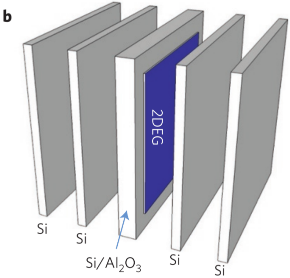
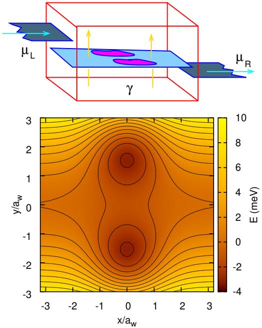
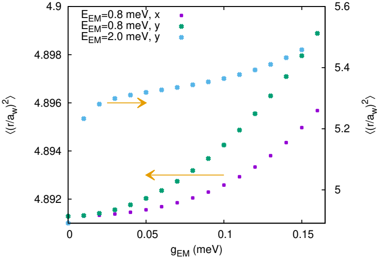


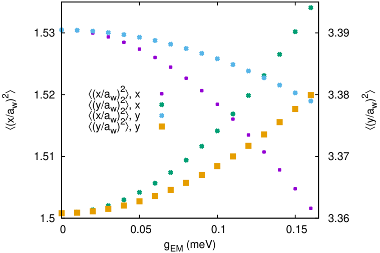




- 2DEG in GaAs-AlGaAs heterostructure
- FIR photon cavity
- External magnetic field

Gudmundsson et al. Ann. Phys. **528**, No. 5, 394–403 (2016)

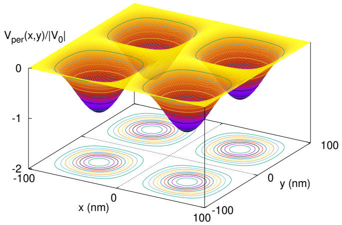


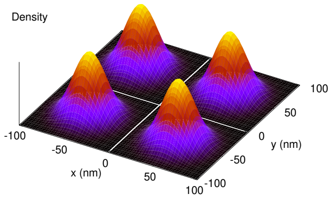




- Exact diagonalization, one photon mode
- $\hbar\omega = 0.8$ meV
- 2 electrons, first photon replica
- **Polarizability**

# Large electron system – 2DEG




- No exact diagonalization possible

  ↓

- QED + DFT = QEDFT
- Use and adapt functional: $E_{\mathrm{xc}}^{\mathrm{GA}}[n_e, \boldsymbol{\nabla} n_e]$, proposed by Johannes Flick, Simple Exchange-Correlation Energy Functionals for Strongly Coupled Light-Matter Systems based on the Fluctuation-Dissipation Theorem (2021), arXiv:2104.06980 [physics.chem-ph]

# Orbital magnetization is sensitive to charge polarizability

- Test for effects on orbital magnetization, $M_o$, of a 2DEG in a quantum dot array $\leftrightarrow$ ground state property

$$M_o + M_s = \frac{1}{2c\mathcal{A}} \int_{\mathcal{A}} d\boldsymbol{r} \ (\mathbf{r} \times \mathbf{j}(\mathbf{r})) \cdot \hat{\mathbf{e}}_z - \frac{g^* \mu_B^*}{\mathcal{A}} \int_{\mathcal{A}} d\boldsymbol{r} \, \sigma_z(\mathbf{r})$$

- EM-field randomly polarized in the 2DEG plane
- External magnetic field, $\boldsymbol{B} \neq 0$
- $\mathcal{A} = L^2$, $L = 100$ nm
- Preprint: https://doi.org/10.48550/arXiv.2203.11029

# Model and EM functional

$$H = H_0 + H_{\mathrm{Zee}} + V_{\mathrm{H}} + V_{\mathrm{per}} + V_{\mathrm{xc}} + V_{\mathrm{xc}}^{\mathrm{EM}}$$

$$E_{\mathrm{xc}}^{\mathrm{GA}}[n_e, \boldsymbol{\nabla} n_e] = \frac{1}{16\pi} \sum_{\alpha=1}^{N_p} |\lambda_\alpha|^2 \int d\boldsymbol{r} \frac{\hbar\omega_p(\boldsymbol{r})}{\sqrt{(\hbar\omega_p(\boldsymbol{r}))^2/3 + (\hbar\omega_g(\boldsymbol{r}))^2} + \hbar\omega_\alpha}$$

$$\left(\hbar\omega_g\right)^2 = C \left| \frac{\boldsymbol{\nabla} n_e}{n_e} \right|^4 \frac{\hbar^2}{m^{*2}}$$

$$\left(\hbar\omega_p(q)\right)^2 = (\hbar\omega_c)^2 + \frac{2\pi n_e^2}{m^*\kappa} q + \frac{3}{4} v_{\mathrm{F}}^2 q^2$$

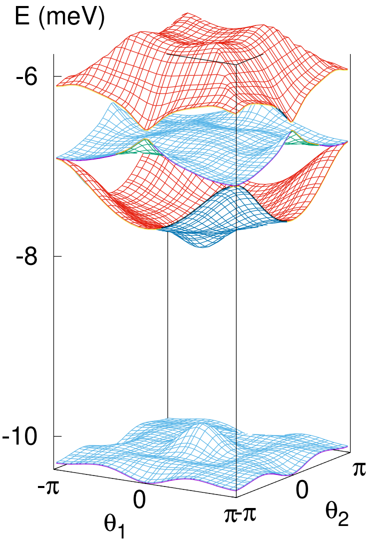
$$\omega_c = \left(\frac{eB}{m^* c}\right), \quad l^2 = \left(\frac{hc}{eB}\right)$$

Select $N_p = 1$, $\hbar\omega_\alpha = 1.0$ meV, $L = 100$ nm, $m^* = 0.067 m_e$, $\kappa = 12.4$, $g^* = 0.44$, and $q \approx k_{\mathrm{F}}/6 \approx |\boldsymbol{\nabla} n_e|/n_e$. $\lambda_\alpha l$ is measured in meV$^{1/2}$

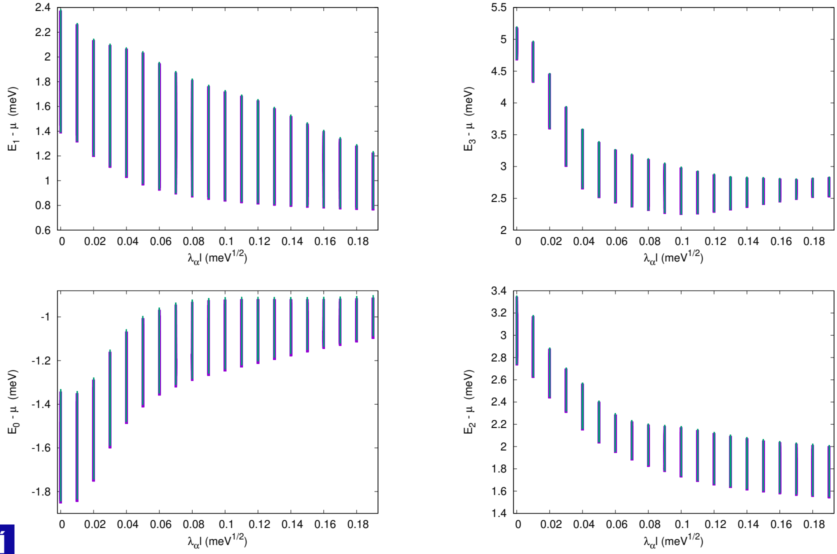
# Commensurability

- $L$ and $l$ are competing length scales - Hofstadter problem (Phys. Rev. B **14**, 2239 (1976))
- Magnetic flux through unit cell: $B\mathcal{A} = pq\Phi_0$, $\Phi_0 = hc/e$, $p, q \in \boldsymbol{N}$

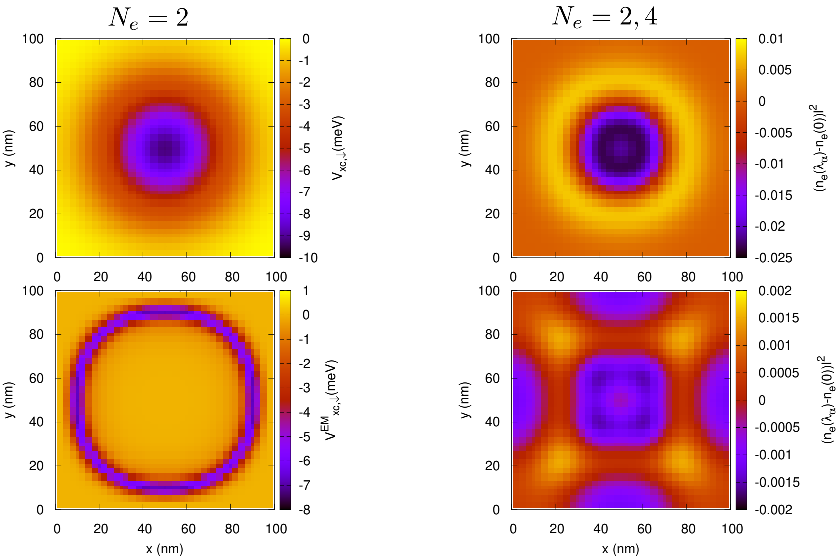
$N_e = 2$, $pq = 1$ $\rightarrow$
$\lambda_\alpha l = 0.050$ meV$^{1/2}$
$\mu = -8.954$ meV
$T = 1.0$ K
$\hbar\omega_\alpha = 1.0$ meV
$E_{\rm Zee} = 1.053 \times 10^{-2}$ meV



# Polaritons emerge, $pq = 1$



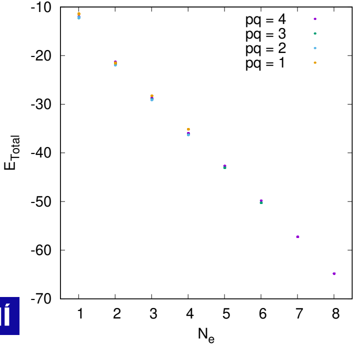
$V_{\rm xc}$, $V_{\rm xc}^{\rm EM}$, $[n_e(\lambda_\alpha) - n_e(0)]$, $pq = 4$, $\lambda_\alpha l = 0.050$ meV$^{1/2}$

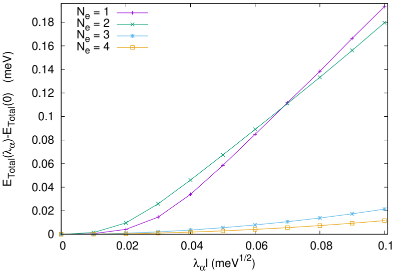


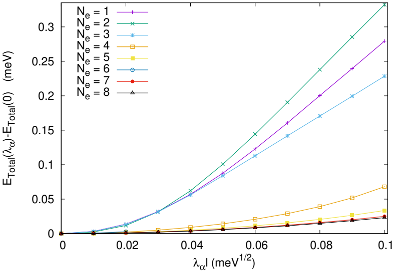
# Total energy

$pq = 1, 4$

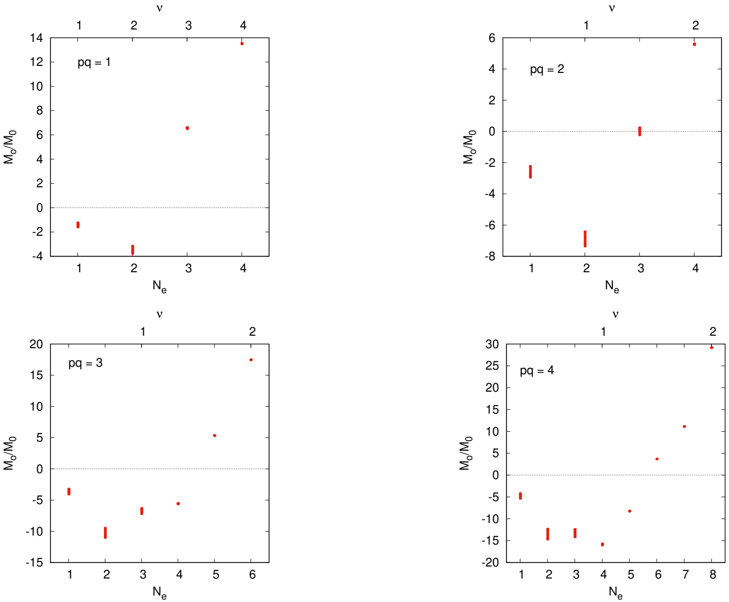
$\lambda_\alpha l = 0 \to 0.1 \text{ meV}^{1/2}$



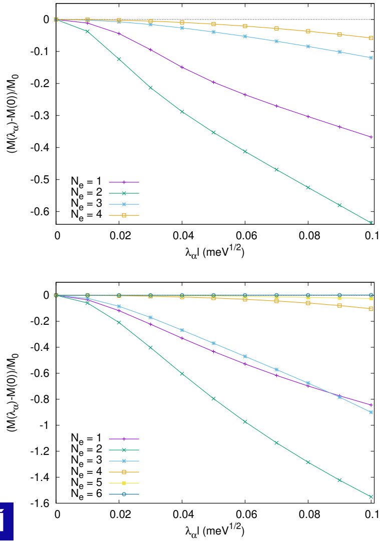



# Orbital magnetization, $M_0 = \mu_B^*/L^2$, $\lambda_\alpha l = 0 \to 0.1$ meV$^{1/2}$

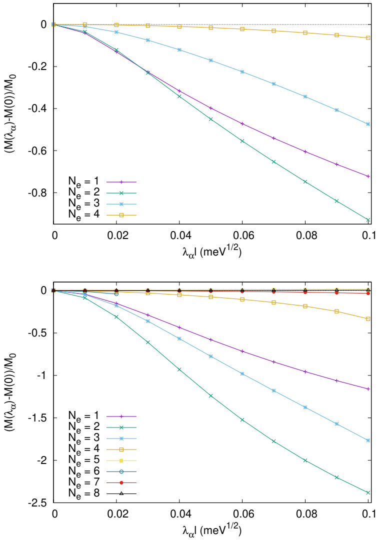


# Cavity-photon influence on orbital magnetization

$pq = 1, 3$




$pq = 2, 4$



# Summary

- QEDFT (GGA), 2DEG
- Electron polarizability
- External magnetic field
- Orbital magnetization, total energy
- Cavity-photon, bandstructure and lattice effects
- Preprint:
  *https://doi.org/10.48550/arXiv.2203.11029*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