Transport of electrons through a photon cavity

Viðar Guðmundsson

Science Institute, University of Iceland, Iceland

vidar@raunvis.hi.is

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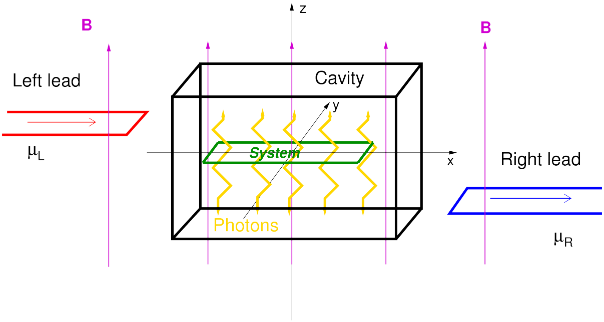
Cooperation

- Andrei Manolescu
- Valeriu Moldoveanu
- Chi-Shung Tang (唐志雄)
- Hsi-Sheng Goan (管希聖)
- Ólafur Jónasson
- Thorsten Arnold
- Nzar Rauf Abdullah
- Kristinn Torfason
- Pei-yi Lin (林佩儀)

- New Journal of Physics 11, 073019 (2009)
- New Journal of Physics 11, 113007 (2009)
- Phys. Rev. B81, 155442 (2010)
- Phys. Rev. B81, 205319 (2010)
- New Journal of Physics, 14, 013036 (2012)
- Phys. Rev. B85, 075306 (2012)
- (arXiv:1203.3048), (arXiv:1203.5980), (arXiv:1207.6797)

Approach

$$H(t) = \underbrace{\underbrace{\underbrace{H_{\rm e} + H_{\rm Coul}}_{\text{diagonalize+cut}} + H_{\rm EM} + H_{\rm e-EM}}_{\text{diagonalize+cut}\rightarrow H_{\rm S}} + H_{\rm LR} + H_{\rm T}(t)}_{\text{project on system}(H_{\rm S})\rightarrow\text{GME}}$$



e-EM coupling

Full electron-photon coupling

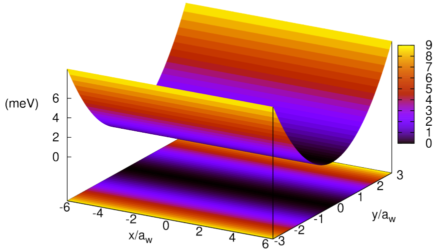
$$
\begin{aligned}
&\int d\mathbf{r}\ \psi^\dagger \left\{ \frac{1}{2m^*} \left(-i\hbar\nabla + \frac{e}{c} \left[\mathbf{A} + \mathbf{A}_{\mathrm{ext}} \right] \right)^2 \right\} \psi \\
&= \int d\mathbf{r}\ \psi^\dagger \left\{ \frac{1}{2m^*} \left(-i\hbar\nabla + \frac{e}{c}\mathbf{A} \right)^2 \right\} \psi \\
&- \frac{1}{c} \int d\mathbf{r}\ \mathbf{j} \cdot \mathbf{A}_{\mathrm{ext}} - \frac{e^2}{2m^* c} \int d\mathbf{r}\ \rho A_{\mathrm{ext}}^2 \\
&= H_{\mathrm{e}} + H_{\mathrm{e-EM}}
\end{aligned}
$$

$$
\mathbf{j} = -\frac{e}{2m^*} \left\{ \psi^\dagger \left(\boldsymbol{\pi}\psi \right) + \left(\boldsymbol{\pi}^* \psi^\dagger \right) \psi \right\}
$$

$$
\rho = -e\psi^\dagger\psi, \qquad \boldsymbol{\pi} = \left(\mathbf{p} + \frac{e}{c}\mathbf{A}_{\mathrm{ext}} \right)
$$

Central system

- Finite parabolic quantum wire $L_x = 300$ nm
- GaAs parameters



- External perpendicular magnetic field, $\mathbf{B} = B\hat{\mathbf{z}}$
  Confinement energy in y-direction $\hbar\Omega_0 = 1.0$ meV
- Semi-infinite leads in magnetic field, parabolic y-confinement

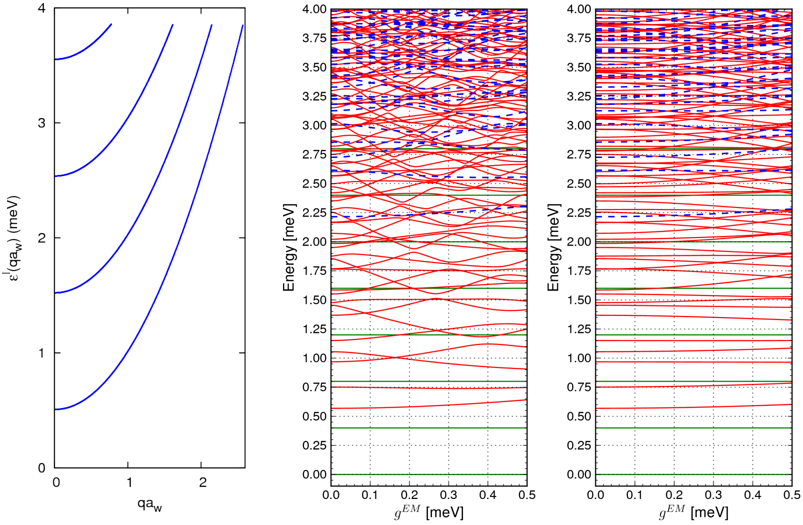
Single cavity mode

$$H_{\rm S} = \sum_a E_a d_a^\dagger d_a + \hbar\nu a^\dagger a + \frac{1}{2}\sum_{abcd}\langle ab|\, V_{\rm Coul}|cd\rangle\, d_a^\dagger d_b^\dagger d_d d_c$$
$$+ \sum_{ab} d_a^\dagger d_b \left\{ g_{ab} a + g_{ab}^* a^\dagger \right\}$$

$$H_{\rm S} = \sum_\mu |\mu)\tilde{E}_\mu(\mu| + \hbar\omega a^\dagger a + g^{\rm EM}\sum_{\mu\nu ij}|\mu)\langle\mu|\mathcal{V}^+ d_i^\dagger d_j \mathcal{V}|\nu\rangle(\nu|\; g_{ij}\left\{a + a^\dagger\right\}$$
$$+ g^{\rm EM}\left(\frac{g^{\rm EM}}{\hbar\Omega_w}\right)\sum_{\mu\nu i}|\mu)\langle\mu|\mathcal{V}^+ d_i^\dagger d_i \mathcal{V}|\nu\rangle(\nu| \left\{\left(a^\dagger a + \frac{1}{2}\right) + \frac{1}{2}\left(aa + a^\dagger a^\dagger\right)\right\}$$

$$\underbrace{|\mu) \otimes |N_{\rm ph}\rangle \longrightarrow |\mu)_{\rm e-EM}}_{\text{diagonalization}}$$

Energy spectra, leads, e-EM central system x- and y-pol., $\hbar\nu = 0.4$ meV, $B = 0.1$ T



Opening the system to the leads $\rightarrow$ GME

- Weak coupling to leads
- Non-Markovian
- Memory effects
- $\mathcal{P} = \rho_L \rho_R \mathrm{Tr}_{LR}$
- $H_{\mathrm{T}}^l(t) = \chi^l(t) \sum_{q,a} \left\{ T_{qa}^l d_{ql}^\dagger c_a + (T_{qa}^l)^* c_a^\dagger d_{ql} \right\}$
- Reduced statistical operator $\rho_{\mathrm{S}}(t) = \mathcal{P}\{W(t)\}$

Liouville-von Neumann equation

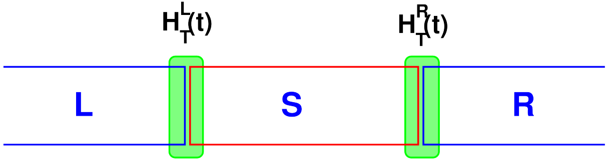
$$\dot{W}(t) = -\frac{i}{\hbar}[H(t), W(t)] = -i\mathcal{L}W(t)$$

$$\langle A(t) \rangle = \mathrm{Tr}\{W(t)A\} = \mathrm{Tr}_{\mathrm{S}}\{\rho_{\mathrm{S}}(t)A\}$$

$$i\hbar\dot{\rho}_{\mathrm{S}}(t) \approx \underbrace{\mathcal{L}_S \rho_{\mathrm{S}}(t)}_{\text{closed system}} + \underbrace{\frac{1}{i\hbar}\mathrm{Tr}_{LR}\left\{\mathcal{L}_T(t)\int_0^t ds\, e^{-i(t-s)\mathcal{L}_0}\mathcal{L}_T(s)\rho_L\rho_R\rho_{\mathrm{S}}(s)\right\}}_{\text{dissipation, memory}}$$

Coupling Hamiltonian

Contact area



Coupling tensor with sensitivity to geometry

$$T^l_{aq} = \int_{\Omega^l_S \times \Omega_l} d\mathbf{r}\, d\mathbf{r}' \left(\psi^l_q(\mathbf{r}')\right)^* \psi^S_a(\mathbf{r}) g^l_{aq}(\mathbf{r}, \mathbf{r}')$$

Nonlocal overlap

$$g^l_{aq}(\mathbf{r}, \mathbf{r}') = g^l_0 \exp\left[-\delta^l_1(x-x')^2 - \delta^l_2(y-y')^2\right] \exp\left(\frac{-|E_a - \epsilon^l(q)|}{\Delta^l_E}\right)$$

Basis transformations

Transform the coupling tensor into the Coulomb interacting many-electron basis $\{|\mu)\}$ and the electron-photon basis $\{|\breve{\mu})\}$

$$\tilde{\mathcal{T}}^l(q) = \mathcal{W}^+\mathcal{V}^+\mathcal{T}^l(q)\mathcal{V}\mathcal{W}, \quad \left(\tilde{\mathcal{T}}^l(q)\right)^* = \mathcal{W}^+\mathcal{V}^+\left(\mathcal{T}^l(q)\right)^*\mathcal{V}\mathcal{W}$$

and the rest of the Hamiltonian after each diagonalization

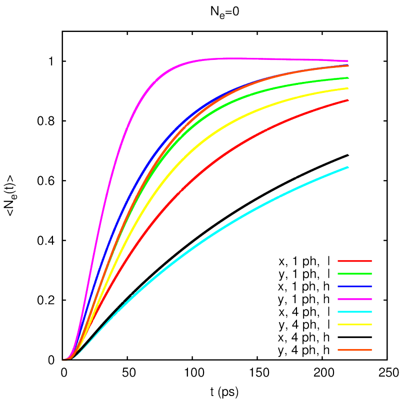
- Fock space construction and truncation schemes

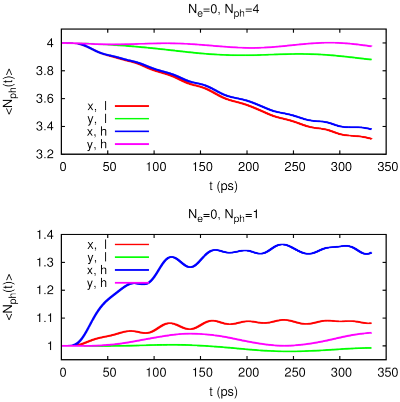
Step by step guide

- http://hartree.raunvis.hi.is/~vidar/Nam/TE/GME-1.pdf
- http://hartree.raunvis.hi.is/~vidar/Nam/TE/GME-2.pdf
- http://hartree.raunvis.hi.is/~vidar/Nam/TE/GME-3.pdf
- http://hartree.raunvis.hi.is/~vidar/Nam/TE/GME-4.pdf

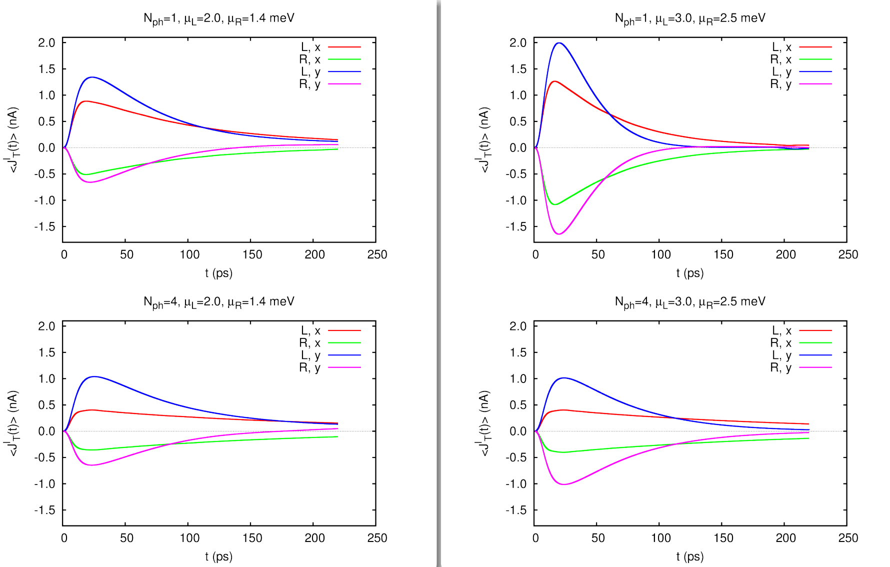
Empty system charged

$\hbar\Omega_0 = 1.0$ meV, $\hbar\omega = 0.4$ meV, $g^{\rm EM} = 0.1$ meV, $B = 0.1$ T,
18 SES, 64 MES + 27 photons $\rightarrow$ 64 MBS, no spin



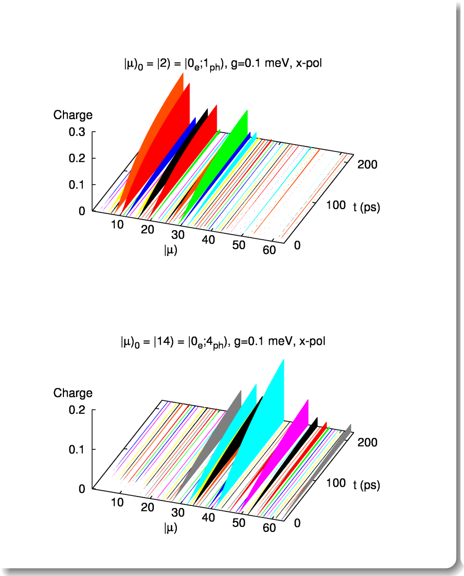


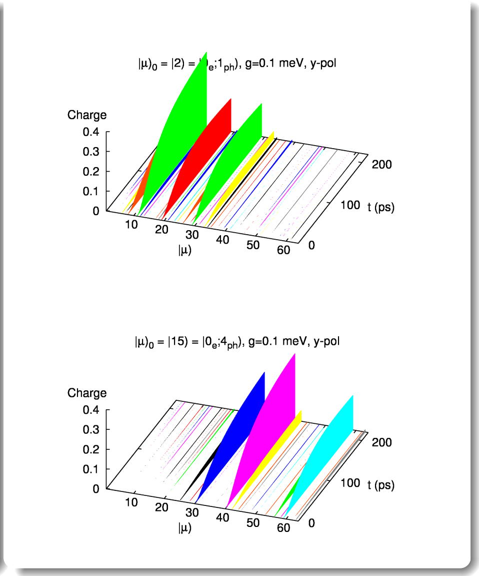
Charging current



Charging of many-body states

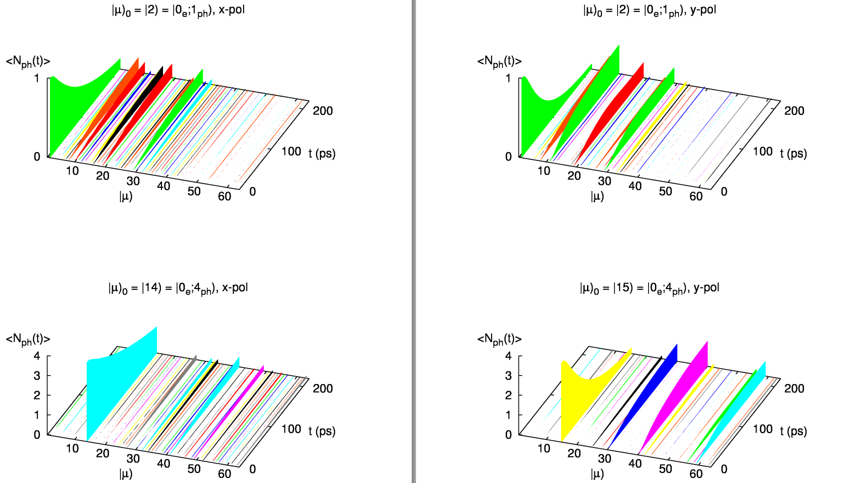
$\mu_L = 2.0$ meV, $\mu_R = 1.4$ meV $\leftrightarrow$ $|\breve{11})$ and $|\breve{21})$





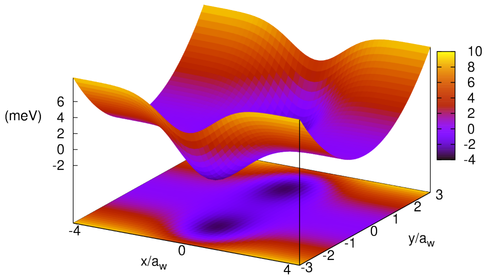
Photon content of many-body states

$\mu_L = 2.0$ meV, $\mu_R = 1.4$ meV $\leftrightarrow$ $|\breve{11})$ and $|\breve{21})$

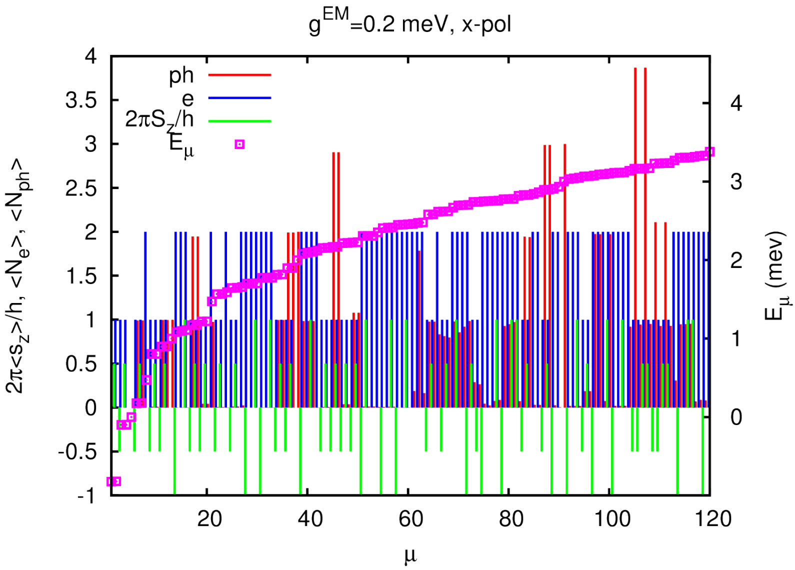


Spin and details

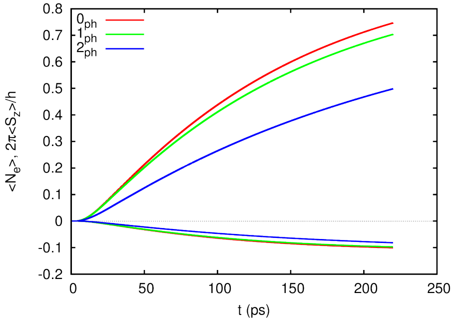
- Spin
- Parallel dots
- $\hbar\Omega_0 = 2.0$ meV
  $\hbar\omega = 1.0$ meV
  $g^{\mathrm{EM}} = 0.2$ meV
  $g = 0.44$
- 36 SES, 120 MES
- 120 MBS

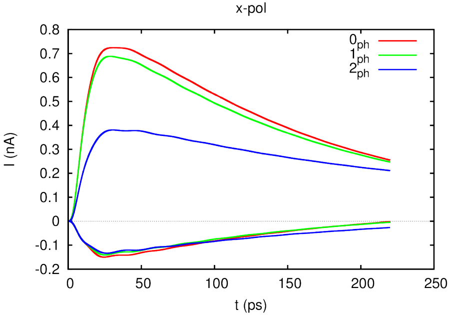


Closed system



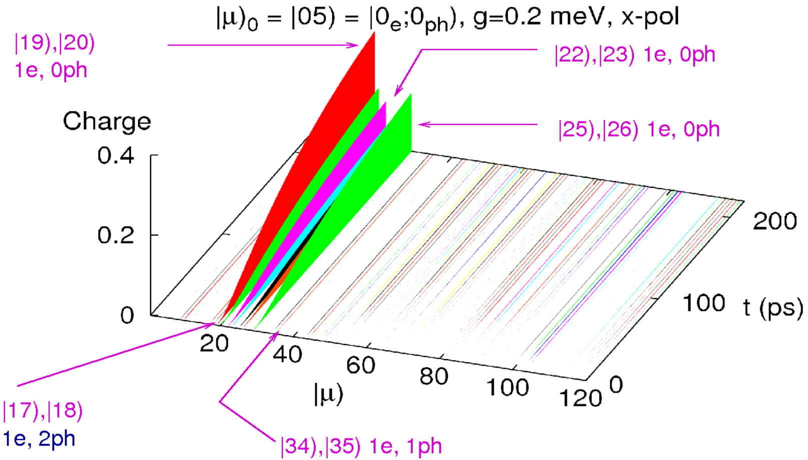
Electron transport



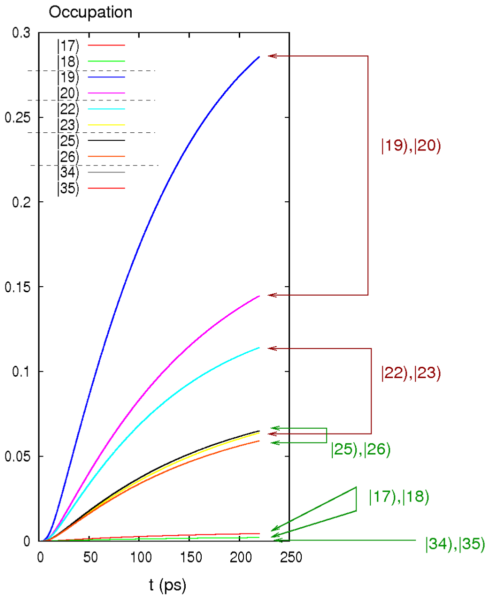


Multi-photon processes

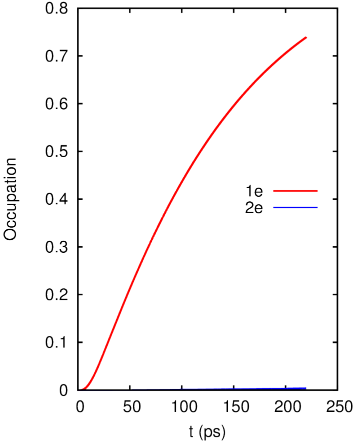
Bias window: |11) --- |21) **Spin pairs**



Spin



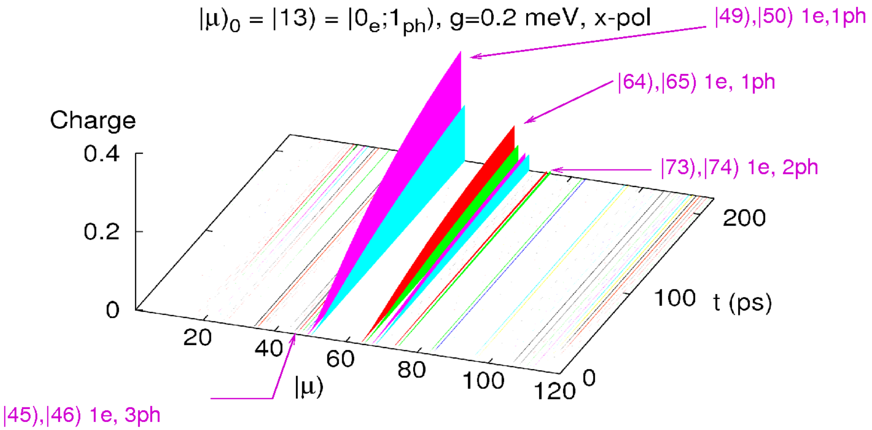
Occupation
0.3
0.25
0.2
0.15
0.1
0.05
0
|17)
|18)
|19)
|20)
|22)
|23)
|25)
|26)
|34)
|35)
0
50
100
150
200
250
t (ps)
|19),|20)
|22),|23)
|25),|26)
|17),|18)
|34),|35)



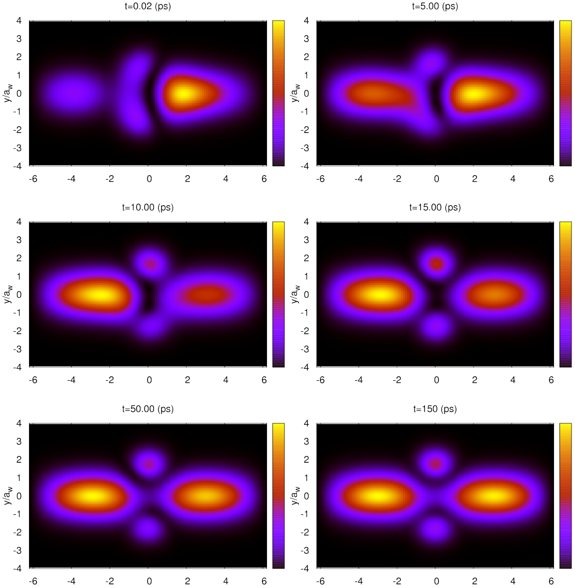
Occupation
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1e
2e
0
50
100
150
200
250
t (ps)

Multi-photon processes

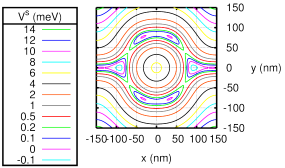
Spin pairs

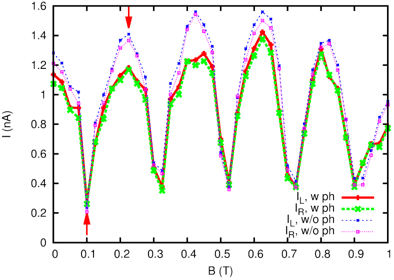


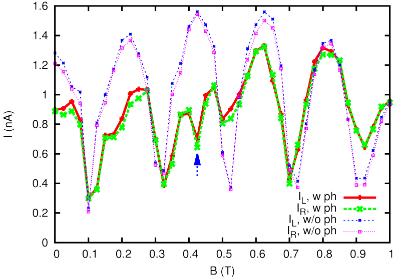
Time-dependent charge



Photon attenuated and assisted transport







Thorsten Arnold, http://arxiv.org/abs/1209.2888

Summary

- Non markovian time-dependent electron transport
- Weak lead-system coupling – strong coupling to a cavity photon mode
- Numerically exact Coulomb and one-photon-mode interaction
- Finite bias, beyond linear response
- Geometrical effects, external homogeneous magnetic field
- Many-body correlations of photons and electrons with spin
- Freedom in choosing initial states
- Parallelization for CPU's and GPU's