Magnetotransport through nanosystems embedded in a two-dimensional quantum wire

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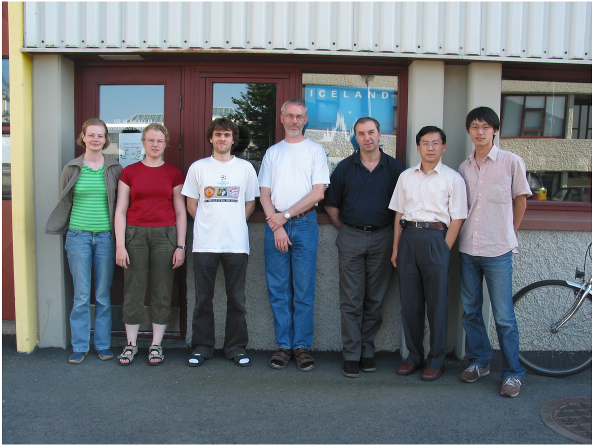
Cooperation



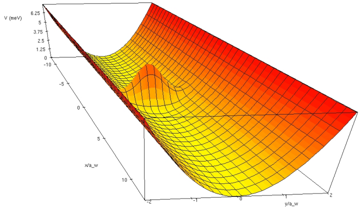
Guðný Guðmundsdóttir, Ingibjörg Magnúsdóttir, Jens H. Bárðarson, Viðar Guðmundsson, Andrei Manolescu, Chi-Shung Tang, Yu-Yu lin (Wing Wa Yu, Valeriu Moldoveanu)

Aim

- Simple $\rightarrow$ complex embedded systems in a quantum wire
  - Dot - antidot
  - Ring
  - Coupled dots
- Scattering formalism
  - Parabolic 2D quantum wire
  - Perpendicular magnetic field
  - General scattering potential
  - Multimode transport
  - Lippmann-Schwinger formalism
- Technical issues
  - Basis, matrix elements
  - Parallel computation
- Physics - application

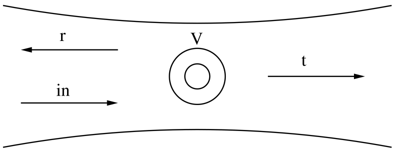
Model

The system



Parabolic confinement

Scattering - asymptotic regions



In scattering out

Asymptotic regions

Free wire, perpendicular magnetic field

$$H_0 = \frac{\hbar^2}{2m^*}\left[-i\boldsymbol{\nabla} - \frac{eB}{\hbar c}y\hat{\boldsymbol{x}}\right]^2 + V_{\rm c}(y)$$

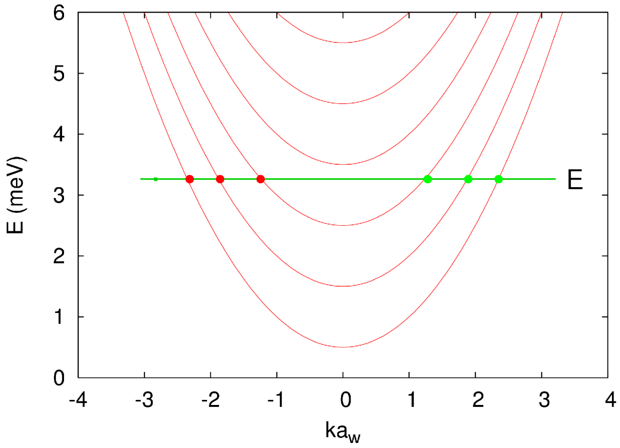
$$\psi^+(x,y,k_n) = e^{ik_nx}\chi_n(y-y_0)$$

$$E = \left(n+\frac{1}{2}\right)\hbar\Omega_w + \mathcal{K}_n(k_n)$$

$$\Omega_w = \sqrt{\omega_c^2+\Omega_0^2}, \quad y_0 = k_n a_w^2\frac{\omega_c}{\Omega_w}, \quad \omega_c = \frac{eB}{m^*c}$$

$$\mathcal{K}_n(k_n) = \frac{(k_na_w)^2}{2}\left(\frac{\hbar^2\Omega_0^2}{\hbar\Omega_w}\right), \quad a_w^2\Omega_w = \frac{\hbar}{m^*}$$

Asymptotic energy spectrum



In-, out- states, energy is conserved

Consequences of $B \neq 0$

- The Lorentz force couples the motion in x- and y-direction
- $\chi_n(y - y_0)$ with different y_0's and n's are not orthogonal
- No simple separation in modes, (k_n and y_0 are related)

Mixed momentum-coordinate representation

$$\Psi_E(p, y) = \int dx\, \psi_E(x, y) e^{-ipx}$$

$$\Psi_E(p, y) = \sum_n \varphi_n(p) \phi_n(p, y)$$

Expansion in terms of eigenfunctions of the shifted harmonic oscillator
$\rightarrow$ transport mode "n"

…transforms the Schrödinger equation

$$\mathcal{K}_n(q)\varphi_n(q) + \sum_{n'} \int \frac{dp}{2\pi} V_{nn'}(q,p)\varphi_{n'}(p) = (E - E_n)\varphi_n(q)$$

$$V_{nn'}(q,p) = \int dy\, \phi_n^*(q,y) V(q-p,y) \phi_{n'}(p,y)$$

$$V(q-p,y) = \int dx\, e^{-i(q-p)x} V_{sc}(x,y)$$

into a set of coupled integral equations

$V_{sc}(x,y)$ is the scattering potential,
(analytic matrix elements)

. . . rewrite

Nonlocal potential

$$\left[-(qa_w)^2 + (k_n(E)a_w)^2\right]\varphi_n(q) = \frac{2\hbar\Omega_w}{(\hbar\Omega_0)^2}\sum_{n'}\int\frac{dp}{2\pi}\,V_{nn'}(q,p)\varphi_{n'}(p)$$

Effective band momentum $(E - E_n) = \frac{[k_n(E)]^2}{2}\frac{(\hbar\Omega_0)^2}{\hbar\Omega_w}$

Free equation $\left[-(qa_w)^2 + (k_n(E)a_w)^2\right]\varphi_n^0(q) = 0$

Suggests an interpretation. . .

. . . a Green function

$$\left[-(qa_w)^2 + (k_n(E)a_w)^2\right] G_E^n(q) = 1$$

Lippmann-Schwinger eq.'s in q-space

$$\varphi_n(q) = \varphi_n^0(q) + G_E^n(q) \sum_{n'} \int \frac{dpa_w}{2\pi} \tilde{V}_{nn'}(q,p)\varphi_{n'}(p)$$

$$\varphi = \varphi^0 + G\tilde{V}\varphi^0 + G\tilde{V}G\tilde{V}\varphi^0 + \cdots = (1 + G\tilde{T})\varphi^0$$

$$\tilde{T}_{nn'}(q,p) = \tilde{V}_{nn'}(q,p) + \sum_{m'} \int \frac{dka_w}{2\pi} \tilde{V}_{nm'}(q,k) G_E^{m'}(k) \tilde{T}_{nm'}(k,p).$$

Transformed into eq's for the T-matrix
(convenient for numerical calulations)

Supplies

Conductance, transmission amplitudes, wavefunctions

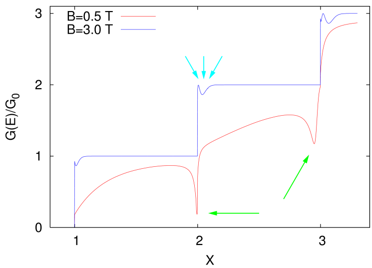
$$G(E) = \frac{2e^2}{h}\mathrm{Tr}[\mathbf{t}^\dagger(E)\mathbf{t}(E)]$$

$$t_{nm}(E) = \delta_{nm} - \frac{i\sqrt{(k_m/k_n)}}{2(k_m a_w)}\left(\frac{\hbar\Omega_0}{\hbar\Omega_w}\right)^2 \tilde{T}_{nm}(k_n, k_m)$$

$$\psi_E(x,y) = e^{ik_n x}\phi_n(k_n, y) + \sum_m \int \frac{dqa_w}{2\pi} e^{iqx} G_E^m(q)\tilde{T}_{mn}(q, k_n)\phi_m(q, y)$$

Elastic scattering

Attractive well - small open quantum dot

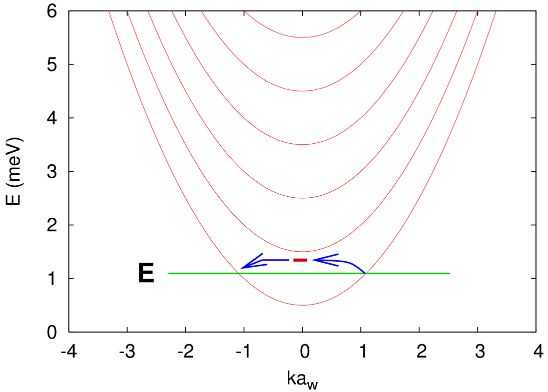


- Broad wire, $\hbar\Omega_0 = 1.0$ meV
- Shallow dot, $V_0 = -0.8$ meV
- $G_0 = \frac{2e^2}{h}$
- $\Omega_w = \sqrt{\omega_c^2 + \Omega_0^2}$
- $X = \frac{E}{\hbar\Omega_w} + \frac{1}{2}$

- Quantization, with or without B
- Lorentz force $\rightarrow$ electrons bypass dot at high B

Pertubative view

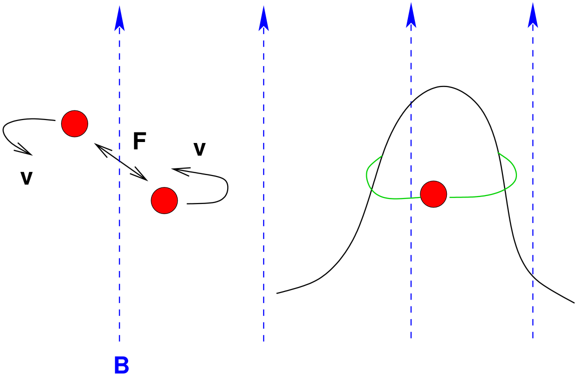
Resonant backscattering caused by an evanescent state



- The Lippmann-Schwinger eq's include scattering to all orders

Quasi bound states

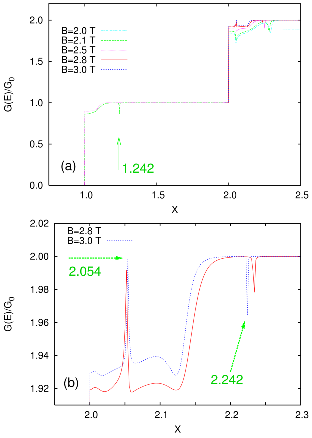
Quasi bound state with negative binding energy



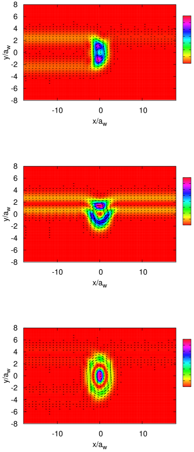
- If $B \neq 0$ then an electron can bind to a potential hill
- Is this quasi bound state seen in transport?

Smooth Gaussian potential hill, $V_0 = 8$ meV

Conductance

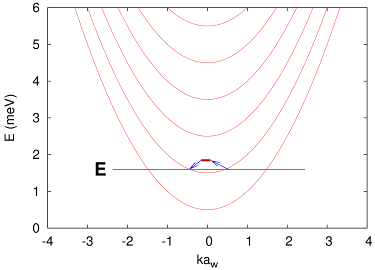


Probability density

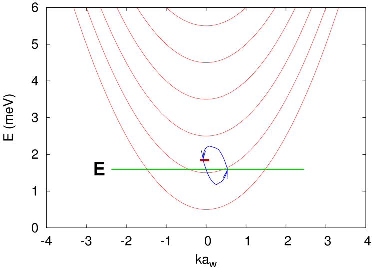


Perturbative view

Backscattering resonance

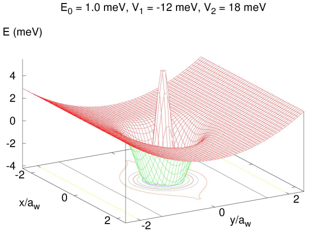


Forward scattering resonance

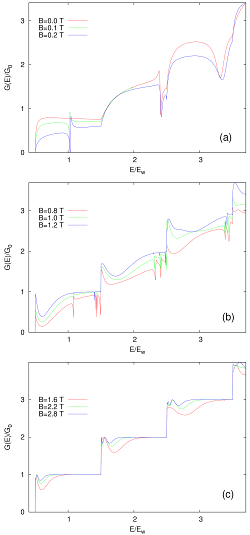


Small ring - Fano resonance

Small ring

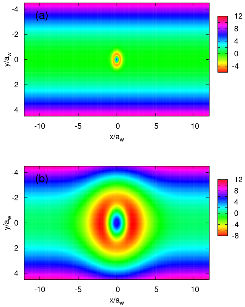


- Forward and backward scattering resonance through a quasi-bound state in the continuum
- Lorentz force

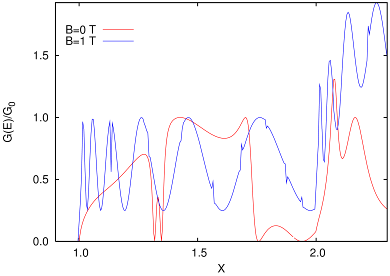


Large quantum ring

System

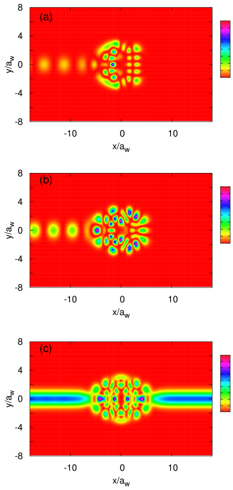


Conductance



- Ahranov-Bohm oscillations
- Quasi-bound state resonances

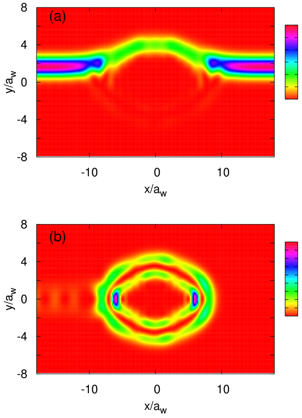
States for $B = 0$



- $X = 1.319$, $n = 0$
- $X = 1.347$, $n = 0$
- $X = 1.425$, $n = 0$

Persistence of eigenstates
Scarring of wave functions

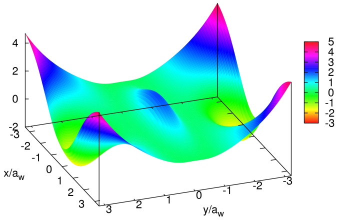
States for $B \neq 0$



- $X = 1.46$, $n = 0$
- $X = 1.135$, $n = 0$, Narrow res.

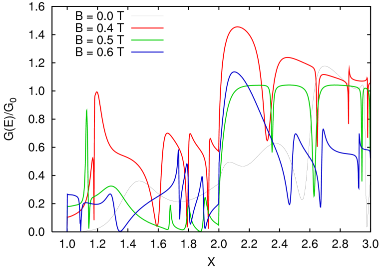
Wire with two parallel quantum dots

The system



Center hill - controlling coupling

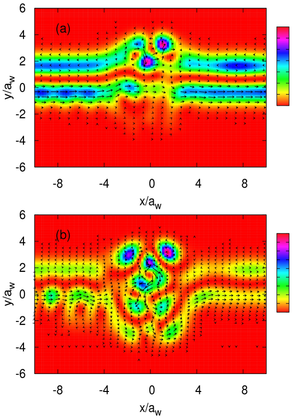
Complex conductance



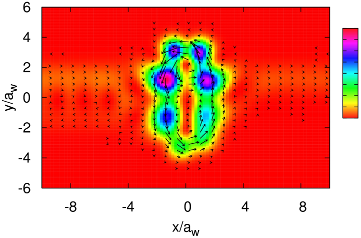
Many resonances,
sensitive to B, and V_0

Probability density and current

Conducting states



Resonance



Designing stable nanosystems?
Hardware for quantum computers

Conclusions

- Flexible model of *B*-Transport
- Quasi-bound states
- Fano resonances
- Electron bound to a hill
- AB oscillations
- Future directions?

Lot of CPU-power

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- Phys. Rev. B**72**, 153306 (2005)
- Phys. Rev. B**72**, Dec.15 (2005)