Time-dependent transport of electrons through nanosystems in a photon cavity

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Macroscopic low temperature assisted transport...

Experimental impetus. . .

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Cavity Photons as a Probe for Charge Relaxation Resistance and Photon Emission in a Quantum Dot Coupled to Normal and Superconducting Continua

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Microwave cavities have been widely used to investigate the behavior of closed few-level systems. Here, we show that they also represent a powerful probe for the dynamics of charge transfer between a discrete electronic level and fermionic continua. We have combined experiment and theory for a carbon nanotube quantum dot coupled to normal metal and superconducting contacts. In equilibrium conditions, where our device behaves as an effective quantum dot-normal metal junction, we approach a universal photon dissipation regime governed by a quantum charge relaxation effect. We observe how photon dissipation is modified when the dot admittance turns from capacitive to inductive. When the fermionic reservoirs are voltage biased, the dot can even cause photon emission due to inelastic tunneling to/from a Bardeen-Cooper-Schrieffer peak in the density of states of the superconducting contact. We can model these numerous effects quantitatively in terms of the charge susceptibility of the quantum dot circuit. This validates an approach that could be used to study a wide class of mesoscopic QED devices.

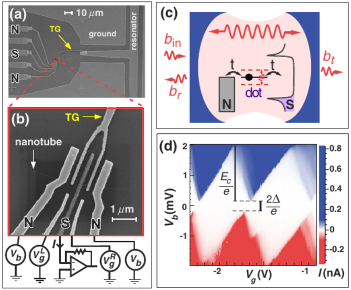
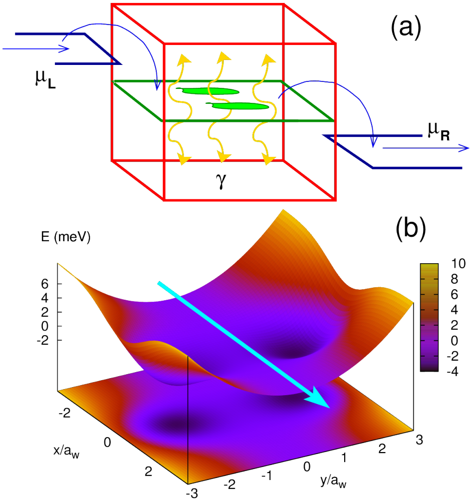


FIG. 1. Panels (a) and (b): Scanning electron micrograph of the microwave resonator and the quantum dot circuit. Panel (c): Principle of our setup. The dot level is tunnel coupled to the N and S reservoirs and modulated by the cavity electric field. Panel (d): Current through the S contact versus the effective gate voltage V_g and the bias voltage V_b.

Coupling to external fermionic reservoirs. . . ,
Gate voltage excitation, $V_{\rm rf}$, . . . ,
Photon pumping, $\langle N_\gamma \rangle \sim 120$ . . .

Transport of electrons through dots in a photon cavity



Exactly interacting electrons and photons, geometry

Time-dependent transport

$\Downarrow$

Time scales

$\Downarrow$

Transient – intermediate – long time – steady state

$\Uparrow$

Density operator

$\Uparrow$

Open Systems

Equation of motion

Liouville-von Neumann

$$\partial_t W = \mathcal{L}W, \quad \mathcal{L}W = -\frac{i}{\hbar}[H, W]$$

$$H = H_{\mathrm{S}} + H_{\mathrm{LR}} + H_{\mathrm{T}}(t), \quad H_{\mathrm{S}} = H_{\mathrm{e}} + H_{\mathrm{EM}}$$

$$\begin{aligned} H_{\mathrm{S}} = &\int d^2r \psi^\dagger(\mathbf{r}) \left\{ \frac{\pi^2}{2m^*} + V(\mathbf{r}) \right\} \psi(\mathbf{r}) + H_{\mathrm{Coul}} + \hbar\omega a^\dagger a \\ &- \frac{1}{c} \int d^2r \, \mathbf{j}(\mathbf{r}) \cdot \mathbf{A}_\gamma - \frac{e}{2m^* c^2} \int d^2r \, \rho(\mathbf{r}) A_\gamma^2 \end{aligned}$$

$$\boldsymbol{\pi} = \left(\mathbf{p} + \frac{e}{c} \mathbf{A}_{\mathrm{ext}} \right), \quad \rho = -e\psi^\dagger \psi, \quad \mathbf{j} = -\frac{e}{2m^*} \left\{ \psi^\dagger \left(\boldsymbol{\pi} \psi \right) + \left(\boldsymbol{\pi}^* \psi^\dagger \right) \psi \right\}$$

Quantized cavity field

$$\mathbf{A}(\mathbf{r}) = \begin{pmatrix} \hat{\mathbf{e}}_x \\ \hat{\mathbf{e}}_y \end{pmatrix} \mathcal{A} \left\{ a + a^\dagger \right\} \begin{pmatrix} \cos\left(\frac{\pi y}{a_\mathrm{c}}\right) \\ \cos\left(\frac{\pi x}{a_\mathrm{c}}\right) \end{pmatrix} \cos\left(\frac{\pi z}{d_\mathrm{c}}\right), \qquad \begin{matrix} \mathsf{TE}_{011}, & x\text{-pol.} \\ \mathsf{TE}_{101}, & y\text{-pol.} \end{matrix}$$

Projection on the central system

Reduced density operator

$$\rho_{\mathrm{S}}(t) = \mathcal{P}W(t) = \rho_{\mathrm{LR}}(0)\mathrm{Tr}_{\mathrm{LR}}\{W(t)\}$$

Liouville-von Neumann $\Rightarrow$ Nakajima-Zwanzig equation (to 2nd order in H_{T})

$$\partial_t \rho_{\mathrm{S}}(t) = \mathcal{L}_{\mathrm{S}}\rho_{\mathrm{S}}(t) + \int_0^t dt' K[t, t-t'; \rho_{\mathrm{S}}(t')]$$

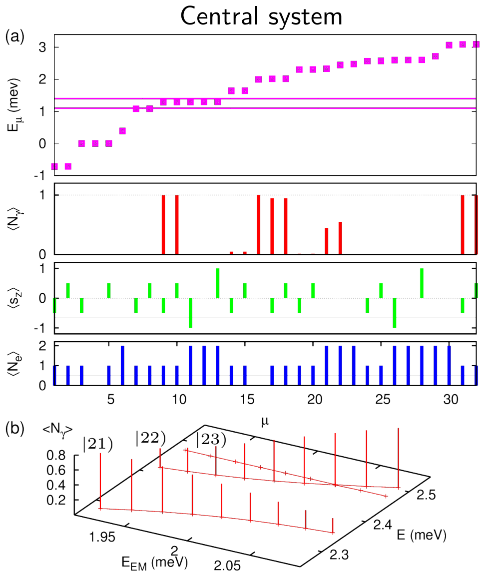
with

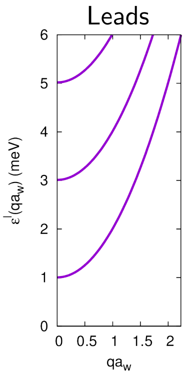
$$K[t, s; \rho_{\mathrm{S}}(t')] = \mathrm{Tr}_{\mathrm{LR}}\left\{\left[H_{\mathrm{T}}(t), \left[U(s)H_{\mathrm{T}}(t')U^{+}(s), U_{\mathrm{S}}(s)\rho_{\mathrm{S}}(t')U_{\mathrm{S}}^{+}(s)\rho_{\mathrm{L}}\rho_{\mathrm{R}}\right]\right]\right\} \tag{1}$$

and

$$H_{\mathrm{T}}(t) = \sum_{i,l}\chi(t)\int dq\ \left\{T_{qi}^{l}c_{ql}^{\dagger}d_i + (T_{qi}^{l})^{*}d_i^{\dagger}c_{ql}\right\} \tag{2}$$

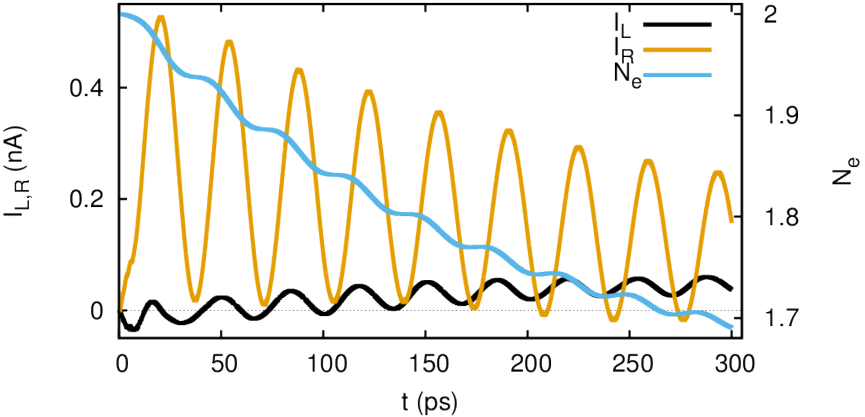
Spectrum of closed systems, y-polarized cavity photons





$$\hbar\omega = 2.0 \text{ meV}$$
$$g_{\mathrm{EM}} = 0.05 \text{ meV}$$
$$B = 0.1 \text{ T}$$
$$a_w = 23.8 \text{ nm}$$
$$V_g = 0.1 \text{ mV}$$

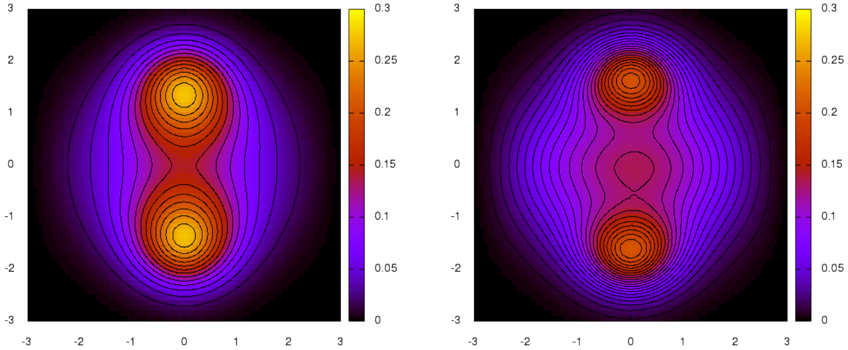
Rabi-oscillations seen in transport current



Initial state: fully entangled 2-e Rabi-split $|21)$ and $|22)$

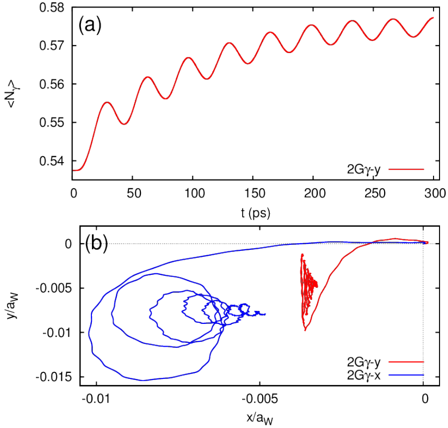
Charge density oscillations

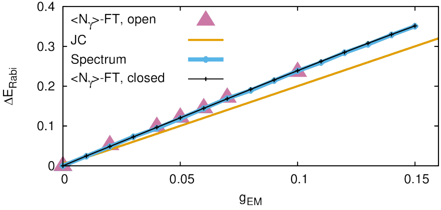
$t = 10$ ps $\qquad$ $t = 60$ ps



Variable probability in contact area $\rightarrow$ variable current

Consequences of geometry





Rabi-splitting

ACS Photonics **2**, 930 (2015)

Dynamical Hall effect

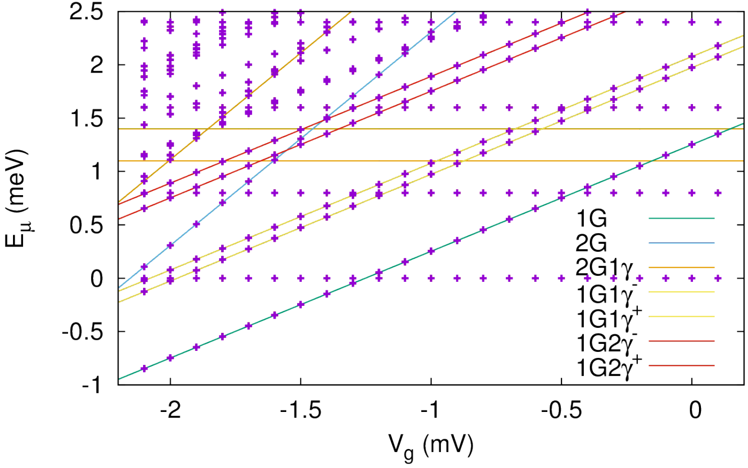
Questions

- What happens beyond 300 ps?
- How long time is needed to get 2 electrons into the system?
- Steady state?
- Are there different time-regimes?

$$\Updownarrow$$

- Time-integration not feasible
- Consider Markovian instead of non-Markovian system
- Continue with no rotating wave approximation
- Start with short quantum wire without embedded dots

Spectrum of closed system vs. plunger gate voltage V_g



x-polarization, $\hbar\omega = 0.8$ meV, $g_{\mathbf{EM}} = 0.05$ meV,
$\hbar\Omega = 2.0$ meV, $B = 0.1$ T

Nakajima-Zwanzig

$$\partial_t \rho = -\frac{i}{\hbar}\left[H_{\mathrm{S}}, \rho\right] - \sum_l \Lambda(\Omega_{ql}, \tau_{ql}, \chi_l, t)$$

with

$$\Lambda(\Omega_{ql}, \tau_{ql}, \chi_l, t) = \frac{1}{\hbar^2}\int dq\ \chi_l(t)\left\{\left[\tau_{ql}, \Omega_{ql}(t)\right] + h.c.\right\}$$

where,

$$\begin{aligned}\Omega_{ql}(t) = \int_0^t ds\ \chi(s) U_{\mathrm{S}}(t-s)\Big\{&\tau_{ql}^{\dagger}\rho(s)(1-f_{ql})\\ &-\rho(s)\tau_{ql}^{\dagger} f_{ql}\Big\} U_{\mathrm{S}}^{\dagger}(t-s) e^{i(s-t)\omega_{ql}}\end{aligned}$$

Change of variable $t - s \to s'$, set $\rho(t-s) \to \rho(t)$

use

$$\int_0^t ds \exp\left[is(E_\nu - E_\mu - \epsilon_{ql})\right] \to \pi\delta(E_\nu - E_\mu - \epsilon_{ql})$$

and

$$\begin{aligned}\int dqA(q)\delta(E_\alpha - E_\beta - \epsilon_{ql}) &= \int d\epsilon(dq/d\epsilon)A(\epsilon)\delta(E_\alpha - E_\beta - \epsilon) \\ &= A^{\alpha\beta}D^{\alpha\beta}\end{aligned}$$

$$\chi_l(t) \to \theta(t)$$

Leads to

$$\Omega_{\alpha\beta} = \left\{ \mathcal{R}[\rho]_{\alpha\beta} - \mathcal{S}[\rho]_{\alpha\beta} \right\} \delta^{\beta\alpha}$$

$$\mathcal{R}[\rho] = \rho\pi f \tau^{\dagger}, \quad \mathcal{S}[\rho] = \pi(1-f)\tau^{\dagger}\rho$$

Introduce

$$\Delta_{\alpha\beta} = \delta^{\alpha\beta} = \delta(E_\alpha - E_\beta - \epsilon)$$

to obtain

$$\mathcal{Z}_{\alpha\beta} = \int DA_{\alpha\lambda}\Omega_{\lambda\sigma}B_{\sigma\beta}d\delta^{\sigma\lambda}$$

$$\Downarrow$$

$$\mathcal{Z} = \int DA \left\{ (\mathcal{R}[\rho] - \mathcal{S}[\rho]) \underset{\uparrow}{\odot} d\Delta^T \right\} B$$

Hadamard product

Fock $\to$ Liouville space

Use vectorization and Kronecker tensor product

$$\mathrm{vec}(\mathbf{AXB}) = \left\{\mathbf{B}^T \otimes \mathbf{A}\right\} \mathrm{vec}(\mathbf{X})$$

$\dim$(Fock-space of states)$\sim N$

$\to \dim$(Liouville-space of transitions)$\sim N^2$

Markovian equation of motion

$$\partial_t \rho_{\mathrm{S}}^{\mathrm{vec}} = \mathcal{L}\rho_{\mathrm{S}}^{\mathrm{vec}}$$

where

$$\mathcal{L} = \left\{ -\frac{i}{\hbar}(I \otimes H - H^T \otimes I) + \sum_{X=R,S} (\mathfrak{Z}_{X_1}\mathfrak{Z}_{X_2}) \right\}$$

and

$$\mathfrak{Z}_{X_1} = \int \left(B^T \otimes DA\right) \mathrm{Diag}(\Delta^T), \qquad X = R, S$$

$$\mathfrak{Z}_{R_2} = \int \mathrm{Diag}(\Delta^T)(I \otimes R)$$

$$\mathfrak{Z}_{S_2} = -\int \mathrm{Diag}(\Delta^T)\left(S^T \otimes I\right)$$

with solution

$$\rho_{\mathrm{S}}^{\mathrm{vec}}(t) = \left[\mathcal{U} \exp\left(\mathcal{L}_{\mathrm{diag}} t\right) \mathcal{V}\right] \rho_{\mathrm{S}}^{\mathrm{vec}}(0)$$

where

$$\mathcal{L}\mathcal{V} = \mathcal{V}\mathcal{L}_{\mathrm{diag}}, \quad \mathcal{U}\mathcal{L} = \mathcal{L}_{\mathrm{diag}}\mathcal{U}, \quad \mathcal{U}\mathcal{V} = \mathcal{V}\mathcal{U} = \mathcal{I}$$

Steady state can be found as the eigenvalue 0 of

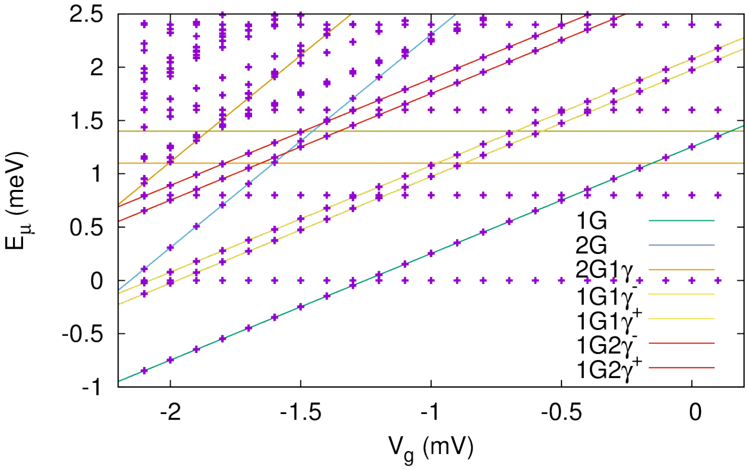
$$0 = \mathcal{L}\rho_{\mathrm{S}}^{\mathrm{vec}}$$

but we use

$$\lim_{t\to\infty} \left[\mathcal{U} \exp\left(\mathcal{L}_{\mathrm{diag}} t\right) \mathcal{V}\right] \rho_{\mathrm{S}}^{\mathrm{vec}}(0)$$

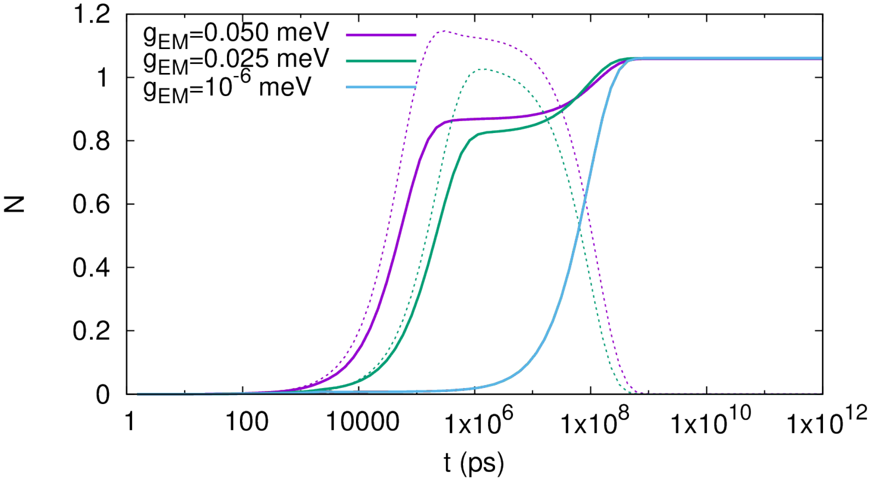
Here, $N = 120$, $V_g = -1.6$ mV

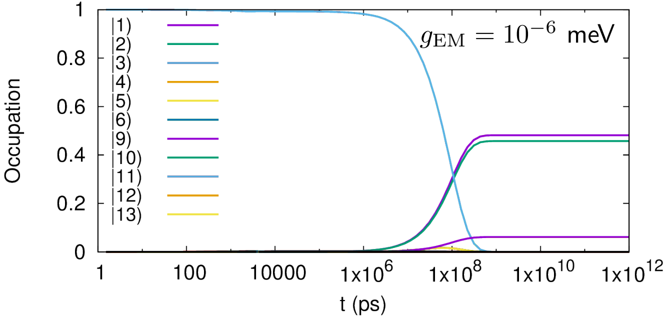
Spectrum of closed system vs. plunger gate voltage V_g



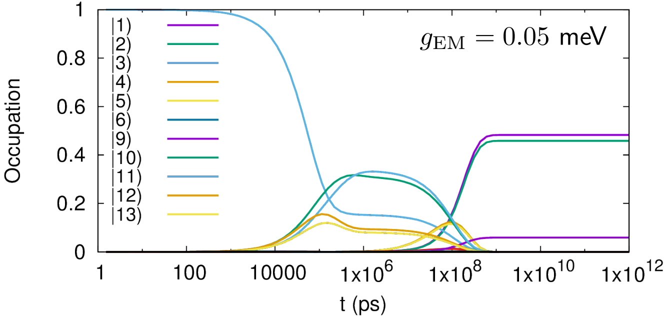
x-polarization, $\hbar\omega = 0.8$ meV, $g_{\mathbf{EM}} = 0.05$ meV,
$\hbar\Omega = 2.0$ meV, $B = 0.1$ T

Mean electron and photon number

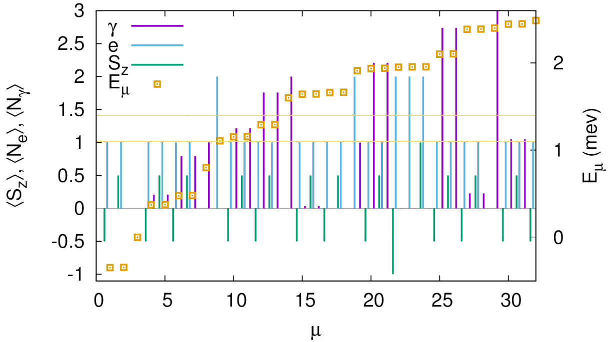


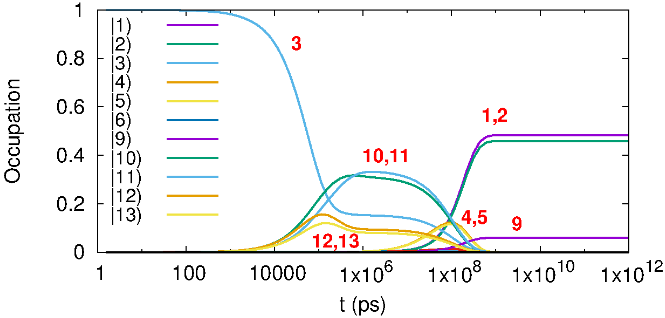


$g_{\rm EM} = 10^{-6}$ meV
Occupation
t (ps)
|1)
|2)
|3)
|4)
|5)
|6)
|9)
|10)
|11)
|12)
|13)

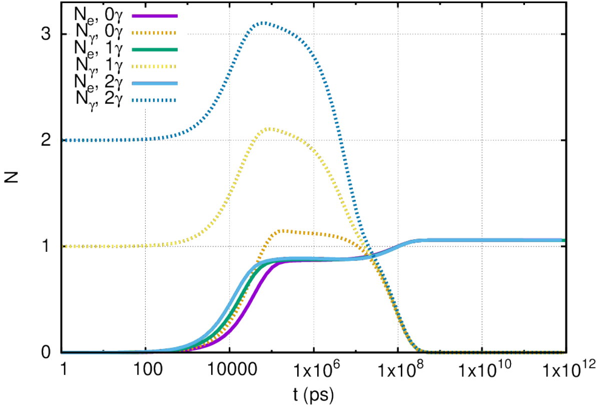


$g_{\rm EM} = 0.05$ meV
Occupation
t (ps)
|1)
|2)
|3)
|4)
|5)
|6)
|9)
|10)
|11)
|12)
|13)

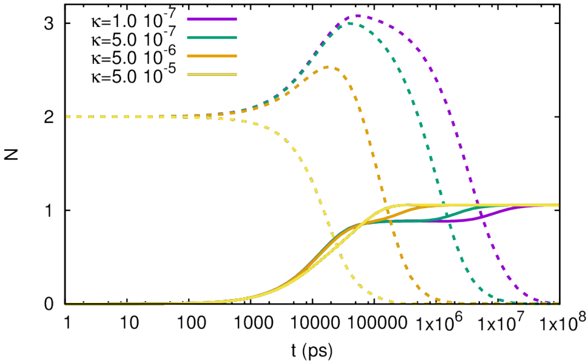

γ
e
S_z
E_μ
$\langle S_z \rangle$, $\langle N_e \rangle$, $\langle N_\gamma \rangle$
E_μ (mev)
μ


|1)
|2)
|3)
|4)
|5)
|6)
|9)
|10)
|11)
|12)
|13)
3
10,11
12,13
1,2
4,5
9
Occupation
t (ps)

Different initial state: 0, 1, or 2 photons

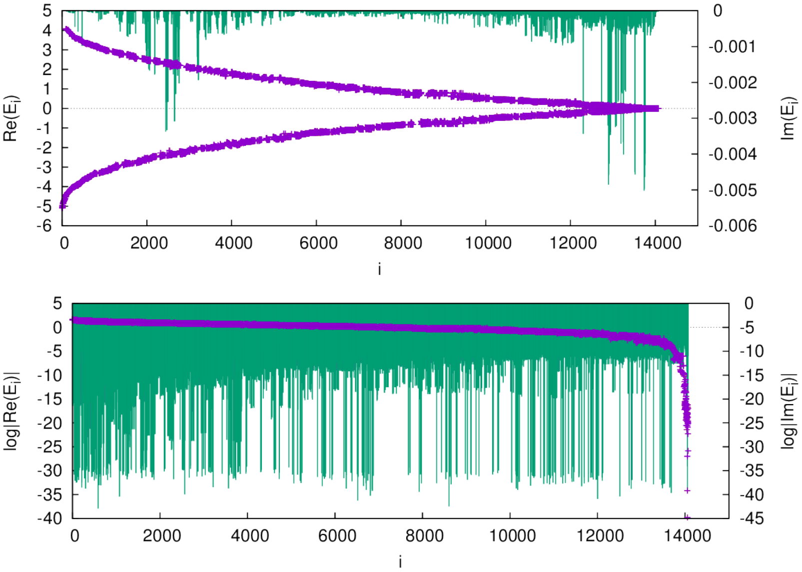


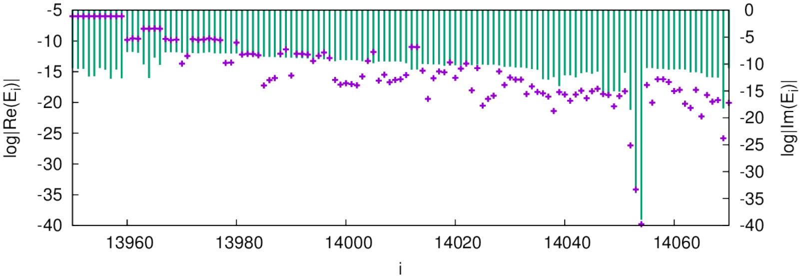
Different cavity photon leakage



$$\cdots + \frac{\kappa}{2\hbar}\left([a\rho, a^\dagger] + [a, \rho a^\dagger]\right)$$

Spectrum of the Liouvillian



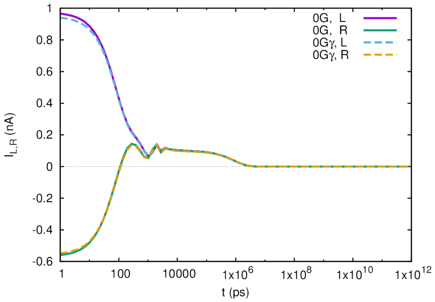


↑

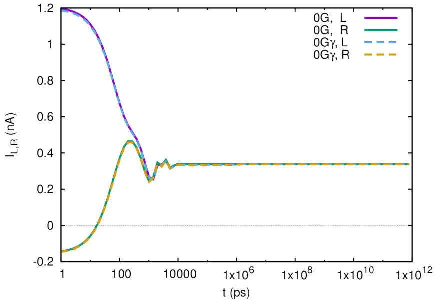
zero eigenvalue

Continuation – current into/through

Two parallel dots



$V_g = +0.1$ mV
2G off-resonant



$V_g = +2.0$ mV
1G resonance

Conclusions

- We can analyze the long time evolution of complex open systems
- We can identify regimes of different types of transitions (relaxation channels), electromagnetic, non-electromagnetic

- *ACS Photonics* **2**, 930 (2015)
- *Annalen der Physik* **528**, 394 (2016)
- `arxiv:1605.08248`, *Annalen der Physik*, in press, `doi:10.1002/andp.201600177`
- `arxiv:1610.03223`, (method, technical...)
- `arxiv:1611.09453`, (current into and through)

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