Orbital magnetization of an array of quantum dots in a photon cavity

Viðar Guðmundsson

Science Institute, University of Iceland

vidar@hi.is

Nordic virtual condensed matter seminar
https://vidargudmundsson.org/Rann/Fyrirlestrar/NVCMS-2023.pdf

April 28, 2023

Experimental impetus. . .

nature physics

LETTERS

PUBLISHED ONLINE: 22 AUGUST 2016 | DOI: 10.1038/NPHYS3850

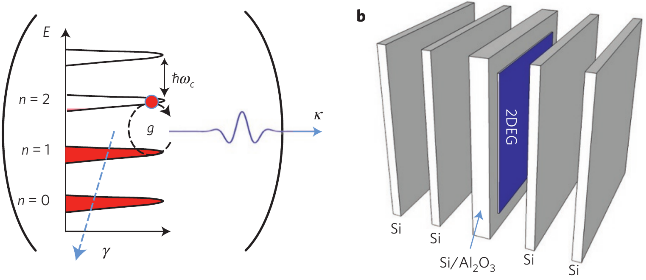
Collective non-perturbative coupling of 2D electrons with high-quality-factor terahertz cavity photons

Qi Zhang[1], Minhan Lou[1], Xinwei Li[1], John L. Reno[2], Wei Pan[3], John D. Watson[4], Michael J. Manfra[4,5] and Junichiro Kono[1,6,7]★

The collective interaction of electrons with light in a high-quality-factor cavity is expected to reveal new quantum phenomena[1-7] and find applications in quantum-enabled technologies[8,9]. However, combining a long electronic coherence time, a large dipole moment, and a high quality-factor has proved difficult[10-13]. Here, we achieved these conditions simultaneously in a two-dimensional electron gas in a high-quality-factor terahertz cavity in a magnetic field. The vacuum Rabi splitting of cyclotron resonance exhibited a square-root dependence on the electron density, evidencing collective interaction. This splitting extended even where the detuning is larger than the resonance frequency. Furthermore, we observed a peak shift due to the normally negligible diamagnetic term in the Hamiltonian. Finally, the high-quality-factor cavity suppressed superradiant cyclotron resonance decay, revealing a narrow intrinsic linewidth of 5.6 GHz. High-quality-factor terahertz cavities will enable new experiments bridging the traditional disciplines of condensed-matter physics and cavity-based quantum optics.

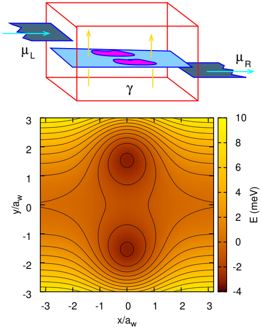
nonresonant matter decay rate, which is usually the decoherence rate in the case of solids. Strong coupling is achieved when the splitting, $2g$, is much larger than the linewidth, $(\kappa+\gamma)/2$, and ultrastrong coupling is achieved when g becomes a considerable fraction of ω_0. The two standard figures of merit to measure the coupling strength are $C \equiv 4g^2/(\kappa\gamma)$ and g/ω_0; here, C is called the cooperativity parameter[18], which is also the determining factor for the onset of optical bistability through resonant absorption saturation[20]. To maximize C and g/ω_0, one should construct a cavity QED set-up that combines a large dipole moment (that is, large g), a small decoherence rate (that is, small γ), a large cavity Q factor (that is, small κ), and a small resonance frequency ω_0.

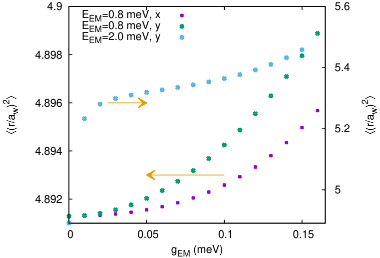
Group III–V semiconductor quantum wells (QWs) provide one of the cleanest and most tunable solid-state environments with quantum-designable optical properties. Microcavity QW-exciton-polaritons represent a landmark realization of a strongly coupled light–condensed-matter system that exhibits a rich variety of coherent many-body phenomena[21]. However, the large values of ω_0 and relatively small dipole moments for interband transitions make it

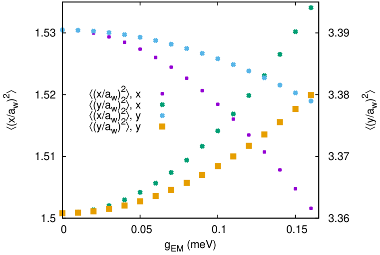


- 2DEG in GaAs-AlGaAs heterostructure
- FIR photon cavity
- External magnetic field

Gudmundsson et al. Ann. Phys. **528**, No. 5, 394–403 (2016)

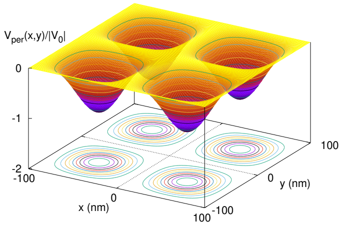


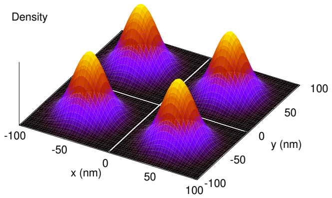




- Exact diagonalization, one photon mode
- $\hbar\omega = 0.8$ meV
- 2 electrons, first photon replica
- **Polarizability**

Large electron system – 2DEG





- No exact diagonalization possible

$\downarrow$

- QED + DFT = QEDFT
- Use and adapt functional: $E_{\mathrm{xc}}^{\mathrm{GA}}[n_e, \boldsymbol{\nabla} n_e]$, proposed by Johannes Flick, Simple Exchange-Correlation Energy Functionals for Strongly Coupled Light-Matter Systems based on the Fluctuation-Dissipation Theorem, *Phys. Rev. Lett.* **129**, 143201 (2022)

Orbital magnetization is sensitive to charge polarizability

- Test for effects on orbital magnetization, M_o, of a 2DEG in a quantum dot array $\leftrightarrow$ ground state property

$$
\begin{aligned}
M_o + M_s =& \frac{1}{2c\mathcal{A}} \int_{\mathcal{A}} d\boldsymbol{r} \; (\mathbf{r} \times \mathbf{j}(\mathbf{r})) \cdot \hat{\mathbf{e}}_z \\
& - \frac{g^* \mu_B^*}{\mathcal{A}} \int_{\mathcal{A}} d\boldsymbol{r} \, \sigma_z(\mathbf{r})
\end{aligned}
$$

- EM-field randomly polarized in the 2DEG plane
- External magnetic field, $\boldsymbol{B} \neq 0$
- $\mathcal{A} = L^2$, $L = 100$ nm
- *Phys. Rev. B* **106**, 115308 (2022)

Model and EM functional,(QED)

$$H = H_0 + H_{\rm Zee} + V_{\rm H} + V_{\rm per} + V_{\rm xc} + V_{\rm xc}^{\rm EM} \quad (\text{LSDA})$$

$$E_{\rm xc}^{\rm GA}[n_e, \boldsymbol{\nabla} n_e] = \frac{1}{16\pi}\sum_{\alpha=1}^{N_p}|\boldsymbol{\lambda}_\alpha|^2 \int d\boldsymbol{r} \frac{\hbar\omega_p(\boldsymbol{r})}{\sqrt{(\hbar\omega_p(\boldsymbol{r}))^2/3 + (\hbar\omega_g(\boldsymbol{r}))^2} + \hbar\omega_\alpha}$$

(AC-FDT)

$$(\hbar\omega_g)^2 = C\left|\frac{\boldsymbol{\nabla} n_e}{n_e}\right|^4 \frac{\hbar^2}{m^{*2}}$$

$$(\hbar\omega_p(q))^2 = (\hbar\omega_c)^2 + \frac{2\pi n_e^2}{m^*\kappa}q + \frac{3}{4}v_{\rm F}^2 q^2$$

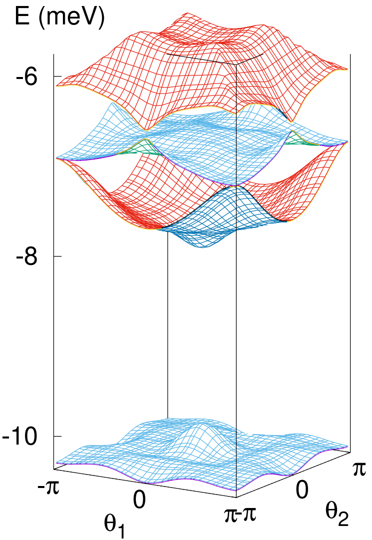
$$\omega_c = \left(\frac{eB}{m^*c}\right), \quad l^2 = \left(\frac{\hbar c}{eB}\right)$$

Select $N_p = 1$, $\hbar\omega_\alpha = 1.0$ meV, $L = 100$ nm, $m^* = 0.067m_e$, $\kappa = 12.4$, $g^* = 0.44$, and $q \approx k_{\rm F}/6 \approx |\boldsymbol{\nabla} n_e|/(6n_e)$. $\lambda_\alpha l$ is measured in meV$^{1/2}$

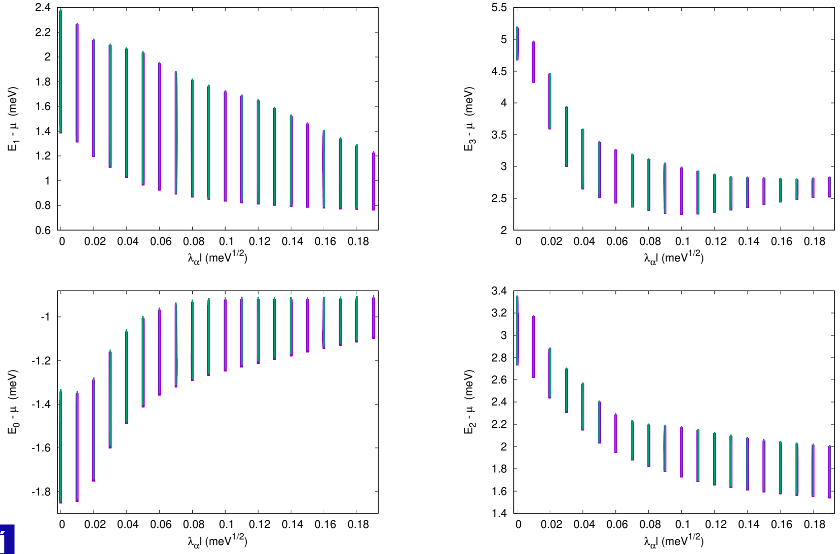
Commensurability

- L and l are competing length scales - Hofstadter problem: *Phys. Rev. B* **14**, 2239 (1976)
- Magnetic flux through unit cell: $B\mathcal{A} = pq\Phi_0$, $\Phi_0 = hc/e$, $p, q \in \boldsymbol{N}$

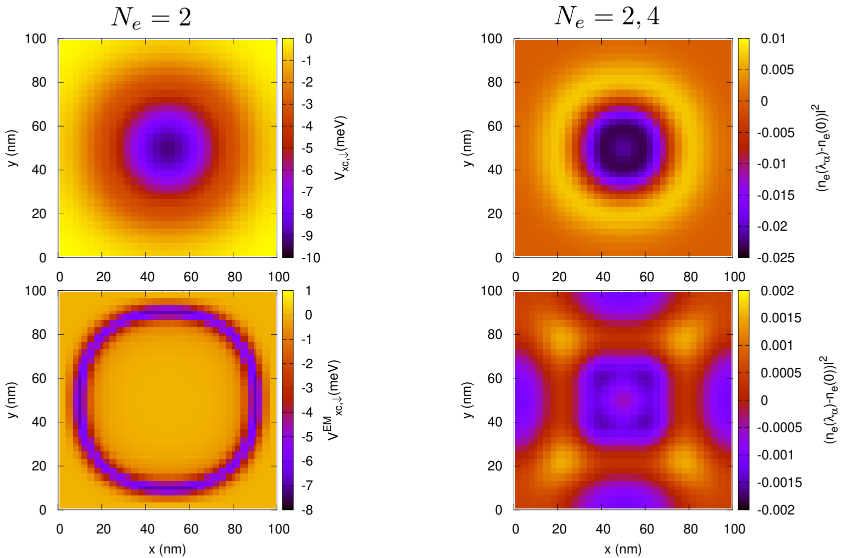
$N_e = 2$, $pq = 1$ $\rightarrow$
$\lambda_\alpha l = 0.050$ meV$^{1/2}$
$\mu = -8.954$ meV
$T = 1.0$ K
$\hbar\omega_\alpha = 1.0$ meV
$E_{\rm Zee} = 1.053 \times 10^{-2}$ meV



Polaritons emerge, $pq = 1$



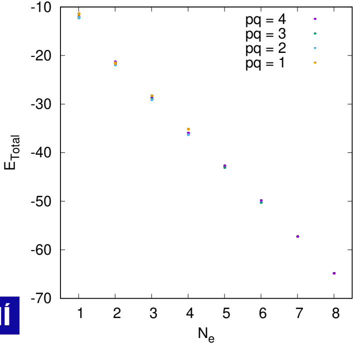
$V_{\rm xc}$, $V_{\rm xc}^{\rm EM}$, $[n_e(\lambda_\alpha) - n_e(0)]$, $pq = 4$, $\lambda_\alpha l = 0.050$ meV$^{1/2}$

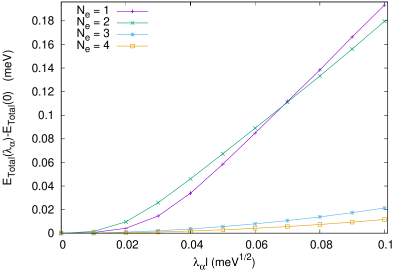


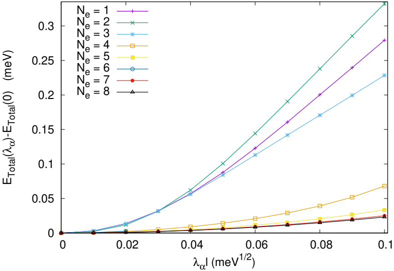
Total energy

$$pq = 1, 4$$

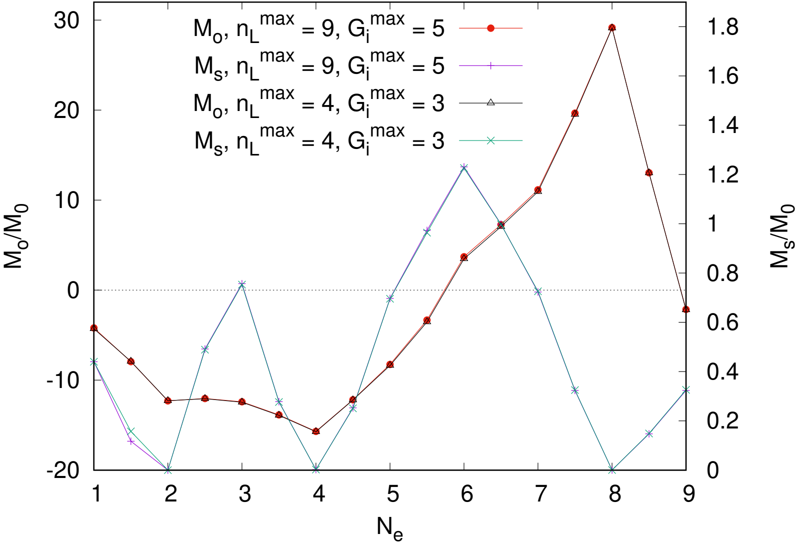
$$\lambda_\alpha l = 0 \to 0.1 \text{ meV}^{1/2}$$



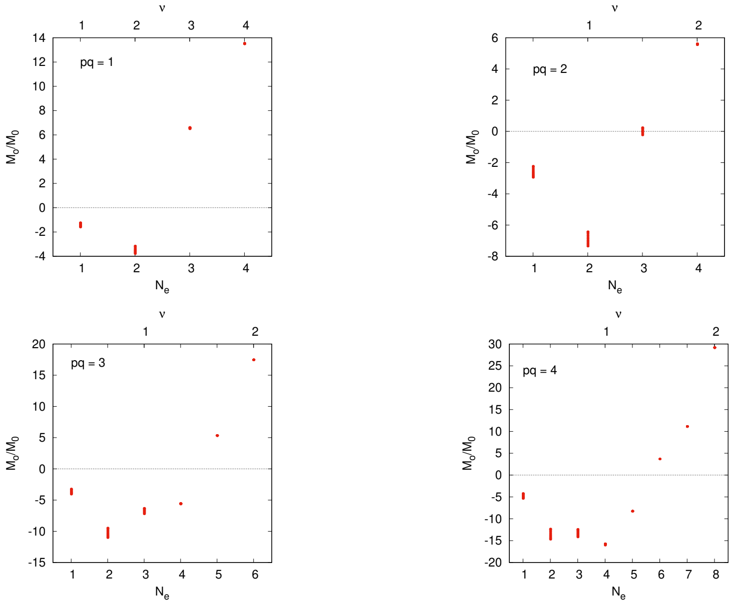




Orbital and spin magnetization, $\lambda_\alpha l = 0$, $pq = 4$

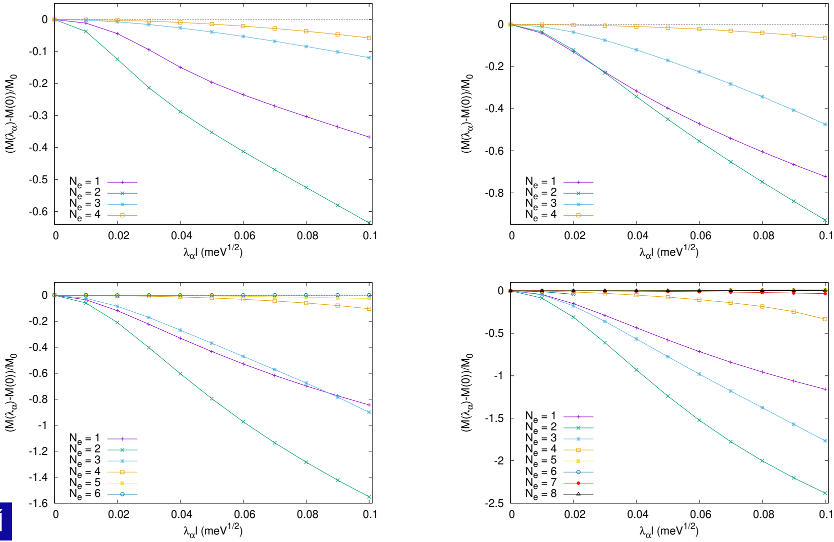


Orbital magnetization, $M_0 = \mu_{\mathrm{B}}^*/L^2$, $\lambda_\alpha l = 0 \to 0.1$ meV$^{1/2}$



Cavity-photon influence on orbital magnetization

$pq = 1, 3$ $\qquad$ $pq = 2, 4$



Summary

- QEDFT (GGA), 2DEG
- Electron polarizability
- External magnetic field
- Orbital magnetization, total energy
- Cavity-photon, bandstructure and lattice effects
- *Phys. Rev. B* **106**, 115308 (2022)

- Andrei Manolescu (RU)
- Valeriu Moldoveanu (NIMP)
- Nzar Rauf Abdullah (US, KUST)
- Chi-Shung Tang (NUU)
- Vram Mughnetsyan (YSU)

Icelandic Infrastructure Fund, ihpc.is, UI, RU, RCP, MOST Taiwan, ASCS

Appendix QED

$$H = \int d\boldsymbol{r}\ \psi^\dagger(\mathbf{r}) \left\{ \frac{\pi^2}{2m^*} + V(\mathbf{r}) \right\} \psi(\mathbf{r})$$
$$+ H_{\mathrm{EM}} + H_{\mathrm{Coul}} + H_{\mathrm{Z}}$$
$$+ \frac{1}{c} \int d\boldsymbol{r}\ \mathbf{j}(\mathbf{r}) \cdot \mathbf{A}_\gamma + \frac{e^2}{2m^* c^2} \int d\boldsymbol{r}\ n_e(\mathbf{r}) A_\gamma^2$$

$$\mathbf{j} = -\frac{e}{2m^*} \left\{ \psi^\dagger \boldsymbol{\pi} \psi + \boldsymbol{\pi}^* \psi^\dagger \psi \right\}, \quad n_e = \psi^\dagger \psi$$

with

$$\boldsymbol{\pi} = \left(\mathbf{p} + \frac{e}{c} \mathbf{A}_{\mathrm{ext}} \right), \qquad \mathbf{A}_{\mathrm{ext}} = \frac{B}{2}(-y, x)$$

$\mathbf{A}_\gamma$: the cavity vector field

(back)

Appendix DFT - LSDA

$$n_e = n_\uparrow + n_\downarrow, \quad \zeta = (n_\uparrow - n_\downarrow)/n_e, \quad \nu(\mathbf{r}) = 2\pi l^2 n_e(\mathbf{r})$$

$$\epsilon_{\rm xc}^B(\nu,\zeta) = \epsilon_{\rm xc}^\infty(\nu)e^{-f(\nu)} + \epsilon_{\rm xc}^0(\nu,\zeta)(1-e^{-f(\nu)})$$

$$f(\nu) = (3\nu/2) + 7\nu^4, \quad \epsilon_{\rm xc}^\infty(\nu) = -0.782\sqrt{\nu}e^2/(\kappa l)$$

$$\epsilon_{\rm xc}^0(\nu,\zeta) = \epsilon_{\rm xc}(\nu,0) + f^i(\zeta)\left[\epsilon_{\rm xc}(\nu,1) - \epsilon_{\rm xc}(\nu,0)\right]$$

$\epsilon_{\rm xc}(\nu,\zeta) = \epsilon_{\rm x}(\nu,\zeta) + \epsilon_{\rm c}(\nu,\zeta)$, with $\epsilon_{\rm x}(\nu,0) = -[4/(3\pi)]\sqrt{\nu}e^2/(kl)$, and $\epsilon_{\rm x}(\nu,1) = -[4/(3\pi)]\sqrt{2\nu}e^2/(kl)$

$$f^i(\zeta) = \frac{(1+\zeta)^{3/2} + (1-\zeta)^{3/2} - 2}{2^{3/2} - 2}$$

$$\epsilon_c(\nu,\zeta) = a_0 \frac{1 + a_1 x}{1 + a_1 x + a_2 x^2 + a_3 x^3} Ry^*$$

$$x = \sqrt{r_s} = (2/\nu)^{1/4}(l/a_B^*)^{1/2}$$

$$V_{xc,\uparrow} = \frac{\partial}{\partial\nu}(\nu\epsilon_{xc}) + (1-\zeta)\frac{\partial}{\partial\zeta}\epsilon_{xc}$$

$$V_{xc,\downarrow} = \frac{\partial}{\partial\nu}(\nu\epsilon_{xc}) - (1+\zeta)\frac{\partial}{\partial\zeta}\epsilon_{xc}$$

B. Tanatar and D. M. Ceperley, *Phys. Rev. B* **39**, 5005 (1989)
U. von Barth and L. Hedin, *J. Phys. C* **5**, 1629 (1972)
J. P. Perdew and A. Zunger, *Phys. Rev. B* **23**, 5048 (1981)
M. I. Lubin, O. Heinonen, and M. D. Johnson, *Phys. Rev. B* **56**, 10373 (1997)
V. Gudmundsson, C.-S. Tang, and A. Manolescu, *Phys. Rev. B* **68**, 165343 (2003)
M. Koskinen, M. Manninen, and S. M. Reimann, *Phys. Rev. Lett.* **79**, 1389 (1997)

(back)

Appendix AC-FDT

J. Flick PRL 129, 143201 (2022), Dipole approximation:

$$\hat{H}_{\text{int}} = \sum_{\alpha=1}^{N_p} \frac{1}{2} \left\{ (\boldsymbol{\lambda}_\alpha \cdot \boldsymbol{R})^2 - \omega_\alpha \hat{q}_\alpha \boldsymbol{\lambda}_\alpha \cdot \boldsymbol{R} \right\}, \quad \hat{q}_\alpha = \sqrt{\frac{1}{2\omega_\alpha}} \left(\hat{a}_\alpha^\dagger + \hat{a}_\alpha \right)$$

$$\boldsymbol{R} = e \int d\boldsymbol{r}\, \boldsymbol{r}\, n_e(\boldsymbol{r}), \quad \boldsymbol{\lambda}_\alpha = 4\pi S(\boldsymbol{k}_\alpha \cdot \boldsymbol{R}) \hat{\boldsymbol{e}}_\alpha$$

$$U = \frac{1}{2} \iint d\boldsymbol{r} d\boldsymbol{r}' \left[\frac{e^2}{4\pi\epsilon} \frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|} + \sum_{\alpha=1}^{N_p} (\boldsymbol{\lambda}_\alpha \cdot \boldsymbol{r})(\boldsymbol{\lambda}_\alpha \cdot \boldsymbol{r}') \right] n(\boldsymbol{r}) n(\boldsymbol{r}') + \frac{1}{2} \sum_{\alpha=1}^{N_p} \int d\boldsymbol{r} (\boldsymbol{\lambda}_\alpha \cdot \boldsymbol{r})^2 n(\boldsymbol{r})$$

$$E_c^{(1)} = \frac{1}{2\pi}\int_0^1 d\gamma \int d\boldsymbol{r} \sum_{\alpha=1}^{N_p} \omega_\alpha (\boldsymbol{\lambda}_\alpha \cdot \boldsymbol{r}) \int d\omega \left[\chi_{n,\gamma}^{q_\alpha}(\boldsymbol{r}, i\omega) - \chi_{n,0}^{q_\alpha}(\boldsymbol{r}, i\omega)\right]$$

$$E_c^{(2)} = -\frac{1}{2\pi}\int_0^1 d\gamma \int d\boldsymbol{r} \left[\frac{e^2}{4\pi\epsilon}\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|} + \sum_{\alpha=1}^{N_p}(\boldsymbol{\lambda}_\alpha \cdot \boldsymbol{r})(\boldsymbol{\lambda}_\alpha \cdot \boldsymbol{r}')\right] \times \int d\omega \left[\chi_{n,\gamma}^{n}(\boldsymbol{r}, i\omega) - \chi_{n,0}^{n}(\boldsymbol{r}, i\omega)\right]$$

...

$$\alpha_{\mu,\nu}(i\omega) = 2\sum_{ia}\frac{(\epsilon_a - \epsilon_e)\langle\phi_a|r_\mu|\phi_i\rangle\langle\phi_i|r_\nu|\phi_a\rangle}{(\epsilon_a-\epsilon_i)^2+\omega^2}$$

$$\longrightarrow \alpha(i\omega) = \frac{1}{4\pi}\int d\boldsymbol{r}\frac{\omega_p(\boldsymbol{r})}{\omega_p^2(\boldsymbol{r})/3 + \omega_g^2(\boldsymbol{r}) + \omega^2}$$

(compare to G. Mahan, chapter 4 for the last step)

(back)