03-04

$$P(x)dx = A e^{-x/\lambda} dx, \quad x \geq 0$$

(a) Finna A, staðla

$$A\int_0^\infty dx\, e^{-x/\lambda} = A\lambda\int_0^\infty \frac{dx}{\lambda} e^{-\frac{x}{\lambda}} = A\lambda\int_0^\infty du\, e^{-u}$$

$$= -A\lambda e^{-u}\Big|_0^\infty = A\cdot 1\cdot\lambda$$

$$\rightarrow A = \frac{1}{\lambda} \quad \text{og} \quad P(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}$$

(b) Meðalgildi $\langle x\rangle$

$$\langle x\rangle = \frac{1}{\lambda}\int_0^\infty dx\, x\, e^{-\frac{x}{\lambda}} = \lambda\int_0^\infty \frac{dx}{\lambda}\frac{x}{\lambda} e^{-\frac{x}{\lambda}}$$

$$\langle x\rangle = \lambda\int_0^\infty du\, u\, e^{-u} = \lambda$$

(c) Staðalfrávik

$$\langle x^2\rangle = \frac{1}{\lambda}\int_0^\infty dx\, x^2 e^{-\frac{x}{\lambda}} = \lambda^2\int_0^\infty \frac{dx}{\lambda}\left(\frac{x}{\lambda}\right)^2 e^{-\frac{x}{\lambda}}$$

$$= \lambda^2\int_0^\infty du\, u^2 e^{-u} = \lambda^2\cdot 2$$

$$\rightarrow \sigma_x = \sqrt{(\langle x^2\rangle - \langle x\rangle^2)} = \sqrt{2\lambda^2 - \lambda^2} = \lambda$$

03-05

Fyrir $\theta \in [0,\pi]$ einsleit dreifing á bilinu

$$\rightarrow P(\theta) = \frac{1}{\pi} \quad \text{fyrir } \theta\in[0,\pi]$$

$$\int_0^\pi P(\theta)d\theta = 1$$

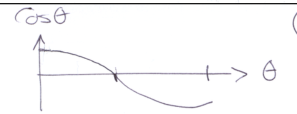
(a) $$\langle\theta\rangle = \frac{1}{\pi}\int_0^\pi d\theta\,\theta = \frac{1}{\pi}\frac{\theta^2}{2}\Big|_0^\pi = \frac{1}{\pi}\frac{\pi^2}{2} = \frac{\pi}{2}$$

(b) $$\langle\theta - \tfrac{\pi}{2}\rangle = \langle\theta\rangle - \frac{\pi}{2} = 0$$

(c) $$\langle\theta^2\rangle = \frac{1}{\pi}\int_0^\pi d\theta\,\theta^2 = \frac{1}{\pi}\frac{\theta^3}{3}\Big|_0^\pi = \frac{1}{\pi}\frac{\pi^3}{3} = \frac{\pi^2}{3}$$

(d) $$\langle\theta^n\rangle = \frac{\pi^n}{(n+1)} \quad \text{fyrir } n\geq 0$$

(e) $$\langle\cos\theta\rangle = \frac{1}{\pi}\int_0^\pi d\theta\,\cos\theta = 0$$



oddstætt á bilinu

(f) $$\langle\sin\theta\rangle = \frac{1}{\pi}\int_0^\pi d\theta\,\sin\theta = \frac{2}{\pi}$$ jafnstætt

(g) $$\langle|\cos\theta|\rangle = \frac{1}{\pi}\left\{\int_0^{\pi/2} d\theta\,\cos\theta - \int_{\pi/2}^{\pi} d\theta\,\cos(\theta)\right\} = \frac{2}{\pi}$$

(h) $$\langle\cos^2\theta\rangle = \frac{1}{\pi}\int_0^\pi d\theta\,\cos^2\theta = \frac{1}{2}$$

verður að vera

(i) $\langle\sin^2\theta\rangle = \frac{1}{2}$, (j) $\langle\underbrace{\cos^2\theta + \sin^2\theta}_{=1}\rangle = 1$

03-08 Vogis framleiðandi fall (5)

$$M(t) = \langle e^{tx} \rangle$$

$$\frac{d^n M(t)}{dt^n} = \langle x^n e^{tx} \rangle \to \left.\frac{d^n M(t)}{dt^n}\right|_{t=0} = \langle x^n 1 \rangle = \langle x^n \rangle$$

Sjáum/Þá tökum sem $\langle x^n \rangle = \underline{M^{(n)}(0)}$

$$\to \langle x \rangle = M^{(1)}(0)$$

$$\sigma_x^2 = M^{(2)}(0) - \{M^{(1)}(0)\}^2 = \langle x^2 \rangle - \langle x \rangle^2$$

(a) Bernoulli til reynu: p eða $1-p$

Ef $M(t) = pe^t + 1 - p \to \langle x \rangle = p$

$M^{(1)}(t) = pe^t$, $M^2(t) = pe^t$ $\quad \langle x^2 \rangle = p$

(b) Tvíliðadreifing (6)

$$M(t) = (pe^t + 1 - p)^n$$

$$M^{(1)}(t) = n(pe^t + 1 - p)^{n-1} \cdot pe^t \to M^{(1)}(0) = np = \langle k \rangle$$

$$M^{(2)}(t) = (n-1)n(pe^t+1-p)^{n-2} \cdot pe^t \cdot pe^t + n(pe^t+1-p)^{n-1} \cdot pe^t$$

$$\to M^{(2)}(0) = (n-1)np^2 + np$$

$$M^{(2)}(0) - \{M^{(1)}(0)\}^2 = (n-1)np^2 + np - n^2p^2 = np - np^2 = np(1-p) = \sigma_k^2$$

(d) (7)

$$M(t) = \frac{\lambda}{\lambda - t}$$

$$M^{(1)}(t) = \frac{+\lambda}{(\lambda - t)^2} \to M^{(1)}(0) = \frac{1}{\lambda}$$

$$M^{(2)}(t) = \frac{2\lambda}{(\lambda - t)^3} \to M^{(2)}(0) = \frac{2}{\lambda^2}$$

Hér fæst greinilega ekki $\langle x \rangle = M^{(1)}(0)$

Giska á að fallið þurfi að vera

$$M(t) = \frac{1/\lambda}{1/\lambda - t} \to M^{(1)}(t) = \frac{1/\lambda}{(1/\lambda - t)^2} \to M^{(1)}(0) = \lambda$$

$$M^2(t) = \frac{2 \cdot 1/\lambda}{(1/\lambda - t)^3} \to M^{(2)}(0) = 2\lambda^2$$

þá passar allt

(7b)

Ég sannreyndi bara, en leiddi ekki út $M(t)$

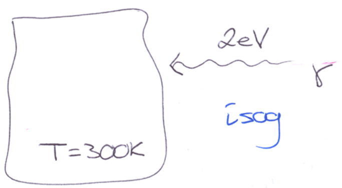
Byrjum úr því fyrir (d)

$$P(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}, \quad M(t) = \langle e^{tx} \rangle$$

$$\to M(t) = \frac{1}{\lambda}\int_0^\infty dx\, e^{tx} e^{-\frac{x}{\lambda}} = \frac{1}{\lambda}\int_0^\infty dx\, e^{x(t - \frac{1}{\lambda})}$$

$$= \boxed{\frac{\lambda^{-1}}{\lambda^{-1} - t}} \quad \text{ef} \quad t > \frac{1}{\lambda}$$

04-05 ⑧



hvernig breytist Ω fyrir stærsoja hlutinn

Notum (4.12) $\Omega(E-\epsilon) = \Omega(E)\, e^{-\epsilon/k_BT}$
og snúum við

$$\frac{\Omega(E)}{\Omega(E-\epsilon)} = e^{\epsilon/k_BT} = \exp\left\{\frac{2eV}{8.617\cdot 10^{-5}\frac{eV}{K}\, 300K}\right\}$$

$$\approx 4\cdot 10^{33}$$

fyrir γ með 100 MHz $\longrightarrow$ $E = h\nu = 4.135\cdot 10^{-15}\, eVs \cdot 100\cdot 10^{6}\,\frac{1}{s}$
$\sim 4.135\cdot 10^{-7}\, eV$

$$\longrightarrow \frac{\Omega(E)}{\Omega(E-\epsilon)} \approx 1.00$$

04-08 ⑨

$k_BT \sim 25.9$ meV við $T = 300K$
eða 0.0259 eV

(a) Jónunarorka H er 13.6 eV
$\longrightarrow$ engin jónun við $T = 300K$

(b) 10^{-4} eV eða 0.1 meV þarf til að auka snúning sameinda $\longrightarrow$ við $T = 300K$ er hring-snúningur sameinda mjög örvaður

20-04 Hreintóna sveifill ①

$$Z = \frac{e^{-\frac{1}{2}\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}}$$

$$U = -\frac{d\ln Z}{d\beta} = \hbar\omega\left\{\frac{1}{2} + \frac{1}{e^{\beta\hbar\omega}-1}\right\}$$

$$F = -\frac{1}{\beta}\ln Z = \frac{\hbar\omega}{2} + \frac{\ln(1-e^{-\beta\hbar\omega})}{\beta}$$

$\beta\hbar\omega = \frac{\hbar\omega}{k_BT}$ $\longrightarrow$ þegar $k_BT << \hbar\omega$ allt aðeins grunnástandið að vera setið

$\beta\hbar\omega \longrightarrow \infty$

grunnástandið hefur þessa orku

$$U \xrightarrow[\beta\hbar\omega\to\infty]{} \frac{\hbar\omega}{2}, \qquad F \xrightarrow[\beta\hbar\omega\to\infty]{} \frac{\hbar\omega}{2}$$

②

$$\frac{S}{k_B} = \frac{U-F}{k_BT} = \beta(U-F)$$

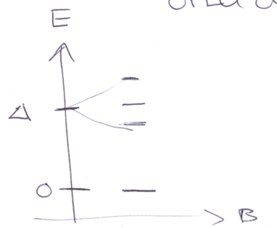
$$= \left\{\frac{\beta\hbar\omega}{e^{\beta\hbar\omega}-1} - \ln(1-e^{-\beta\hbar\omega})\right\}$$

og

$$\lim_{x\to\infty}\left\{\frac{x}{e^{x}-1} - \ln(1-e^{-x})\right\} = 0 \quad \longrightarrow \frac{S}{k_B} \longrightarrow 0$$

þegar $k_BT << \hbar\omega$

20-06 þéttleiki n óháðra sameinda með fjögur orkuástönd $0,\ \Delta - g\mu_BB,\ \Delta,\ \Delta + g\mu_BB$



finna Z, F og M, χ

$n = \frac{N}{V}$

fyrir eina sameind

$$Z = \sum_{\alpha} e^{-\beta E_\alpha} = \Big\{1 + \exp(-\beta\Delta) + \exp(-\beta(\Delta - g\mu_B B)) + \exp(-\beta(\Delta + g\mu_B B))\Big\}$$

$$= \Big\{1 + \exp(-\beta\Delta)\big[1 + 2\,\mathrm{Cosh}(\beta g\mu_B B)\big]\Big\}$$

$U = -\frac{d}{d\beta}\ln Z^N$ fyrir N óháðar sameindir

$$= -N\,\frac{-\Delta\exp(-\beta\Delta)\Big\{\big[1+2\,\mathrm{Cosh}(\beta g\mu_B B)\big] + 2g\mu_B B\,\mathrm{Sinh}(\beta g\mu_B B)\Big\}}{1+\exp(-\beta\Delta)\big[1+2\,\mathrm{Cosh}(\beta g\mu_B B)\big]}$$

$$F = -\frac{1}{\beta}\ln Z^N = -\frac{N}{\beta}\ln\Big\{1+\exp(-\beta\Delta)\big[1+2\,\mathrm{Cosh}(\beta g\mu_B B)\big]\Big\}$$

$$m = -\left(\frac{\partial F}{\partial B}\right)_T = +\frac{N}{\beta}\,\frac{e^{-\beta\Delta}\,2\beta g\mu_B\,\mathrm{Sinh}(\beta g\mu_B B)}{1+e^{-\beta\Delta}\big[1+2\,\mathrm{Cosh}(\beta g\mu_B B)\big]}$$

$$M = \frac{m}{V} = n\,\frac{e^{-\beta\Delta}\,2g\mu_B\,\mathrm{Sinh}(\beta g\mu_B B)}{1+e^{-\beta\Delta}\big\{1+2\,\mathrm{Cosh}(\beta g\mu_B B)\big\}}$$

$\mathrm{Sinh}(x) \simeq x + \frac{x^3}{6} + \dots$

$\mathrm{Cosh}(x) \simeq 1 + \frac{x^2}{2} + \dots$

$$\lim_{B\to 0} M = \frac{n\,e^{-\beta\Delta}\,2g\mu_B(\beta g\mu_B B)}{1+3e^{-\beta\Delta}}$$

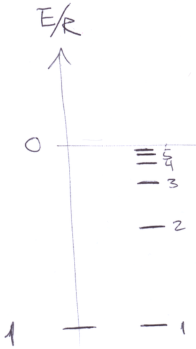
$$\chi = \lim_{B\to 0}\frac{\mu_0 M}{B} = \frac{\mu_0 n\,e^{-\beta\Delta}\,2g\mu_B\,\beta g\mu_B}{1+3e^{-\beta\Delta}} = \frac{2n\mu_0 g^2\mu_B^2\,e^{-\beta\Delta}}{k_BT(1+3e^{-\beta\Delta})}$$

$$\boxed{\chi = \frac{2n\mu_0 g^2\mu_B^2}{k_BT\left(3+e^{\frac{\Delta}{k_BT}}\right)}}$$

eins og ~~betra~~ ver um

20-08 Vetnis atóm $E = -\frac{R}{n^2}$, $R = 13,6$ eV

með margfeldni $2n^2$. styrjála orkuróf bundinna ástanda. Líka er til samfellt róf með $E > 0$



$$Z = \sum_{\alpha} e^{-\beta E_\alpha} = \sum_{n=1}^{\infty} 2n^2\exp\left\{\frac{R}{n^2k_BT}\right\}$$

Nálgun

$$Z \simeq \sum_{n=1}^{2} 2n^2\exp\left\{\frac{R}{n^2k_BT}\right\}$$

$$\langle E\rangle = -\frac{2Re^{\beta R} + \frac{8R}{4}e^{\frac{\beta R}{4}}}{2e^{\beta R} + 8e^{\frac{\beta R}{4}}}$$

Við $T = 300$K

er $\beta = 25.9$ meV

$\rightarrow \beta R = \frac{13.6}{0,0259} \simeq 525,1\dots$

$= 13.600$ eV

hvertandi örum úr grunnástandinu

05-03 Hve mikil skekkja er að nota $\langle v\rangle$ í stað $\sqrt{\langle v^2\rangle}$

$$\langle v\rangle = \sqrt{\frac{8k_BT}{\pi m}} \qquad \langle v^2\rangle = \frac{3k_BT}{m}$$

$$\rightarrow \frac{\langle v\rangle}{\sqrt{\langle v^2\rangle}} = \sqrt{\frac{8}{\pi\cdot 3}} \simeq 0,922$$

7,8 %

05-04

7

Vitum að $\langle v \rangle = \sqrt{\frac{8k_BT}{\pi m}}$

$$f(v)dv = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2} v^2 dv\, e^{-mv^2/(2k_BT)}$$

Finnum $\langle 1/v \rangle$

$$\left\langle \frac{1}{v} \right\rangle = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2} \underbrace{\int_0^\infty dv\, v\, e^{-mv^2/(2k_BT)}}$$

$$= \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2} \cdot \frac{k_BT}{m} = \frac{4}{\sqrt{\pi}\sqrt{8}}\sqrt{\frac{m}{k_BT}} = 4\sqrt{\frac{m}{\pi k_BT \cdot 8}}$$

$$\to \langle v \rangle \left\langle \frac{1}{v} \right\rangle = 4\sqrt{\frac{8k_BT}{\pi m}}\sqrt{\frac{m}{8\pi k_BT}} = \frac{4}{\pi}$$

06-03

1

Fourier í herbergi

$T = 18°C \to 25°C$

Hvernig breytist heildarorka loftsins í herberginu?

p er fasti ~ ein loft þyngd

Kjörgas $\to p = \frac{1}{3} nm\langle v^2 \rangle$ (6.15)

Orkuþéttleiki $u = \frac{1}{2} nm\langle v^2 \rangle$ (6.24)

$\to p = \frac{2}{3} u$ (6.25)

$\to$ fastur $p \to$ fast u

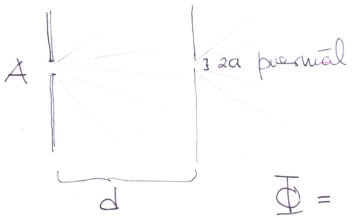
heildarorkan $U = uV$ líka fasti!

07-05

2

Útstreymi

Sá hluti flæðisins um „A" sem hittir á sama gatið



Notum því (7.5) $\quad \sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$

$$\Phi = \frac{n}{2}\int_0^\infty dv\, v f(v) \int_0^{\theta_{max}} d\theta \cos\theta \sin\theta$$

$\cos(2x) \sim 1 - 2x^2 + \dots$

$\sin\theta_{max} = \frac{a}{d}$, $\quad a \ll d \to \theta_{max} \simeq \frac{a}{d}$

$$\Phi = \frac{n}{4}\langle v \rangle \int_0^{a/d} d\theta \sin(2\theta) = \frac{n}{4}\langle v \rangle \left[-\frac{\cos(2x)}{2}\Big|_0^{a/d} \right]$$

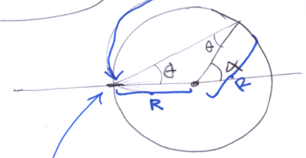
$$= \frac{n}{4}\langle v \rangle \left\{ \frac{1}{2} - \frac{1}{2}\left(1 - 2\frac{a^2}{d^2}\right) \right\} = \frac{n}{4}\langle v \rangle \frac{a^2}{d^2} = \frac{n}{4}\langle v \rangle \frac{a^2}{d^2}$$

$$\to A\Phi = \frac{n}{4}\langle v \rangle A \frac{a^2}{d^2}$$

07-06

3

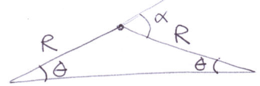
útstreymi inn í kúlu



Hver er dreifingin ef sameindirnar festast þar sem þær lenda?

(7.5) horndreifingin er $\cos\theta\sin\theta = \frac{1}{2}\sin(2\theta)$ út um gatið

jafnarma þríhyrningur



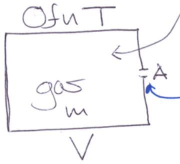
$\to \theta + \theta + (\pi - \alpha) = \pi \to \alpha = 2\theta$

Rúmhornið milli α og $\alpha + d\alpha$ $\sim \sin\alpha\, d\alpha$ $\sim \sin(2\theta)d\theta$

Dreifingin á horn út úr gatinu er sama og flötur kúluyfirborðsins í kúlunni undir sama horni $\to$ jafndreift

07-08

finna $p(t)$



fjöldi einda sem leka ΦA

inni í ofni fækkar þá eindum um $V\frac{d}{dt}n$

$\rightarrow$ varðveisla einda $\quad V\frac{d}{dt}n = -\Phi A$

Kjörgas $\rightarrow p = nk_BT \rightarrow n = \frac{p}{k_BT}$

fast T $\longrightarrow \frac{dn}{dt} = \frac{dp}{dt}\cdot\frac{1}{k_BT}$

og $\quad \Phi = \frac{p}{\sqrt{2\pi m k_BT}}$

$$\frac{V}{k_BT}\frac{dp}{dt} = -A\frac{p}{\sqrt{2\pi m k_BT}}$$

$$\rightarrow \frac{dp}{dt} = -\frac{A}{V}\sqrt{\frac{k_BT}{2\pi m}}\,p$$

$$\frac{dp}{dt} + \underbrace{\frac{A}{V}\sqrt{\frac{k_BT}{2\pi m}}}\,p = 0$$

með vídd $\frac{1}{T}$, köllum $\tau = \frac{V}{A}\sqrt{\frac{2\pi m}{k_BT}}$

$$\rightarrow \frac{dp}{dt} + \frac{1}{\tau}p = 0 \rightarrow \frac{dp}{p} = -\frac{dt}{\tau}$$

$$\rightarrow \int_{p(0)}^{p(t)}\frac{dp'}{p'} = -\frac{1}{\tau}\int_0^t dt'$$

$$\rightarrow \ln(p(t)) - \ln(p(0)) = -\frac{t}{\tau} \quad \text{eða} \quad \ln\left[\frac{p(t)}{p(0)}\right] = -\frac{t}{\tau}$$

$$\frac{p(t)}{p(0)} = e^{-t/\tau} \quad \text{eða} \quad \boxed{p(t) = p(0)e^{-t/\tau}}$$

τ er náttúrulegi tímastuðull þrýstingsbreytingarinnar

08-01 Finna λ fyrir N_2-sameind í $p = 10^{-10}$ mbar

Hólkur með $d = 0{,}5$ m

$\lambda = \frac{1}{\sqrt{2}n\sigma}$ $\qquad$ Ex 8.1 $\rightarrow \pi d^2 = \sigma = 4.3\cdot10^{-19}\,m^2$

Gerum ráð fyrir $T \approx 300K$

$n = \frac{p}{k_BT}$ $\qquad p = 10^{-10}$ mbar $= 10^{-10}\cdot 10^2$ Pa

$$\rightarrow \lambda = \frac{k_BT}{\sqrt{2}\,p\,\pi d^2} = \frac{1{,}38\cdot10^{-23}\,JK^{-1}\cdot 300K}{\sqrt{2}\,10^{-8}\,Pa\cdot 4{,}3\cdot10^{-19}\,m^2}$$

$$= 6{,}8\cdot10^5\,m$$

$$\langle v\rangle = \sqrt{\frac{8k_BT}{\pi m}} = \sqrt{\frac{8\cdot1.38\cdot10^{-23}\,JK^{-1}\,300K}{\pi\,\frac{0.028\,kg}{6.022\cdot10^{23}}}} \approx 476\ m/s$$

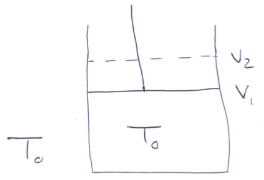
$$\rightarrow \tau = \frac{\lambda}{\langle v\rangle} \approx \frac{6{,}8\cdot10^5\,m}{476\ m/s} \approx 1428\,s \sim 24\text{ mín}$$

Meðal fjöldi árekstra við veggi / árekstrar við sameindir

$\sim \frac{6{,}8\cdot10^5\,m}{0{,}6\,m} \sim 10^6$ $\qquad$ milljón sinnum fleiri

11-01 Kjörgas í strokki þenst hægt frá $V_1 \to V_2$ (eitt mól, $pV = RT$) við fast $T = T_0$

1



Hversvegna breytist U ekki?

$$U = \frac{3}{2}RT \longrightarrow \left(\frac{\partial U}{\partial V}\right)_T = 0$$

U er ekki fall af V

Reikna ΔW og ΔQ

Vinna gasins $\quad dW = p\,dV \quad$ (á umhverfið)

$$\Delta W = \int_{V_1}^{V_2} dV\,p = RT_0\int_{V_1}^{V_2}\frac{dV}{V} = RT_0 \ln\left(\frac{V_2}{V_1}\right)$$

2

$$\Delta U = \Delta Q - \Delta W$$

vinna kerfisins á umhverfið

varma flæði inn í kerfið

$$\Delta U = 0 \longrightarrow \Delta Q = \Delta W = RT_0 \ln\left(\frac{V_2}{V_1}\right)$$

11-02 Sýna að fyrir kjörgas gildi

$$\frac{R}{C_V} = \gamma - 1 \quad \text{og} \quad \frac{R}{C_p} = \frac{\gamma-1}{\gamma}$$

C_V, C_p eru varmarýmdir á mól

Höfum $\quad C_p - C_V = R \quad$ og $\quad \gamma = \frac{C_p}{C_V}$

$$1 - \frac{C_V}{C_p} = \frac{R}{C_p} \longrightarrow 1 - \frac{1}{\gamma} = \frac{R}{C_p} \quad \text{eða} \quad \boxed{\frac{R}{C_p} = \frac{\gamma-1}{\gamma}}$$

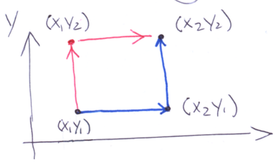
$$\frac{C_p}{C_V} - 1 = \frac{R}{C_V} \longrightarrow \boxed{\gamma - 1 = \frac{R}{C_V}}$$

11-03

3

$$df = 2xy\,dx + (x^2 + 2y)\,dy$$

(i) heildunarvegur $\quad (x_1, y_1) \to (x_2, y_1) \quad$ og $\quad (x_2, y_1) \to (x_2, y_2)$



(ii) $\quad (x_1, y_1) \to (x_1, y_2) \quad$ og $\quad (x_1, y_2) \to (x_2, y_2)$

$$\text{(i)} \quad \int_i df = 2y_1\int_{x_1}^{x_2} x\,dx + \int_{y_1}^{y_2}(x_2^2 + 2y)\,dy$$

$$= 2y_1 \frac{x_2^2 - x_1^2}{2} + x_2^2\cdot(y_2 - y_1) + 2\,\frac{y_2^2 - y_1^2}{2}$$

$$= y_1(x_2^2 - x_1^2) + x_2^2(y_2 - y_1) + y_2^2 - y_1^2$$

$$= -y_1x_1^2 + x_2^2y_2 + y_2^2 - y_1^2$$

4

$$\int_{ii} df = \int_{y_1}^{y_2}(x_1^2 + 2y)\,dy + 2y_2\int_{x_1}^{x_2} x\,dx$$

$$= x_1^2(y_2 - y_1) + y_2^2 - y_1^2 + y_2(x_2^2 - x_1^2)$$

$$= -x_1^2y_1 + x_2^2y_2 + y_2^2 - y_1^2$$

$$\longrightarrow \int_i dz = \int_{ii} dz$$

afleiðan er nákvæm

Reynum

$$\left.\begin{array}{l} f(x,y) = x^2y + y^2 \\ \longrightarrow \left(\frac{\partial f}{\partial x}\right)_y = 2xy \\ \left(\frac{\partial f}{\partial y}\right)_x = x^2 + 2y \end{array}\right\} \longrightarrow df = 2xy\,dx + (x^2 + 2y)\,dy$$

11-04 (5)

$$x = r\cos\theta \equiv x(r,\theta)$$
$$y = r\sin\theta \equiv y(r,\theta)$$

$$\rightarrow \left(\frac{\partial x}{\partial r}\right)_\theta = \cos\theta = \frac{x}{r}$$

$$x^2 + y^2 = r^2 \rightarrow x^2 = r^2 - y^2 \equiv x^2(r,y)$$

$$\rightarrow 2x\left(\frac{\partial x}{\partial r}\right)_y = 2r \rightarrow \left(\frac{\partial x}{\partial r}\right)_y = \frac{r}{x}$$

$$\rightarrow \left(\frac{\partial x}{\partial r}\right)_\theta = \frac{1}{\left(\frac{\partial x}{\partial r}\right)_y} = \left(\frac{\partial r}{\partial x}\right)_y$$

ekkert rangt við þetta, þar sem afleiðurnar eru teknar með mismunandi stærðir.

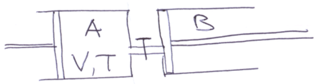
11-05 (6)

Samgild: varmi er vinna
vinna er varmi

ekki rétt því ekki er hægt að breyta varma algerlega í vinnu

12-05 (1)

finna T (ekkert varmaflæði frá umhverfi)



A Bulla A föst, ventill opnaður
Bulla B dregin hægt út $\rightarrow$ $V_A + V_B$ = 2V

Óvermið $đQ = 0$, kjörgas

$$dU = đQ + đW \rightarrow dU = đW$$
$$dU = C_V dT + \underbrace{\left(\frac{\partial U}{\partial V}\right)_T}_{=0} dT$$

$$đW = -p\,dV$$
$$\rightarrow C_V dT = -p\,dV = -\frac{RT}{V}dV$$
$$\rightarrow C_V \frac{dT}{T} = -R\frac{dV}{V}$$

$$C_V \int_T^{T_f} \frac{dT'}{T'} = -R\int_{V_A}^{2V} \frac{dV'}{V'}$$
$$C_V \ln\left(\frac{T_f}{T}\right) = -R\ln\left(\frac{2V}{V}\right)$$
$$C_V \ln\left(\frac{T}{T_f}\right) = R\ln(2)$$
$$\ln\left(\frac{T}{T_f}\right) = \frac{R}{C_V}\ln 2$$

(2)

$$\text{Kjörgas} \rightarrow \gamma = \frac{5}{3} \text{ og } \frac{R}{C_V} = \gamma - 1 = \frac{2}{3}$$

$$\ln\left(\frac{T}{T_f}\right) = (\gamma-1)\ln 2 \rightarrow \frac{T}{T_f} = 2^{\gamma-1}$$

$$\rightarrow T_f = \frac{T}{2^{\gamma-1}} = \frac{T}{2^{2/3}}$$

Hitastigið lækkar
Innri orkan breytt

B Bulla B dregin alveg út
ventill opnaður aðeins
Bulla A ýtt
þ.e. p haldist fastur eins langt og gengur

Bullustrokkarnir leyfa varmaflæði milli sín.

Þrýstingur í lokin og upphafsástandi er sá sami
Rúmmál B verður V, en bætist nú við $V_A \neq V$ í A

Upphaf : $pV = RT$, T er hitinn í upphafi

Lok : $p(V+V_A) = RT_f$

Varma einangrun frá umhverfi $\to dU = dW$

Engin vinna vegna hreyfingar bullu í B (engin hreyfing)
A þjappað

$$C_V\int_T^{T_f} dT = -p\int_V^{V_A} dV \to C_V(T_f-T) = -p(V_A-V)$$

$U = U(T)$, ekki háð V

$$\to C_V(T_f-T) = +p(V-V_A)$$

Ástandsjöfnurnar gefa saman

$$\frac{V+V_A}{V} = \frac{T_f}{T} \to \frac{V_A}{V} = \frac{T_f}{T} - 1$$

$$\to C_V(T_f-T) = \frac{RT}{V}(V-V_A) = RT\left(1-\frac{V_A}{V}\right)$$

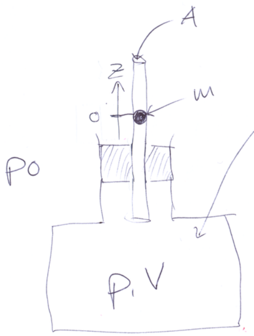
$$\frac{C_V}{R}\left(\frac{T_f-T}{T}\right) = \left(1-\frac{T_f}{T}+1\right) = \left(2-\frac{T_f}{T}\right)$$

$$\frac{C_V}{R}(T_f-T) = (2T-T_f) \qquad \frac{C_V}{R} = \frac{3}{2}$$

$$T_f\left(1+\frac{C_V}{R}\right) = T\left(2+\frac{C_V}{R}\right) \to T_f = \frac{7}{5}T$$

12-06

Hraðar sveiflur – óvermið ferli



$$p = p_0 + \frac{mg}{A}$$

"Örlítil hnikun kúlu um dz leiðir til rúmmálsbreytingar $dV = A\,dz$ og breytingar á kraftinum á kúluna $A\,dp = m\ddot{z}$

fyrir óvermið ferli gildir pV^γ = fasti

$$\to \frac{dp}{dV} = -\gamma\frac{p}{V} \quad \text{eða} \quad \frac{dp}{p} = -\gamma\frac{dV}{V}$$

$$m\ddot{z} = A\,dp = -A\gamma p\frac{dV}{V} = -A\gamma p\frac{A\,dz}{V}$$

$$\to \ddot{z} = -\frac{\gamma p A^2}{mV}dz$$

miðum z frá jafnvægisstöðunni

$$\to \ddot{z} + \frac{\gamma p A^2}{mV}z = 0$$

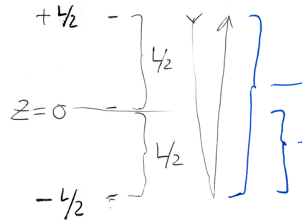
$$\ddot{z} + \omega^2 z = 0$$

með $\omega^2 = \frac{\gamma p A^2}{mV}$

$$\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mV}{\gamma p A^2}}$$

b) Útslag kúlunnar frá punktinum þ.s. $p = p_0$

Stöðuorka : $mgL = \frac{1}{2}\omega^2\left(\frac{L}{2}\right)^2$: fjaðurorkan

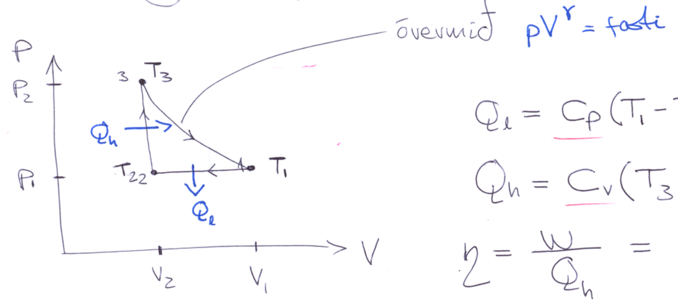


$$\to mgL = \frac{1}{8}\frac{\gamma p A^2}{mV}L^2$$

$$\to L = \left(\frac{8m^2Vg}{\gamma p A^2}\right)$$

7

13-04 Kjörgas hringur



$$Q_\ell = C_p(T_1-T_2)$$

$$Q_h = C_v(T_3-T_2)$$

$$\eta = \frac{W}{Q_h} = \frac{Q_h-Q_\ell}{Q_h} = 1-\frac{Q_\ell}{Q_h}$$

$$\rightarrow \eta = 1-\frac{C_p}{C_v}\frac{T_1-T_2}{T_3-T_2}$$

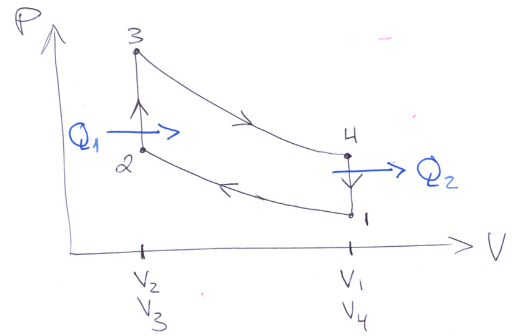
fyrir 1->2 og 2->3 notum við $pV \sim T$, það er ekki hægt fyrir 3->1, en þar er ferlið óvermið

$$= 1-\underbrace{\frac{C_p}{C_v}}_{\gamma}\frac{p_1V_1-p_1V_2}{p_2V_2-p_1V_2} = 1-\gamma\frac{V_1-V_2}{\frac{p_2}{p_1}V_2-V_2}$$

$$= 1-\gamma\frac{\frac{V_1}{V_2}-1}{\frac{p_2}{p_1}-1}$$

8

13-05 Otto-hringurinn



$$\eta = \frac{W}{Q_1} = \frac{Q_1-Q_2}{Q_1}$$

$$= 1-\frac{Q_2}{Q_1}$$

$$Q_1 = C_v(T_3-T_2)$$

$$Q_2 = C_v(T_4-T_1)$$

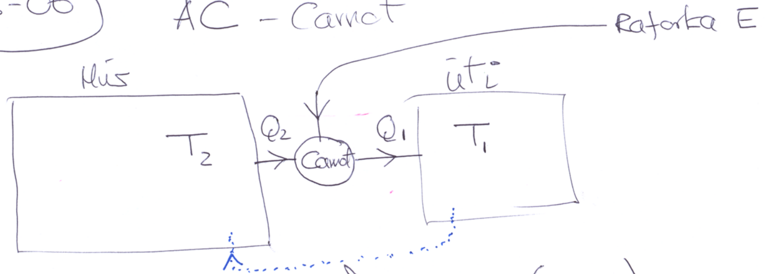
notum tengsl um óvermu bútana $TV^{\gamma-1}$ = fasti

$$\eta = 1-\frac{T_4-T_1}{T_3-T_2} = 1-\frac{\frac{C_2}{V_1^{\gamma-1}}-\frac{C_1}{V_1^{\gamma-1}}}{\frac{C_2}{V_2^{\gamma-1}}-\frac{C_1}{V_2^{\gamma-1}}}$$

$$= 1-\left(\frac{V_2}{V_1}\right)^{\gamma-1} = 1-\left(\frac{V_1}{V_2}\right)^{1-\gamma} = 1-r^{1-\gamma}$$

9

13-06 AC - Carnot



Varmaleki inn í húsið $Q = A(T_1-T_2)$

~~Staða~~ stöðuga ástandið, þegar $Q = Q_2$

Orkuvarðveisla samkvæmt 1. lögmálinu -> $Q_1 = E+Q_2$

Carnot vél $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

finna jöfnu fyrir $T_2(T_1, E, A)$

losa okkur við Q, Q_1 og Q_2

10

$Q = Q_2$, $Q = A(T_1-T_2)$ → $Q_2 = A(T_1-T_2)$

$Q_1 = E+Q_2$ → $Q_1 = E+A(T_1-T_2)$

$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$ → $\frac{A(T_1-T_2)}{T_2} = \frac{E+A(T_1-T_2)}{T_1}$

$$AT_1(T_1-T_2) - ET_2 - AT_2(T_1-T_2) = 0$$

$$T_2^2A + T_2\{-AT_1-E-AT_1\} + \{AT_1^2\} = 0$$

$$T_2^2A - T_2\{2AT_1+E\} + AT_1^2 = 0$$

$$T_2^2 - T_2\left\{2T_1+\frac{E}{A}\right\} + T_1^2 = 0$$

11

Sem hefur lausnir

$$T_2 = T_1 + \frac{E}{2A} \pm \sqrt{\left(\frac{E}{2A}\right)^2 + \frac{ET_1}{A}}$$

fyrir $T_1 = 30°C$ og $T_2 = 20°C$ er $E = 0{,}3\,E_{max}$

Notum $T_2^2 - T_2\left\{2T_1 + \frac{E}{A}\right\} + T_1^2 = 0$

sem $T_2 - 2T_1 - \frac{E}{A} + \frac{T_1^2}{T_2} = 0$

og $\frac{E}{A} = -2T_1 + T_2 + \frac{T_1^2}{T_2} = T_1\left\{-2 + \frac{T_2}{T_1} + \frac{T_1}{T_2}\right\}$

$$= 303\left\{-2 + \frac{293}{303} + \frac{303}{293}\right\} = 0{,}341$$

12

Notum síðan aftur $T_2^2 - T_2\left\{2T_1 + \frac{E}{A}\right\} + T_1^2 = 0$ til að finna

$$T_1 = T_2 \pm \frac{1}{2}\sqrt{(2T_2)^2 - 4T_2\left(-\frac{E}{A} + T_2\right)}$$

og reiknum $* 3{,}3333$

og fáum þá $T_1 \simeq 311.3\,K = 38.3\,°C$

1

14-03 $R = 10\,\Omega$ viðnám við 300K ← fast

$I = 5A$ send um það í 2 mínútur $= \Delta t$

Gleymum straumgjafanum

a) ΔS í viðnáminu ↔ Kerfi

b) ΔS í Alheiminum

b) varmi tekinn úr viðnáminu yfir í umhverfið $\Delta Q = \overbrace{I^2 R \cdot \Delta t}^{A t L}$

$= 5^2 \cdot 10 \cdot 120 = 3000\,J$

$S = \frac{\Delta Q}{T} = \frac{30000\,J}{300K} = 100\,J/K$

a) viðnámið er kalt, engin varmi helst í því $\rightarrow \Delta Q = 0$ og $\Delta S = 0$.

Upphafsástand viðnáms er sama og lokaástandið $\rightarrow \Delta S$. Það er ekki svo um geyminn sem tekur við ΔQ

2

14-04 ΔS fyrir

a) bolli með vatni $T_i = 20°C$, tengt við geymi með $T = 80°C$ $\quad T_i = 293\,K$, $T = 353\,K$

$T_f = T$

Gera ráð fyrir varmarýmd bollans $C = 10^4\,J/K$

(Nýtum Example 14.1 í bók)

$$\Delta S_{system} = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{C\,dT}{T} = C\ln\left(\frac{T_f}{T_i}\right) = 10^4 \ln\left(\frac{353}{293}\right)$$

$$= 1{,}86 \cdot 10^3\,J/K$$

b) fyrir geyminn

$$\Delta S_{res} = \int \frac{dQ}{T} = \frac{1}{T}\int dQ = -\frac{\Delta Q}{T} = \frac{C(T_i - T)}{T}$$

$$= \frac{10^4 \cdot (293-353)}{353} = -1{,}7 \cdot 10^3\,J/K$$

3

c) ΔS ef Carnotvél er notuð fyrir varmaflutningum milli þeirra

Carnot vélin er jafngeng $\rightarrow \Delta S = 0$

en meiri varmi er tekinn út úr geyminum en áður $\rightarrow \Delta S_{res}$ er stærri vegna lægrar nýtni vélarinnar. Hún getur þá séð W sem við getum ekki sagt mikið um....

14-05

Blýklumpur $C = 1\,kJ/K$ $\quad T_i = 200K \quad T_f = 100K$

a) hent út í vökvageymi með $T = 100K$

$$\Delta S_{blý} = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{C\,dT}{T} = C\ln\left(\frac{T_f}{T_i}\right) = C\ln\left(\tfrac{1}{2}\right)$$

$$\Delta S_{res} = \frac{\Delta Q}{T} = C\frac{(T_i - T_f)}{T_f} = C$$

$$\rightarrow \Delta S_{tot} = C - C\ln(2) = C(1-\ln(2))$$

4

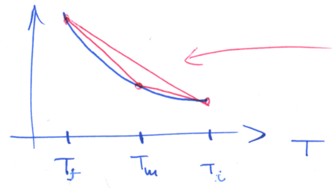
b) $T_i = 200K, \quad T_m = 150K, \quad T_f = 100K$

$$\Delta S_{blý} = \int \frac{dQ}{T} = C\int_{T_i}^{T_m} \frac{dT}{T} + C\int_{T_m}^{T_f} \frac{dT}{T} = \Delta S_{blý} \text{ áður}$$

$$\Delta S_{res} = \frac{C(T_i - T_m)}{T_m} + \frac{C(T_m - T_f)}{T_f}$$

$$= C\,\frac{50}{150} + C\,\frac{50}{100} = C\left(\tfrac{1}{3} + \tfrac{1}{2}\right) = \tfrac{5}{6}C$$

Ef við fjölgum $T_m \rightarrow \infty$ nálgumst við betur og betur heildið fyrir $\Delta S_{blý}$ $\rightarrow \Delta S_{total} = 0$



betri og betri nálgun á heildinu fyrir $\frac{dQ}{T}$

5

14-06

C $= 1\mu F$
$V = 100\,V$
$T = 273\,K$

ΔS þegar C er hlaðinn

a)

Hleðslan á þéttinum er $Q = CV$, stöðuorka hennar er $E_Q = \frac{1}{2}CV^2 = \frac{1}{2}QV$

Vinna rafhlöðunnar er $W_b = QV$

$\rightarrow$ varmi $\Delta Q = E_Q$ myndast í rafhlöðunni

$$\Delta S_{total} = \frac{CV^2}{2T} = 1.8\cdot 10^{-5}\ J/K$$

b) sama, því nú fer orkan í þéttinum öll í varma í viðnáminu við fast hitastig T

c) 1 mól gas við $T = 273K$ þanið við jafnan hita í 2V og jafngengt

$\rightarrow dU = 0$ $\quad dU = Tds - pdV$ 1. lögmálið $\rightarrow Tds = pdV$

6

$$\Delta S = \int_i^f ds = \int_V^{2V} \frac{p\,dV'}{T} = R\int_V^{2V} \frac{dV'}{V'} = R\ln 2$$

gert ráð fyrir kjörgasi $p = \frac{RT}{V}$

jafnhita – jafngengt $\rightarrow \Delta S_{tot} = 0$, og $\Delta S_{res} = -R\ln 2$

d) jafngengt – óvermið: $dQ = 0 \leftarrow \Delta S_{res} = 0$

$\Delta S_{tot} = 0 \rightarrow \Delta S = 0$

e) Joule þensla

$\Delta S = R\ln 2$
$\Delta S_{res} = 0$ $\Big\} \Rightarrow \Delta S_{tot} = R\ln 2$

14-07 (7)

n mól gass V, T > þenst í αV — kjörgas → $pV = nRT$

a) jafngeng jafnhitaþensla → $dU = 0$

$dU = Tds - pdV$ → $Tds = pdV$

$ds = \frac{p}{T} dV$ → $\Delta S = \int_{V_i}^{V_f} \frac{pdV}{T} = \int_{V_i}^{V_f} \frac{nR}{V} dV$

$= nR \ln\left(\frac{V_f}{V_i}\right) = nR\ln\left(\frac{\alpha V}{V}\right) = nR\ln\alpha$

b) Joule þensla. Reiknað í bók, en S er ástandsbreyta → ΔS er óháð leið → $\Delta S = nR\ln\alpha$

Athugum a) fyrir van der Waals ástandsjöfnu $(p + \frac{n^2 a}{V^2}) = \frac{nRT}{V-nb}$

$(p + \frac{n^2 a}{V^2})(V-nb) = nRT$ → $p = \frac{nRT}{V-nb} - \frac{n^2 a}{V^2}$

(8)

jafnhitaþensla $S = S(V,T)$ — $\left(\frac{\partial p}{\partial T}\right)_V$ ← Maxwell venst

$ds = \underbrace{\left(\frac{\partial S}{\partial T}\right)_V dT}_{=0} + \left(\frac{\partial S}{\partial V}\right)_T dV = \left(\frac{\partial S}{\partial V}\right)_T dV$

→ $\Delta S = \int_{V_i}^{V_f} \left(\frac{\partial p}{\partial T}\right)_V dV = \int_{V_i}^{V_f} \frac{nR\,dV}{V-nb} = nR\ln(V-nb)\Big|_{V_i}^{V_f}$

$= nR\ln\left(\frac{V_f - nb}{V_i - nb}\right) = nR\ln\left(\frac{\alpha V - nb}{V-nb}\right)$

í b) hve mikið breytist T?

Joul þensla adiabatisk (óvenuin)

$ds = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$

(9)

óvenuið → $ds = 0$

ATH

→ $\left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV = 0$

→ $dT = -\left(\frac{\partial T}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_T dV$, $\frac{C_V}{T} = \left(\frac{\partial S}{\partial T}\right)_V$

$= -\frac{T}{C_V}\left(\frac{\partial S}{\partial V}\right)_T dV$ — Maxwell $\left(\frac{\partial p}{\partial T}\right)_V$

$= -\frac{T}{C_V}\left(\frac{\partial p}{\partial T}\right)_V dV$

van der Waals $\left(\frac{\partial p}{\partial T}\right)_V = \frac{nR}{V-nb}$

→ $\frac{dT}{T} = -\frac{1}{C_V}\frac{nR\,dV}{V-nb}$

→ $\int_{T_i}^{T_f} \frac{dT}{T} = -\frac{nR}{C_V}\int_{V_i}^{V_f} \frac{dV}{V-nb}$

Ekki rétt hér að $ds = 0$. Óvenuið, en ekki jafngengt → $dU = 0$. Rétta lausnin er í næsta staumi í dæmi 16-02. Þessi lausn er fyrir jafngengt óvenuið ferli

(10)

$\ln\left(\frac{T_f}{T_i}\right) = -\frac{nR}{C_V}\ln(V-nb)\Big|_{V_i}^{V_f} = -\frac{nR}{C_V}\ln\left(\frac{V_f - nb}{V_i - nb}\right)$

$= -n(\gamma-1)\ln\left(\frac{\alpha V - nb}{V-nb}\right)$

→ $\frac{T_f}{T_i} = \left(\frac{V-nb}{\alpha V - nb}\right)^{n(\gamma-1)}$

ef $b \to 0$ $(a \to 0)$ fæst kjörgas, um það gildir $TV^{\gamma-1} = $ fasti

svarið okkar hefur þetta markgildi

(1)

16-02

(i) Sýnið að $\left(\frac{\partial T}{\partial V}\right)_U = -\frac{1}{C_V}\left\{T\left(\frac{\partial p}{\partial T}\right)_V - p\right\}$

notum (16.64)

$$\left(\frac{\partial T}{\partial V}\right)_U = -\left(\frac{\partial T}{\partial U}\right)_V \left(\frac{\partial U}{\partial V}\right)_T \qquad \text{og} \qquad \left(\frac{\partial U}{\partial T}\right)_V = C_V$$

sem kemur

$$dU = TdS - pdV \to \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p$$

$$\left(\frac{\partial T}{\partial V}\right)_U = -\frac{1}{C_V}\left\{T\left(\frac{\partial S}{\partial V}\right)_T - p\right\}$$

og Maxwell gefur $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$

(2)

þannig fæst að lokum

$$\left(\frac{\partial T}{\partial V}\right)_U = -\frac{1}{C_V}\left\{T\left(\frac{\partial p}{\partial T}\right)_V - p\right\} \qquad (*)$$

Sem minnir á að lausn mín á 14-07 er röng
Ég gerði ráð fyrir að $dS = 0$ þar (í stað að U = fasti)
$dQ_{rev} = 0$ → jafngengt óvermið ferli er líka jafn óreiðu ferli, en Joule-þenslan er ekki jafngeng því þarf að nota í dæminu (*)

$$\Delta T = -\frac{1}{C_V}\int_V^{\alpha V} dV \left\{T\left(\frac{\partial p}{\partial T}\right)_V - p\right\}$$

(3)

og þegar ástandsjafnan

$$p = \frac{nRT}{V - nb} - \frac{n^2 a}{V^2}$$

er notuð fæst

$$\left\{T\left(\frac{\partial p}{\partial T}\right)_V - p\right\} = \frac{n^2 a}{V^2}$$

og því

$$\Delta T = -\frac{n^2 a}{C_V}\int_V^{\alpha V} \frac{dV'}{V'^2} = +\frac{n^2 a}{C_V}\frac{1}{V'}\Big|_V^{\alpha V}$$

$$= \frac{n^2 a}{C_V}\left\{\frac{1}{\alpha V} - \frac{1}{V}\right\} = \frac{n^2 a}{C_V V}\left\{\frac{1-\alpha}{\alpha}\right\}$$

$$= -\frac{n^2 a}{C_V V}\left\{\frac{\alpha - 1}{\alpha}\right\}$$

(4)

(ii) Sýna að

$$\left(\frac{\partial T}{\partial V}\right)_S = -\frac{1}{C_V} T \left(\frac{\partial p}{\partial T}\right)_V$$

sem var einmitt það sem ég reiknaði í 14-07

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

$$dS = 0 \to \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV = 0$$

$$\to \left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial T}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_T$$

notum $\frac{C_V}{T} = \left(\frac{\partial S}{\partial T}\right)_V$

$$= -\frac{T}{C_V}\left(\frac{\partial S}{\partial V}\right)_T$$

og síðan Maxwell $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$

$$= -\frac{T}{C_V}\left(\frac{\partial p}{\partial T}\right)_V$$

5

(iii) Sýna að

$$\left(\frac{\partial T}{\partial p}\right)_H = \frac{1}{C_p}\left\{T\left(\frac{\partial V}{\partial T}\right)_p - V\right\}$$

Byrjum með

$$\left(\frac{\partial T}{\partial p}\right)_H = -\left(\frac{\partial T}{\partial H}\right)_p\left(\frac{\partial H}{\partial p}\right)_T$$

notum $dH = TdS + Vdp$

$$\to \left(\frac{\partial H}{\partial T}\right)_p = T\left(\frac{\partial S}{\partial T}\right)_p = C_p$$

$$\to \left(\frac{\partial H}{\partial p}\right)_T = T\left(\frac{\partial S}{\partial p}\right)_T + V$$

því fæst

$$\left(\frac{\partial T}{\partial p}\right)_H = -\frac{1}{C_p}\left\{T\left(\frac{\partial S}{\partial p}\right)_T + V\right\}.$$

6

Maxwell gefur $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$

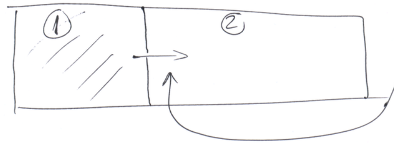
$$\to \left(\frac{\partial T}{\partial p}\right)_H = +\frac{1}{C_p}\left\{T\left(\frac{\partial V}{\partial T}\right)_p - V\right\}$$

(i) ← Joul þensla $\left(\frac{\partial T}{\partial V}\right)_U$ ekki jafnvægi

(ii) ← óvermin jafnvægi þensla $\left(\frac{\partial T}{\partial V}\right)_S$

(iii) ← Joule-Kelvin þensla (Joule-Thomson)

óvermin, ekki jafnvægi



í gegnum ventil

tvær bullur eins og í

dæmi 12-05

7

b) kjörgas

$$\left(\frac{\partial T}{\partial V}\right)_U = -\frac{1}{C_V}\left\{T\left(\frac{\partial p}{\partial T}\right)_V - p\right\} \qquad p = \frac{nRT}{V}$$

$$= 0$$

$$\left(\frac{\partial T}{\partial p}\right)_H = +\frac{1}{C_p}\left\{T\left(\frac{\partial V}{\partial T}\right)_p - V\right\} = 0$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\frac{1}{C_V}T\left(\frac{\partial p}{\partial T}\right)_V = -\frac{T}{C_V}\frac{nR}{V}$$

$$\to dT = -\frac{nRT}{C_V}\frac{dV}{V} = -\frac{2}{3}T\frac{dV}{V}$$

$$\to \frac{dT}{T} = -(\gamma-1)\frac{dV}{V}$$

$\frac{C_V}{Rn} = \frac{3}{2}$

$\gamma = \frac{5}{3}$

8

16-03 Sýna að

$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{C_p - C_V}{V\beta_p} - p$$

þ.s. $\beta_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$

Notum að $U(V,T)$

$$dU = đQ + đW = đQ - pdV$$

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

$$\to đQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left\{\left(\frac{\partial U}{\partial V}\right)_T + p\right\}dV$$

$$C_V = \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$C_p = \left(\frac{\partial Q}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_V + \left\{\left(\frac{\partial U}{\partial V}\right)_T + p\right\}\left(\frac{\partial V}{\partial T}\right)_p$$

(9)

$$\rightarrow \left(\frac{\partial U}{\partial V}\right)_T + p = C_p \underbrace{\left(\frac{\partial T}{\partial V}\right)_p}_{\frac{1}{\beta_p V}} - \underbrace{\left(\frac{\partial U}{\partial T}\right)_V}_{C_V} \underbrace{\left(\frac{\partial T}{\partial V}\right)_p}_{\frac{1}{\beta_p V}}$$

$$\rightarrow \boxed{\left(\frac{\partial U}{\partial V}\right)_T = \frac{C_p - C_V}{\beta_p V} - p}$$

16-04 $U = U(S,V)$ náttúrulegar breytur

finnið pá jöfnur fyrir T og p

1. Løgmálið $dU = TdS - pdV$

$$dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV$$

(10)

$$\rightarrow T = \left(\frac{\partial U}{\partial S}\right)_V \quad \text{og} \quad p = -\left(\frac{\partial U}{\partial V}\right)_S$$

b) Setjum sum svo ~~at vit~~ þekkjum V, T og $U(V,T)$

Hvør er pá jafnan fyrir p?

nýtum Maxwell

$$dU = TdS - pdV$$

$$\rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

Komum yfir í eina afleiðu með

$$\left(\frac{\partial U}{\partial V}\right)_T = T^2\left(\frac{\partial (\frac{p}{T})}{\partial T}\right)_V \rightarrow \left(\frac{\partial (\frac{p}{T})}{\partial T}\right)_V = \frac{1}{T^2}\left(\frac{\partial U}{\partial V}\right)_T$$

heildum

$$\rightarrow \frac{p}{T} = \int \frac{dT}{T^2}\left(\frac{\partial U}{\partial V}\right)_T + f(V)$$

(11)

16-07 Höfum (16.79) Kjörgas

$$S = C_v \ln T + R \ln V + \text{fasti}$$

nýtum $pV = RT$ (fyrir eitt mól)

$$\rightarrow S = C_v \ln(pV) + R\ln V + C_1 \qquad \Big| \; \frac{R}{\gamma - 1} = C_v$$

$$= C_v \ln(pV) + C_v(\gamma-1)\ln V + C_1$$

$$= C_v \ln(pV) + C_v \ln(V^{\gamma-1}) + C_1$$

$$= C_v \ln(pV^{\gamma}) + C_1 , \qquad \rho = \frac{M}{V}$$

$$= C_v \ln\left(\frac{p}{\rho^{\gamma}}\right) + C_2$$

(12)

18-02

$$H = G - T\left(\frac{\partial G}{\partial T}\right)_p$$

$$\hookrightarrow G - H = T\left(\frac{\partial G}{\partial T}\right)_p$$

$$\rightarrow \Delta G - \Delta H = T\left(\frac{\partial \Delta G}{\partial T}\right)_p$$

$$dG = Vdp - SdT \rightarrow \Delta G - \Delta H = -T\Delta S$$

Ef $T \rightarrow 0 \rightarrow \Delta S \rightarrow 0$

$$\rightarrow \Delta G - \Delta H \rightarrow 0$$

(19.2) (1)

Gasfastinn $R = 8.314 \frac{J}{mol \cdot K}$

Kjörgas

Við fast p, T = 298K $C_p = C_v + R = \frac{5}{2}R \simeq 20.8 \frac{J}{mK}$

→ einatóma gas: $C_p = 20.8 \frac{J}{mK}$

tvíatóma sameind gas með snúning: $C_p = \frac{7}{2}R \simeq 29.1 \frac{J}{mK}$

+ titring: $C_p = \frac{9}{2}R \simeq 37.4 \frac{J}{mK}$

fast efni (kristallar): $C_p = 3R = 24.9 \frac{J}{mK}$

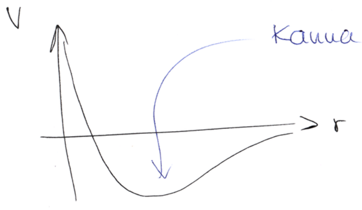
Al	= 24.35	fast	H_2	= 28.82	tvíatóma gas
Ar	= 20.79	einatóma gas	Fe	= 25.10	fasti
Au	= 25.42	fast	Pb	= 26.44	fasti
Cu	= 24.44	fast	Ne	= 20.79	einatóma gas
He	= 20.79	einatóma gas	⋮		

(19-03) (2)

$$V(r) = \frac{A}{r^n} - \frac{B}{r} \qquad A, B > 0 \quad \text{og} \quad n > 2$$

harðkjarna fráhrinding

Coulomb aðdráttur



Kanna mættið nærri lágmarkinu. Aðeins radial þátturinn (út þátturinn) skiptir máli. Lítum þess vegna á þetta sem „einvítt" mætti

Finnum lágmarkið: $\frac{dV}{dr} = -\frac{nA}{r^{n+1}} + \frac{B}{r^2} = 0$

$$\to \frac{r_0^{n+1}}{r^2} = \frac{nA}{B} \to r_0^{n-1} = \frac{nA}{B}$$

Beitum Taylor-liðun fyrir $r = r_0 + \Delta r$

$$V(r) \simeq V(r_0) + V'(r_0)\cdot \Delta r + \frac{1}{2}V''(r_0)\cdot(\Delta r)^2 + \dots$$

(3)

$$V(r) \simeq V(r_0) + \frac{1}{2}V''(r_0)\cdot(\Delta r)^2$$

$V(r_0)$ skiptir ekki máli, heldur ekki gildið á $V''(r_0)$, nema að það sé jákvætt. Um lögun pantsins er mættið fleygbogið

$$\to \langle E \rangle = \left\{ \underbrace{\frac{1}{2}k_BT}_{\text{hreyfiorka}} + \underbrace{\frac{1}{2}k_BT}_{\text{mættisorka}} \right\} = k_BT$$

(19-04) $E_i = \alpha_i x_i^2$ sýna að $\langle x_i^2 \rangle = \frac{k_BT}{2\alpha_i}$

$$\langle x_i^2 \rangle = \frac{\int_{-\infty}^{\infty} dx_i\, x_i^2 \exp\{-\beta\alpha_i x_i^2\}}{\int_{-\infty}^{\infty} dx_i \exp\{-\beta\alpha_i x_i^2\}} = \frac{\frac{1}{2}\sqrt{\frac{\pi}{(\beta\alpha_i)^3}}}{\sqrt{\frac{\pi}{\beta\alpha_i}}} = \frac{1}{2}\frac{1}{\beta\alpha_i} = \frac{k_BT}{2\alpha_i}$$

(21-01) (4)

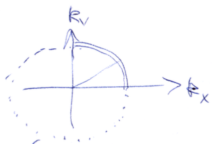
Sýna að Z_1^{2D} fyrir tvívítt gas á fleti A sé

$$Z_1^{2D} = \frac{A}{\lambda_{th}^2} \qquad \text{þ.s.} \quad \lambda_{th} = \frac{h}{\sqrt{2\pi m k_BT}}$$

Til þess þarf að finna ástandaþéttleikann í 2D

Samskonar bylgjufall, en aðeins tvær víddir

$$k_x = \frac{n_x\pi}{L}, \quad k_y = \frac{n_y\pi}{L}, \quad L^2 = A$$



Fyrir 3D var $g^{3D}(k)dk = \frac{\frac{1}{8}\cdot 4\pi k^2 dk}{\left(\frac{\pi}{L}\right)^3} = \frac{Vk^2dk}{2\pi^2}$

en fyrir 2D er

$$g^{2D}(k)dk = \frac{\frac{1}{4}\cdot 2\pi k dk}{\left(\frac{\pi}{L}\right)^2} = \frac{L^2 k dk}{2\pi} = \frac{A k dk}{2\pi}$$

5

$$Z_1^{2D} = \int_0^\infty dk\, g^{2D}(k)\, e^{-\beta E(k)} \quad , \quad E(k) = \frac{\hbar^2 k^2}{2m} \quad , \quad k^2 = k_x^2 + k_y^2$$

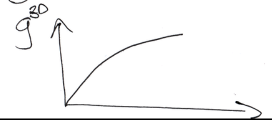
$$= \int_0^\infty dk \frac{Ak}{2\pi} \exp\left\{-\beta \frac{\hbar^2 k^2}{2m}\right\} \overset{(GR\text{-}3.461.3)}{=} \frac{A}{2\pi}\frac{2m}{\beta \hbar^2} = \frac{A 2\pi m k_B T}{h^2}$$

$$\Rightarrow \quad Z_1^{2D} = \frac{A}{\lambda_{th}^2} \quad \text{með} \quad \lambda_{th} = \frac{h}{\sqrt{2\pi m k_B T}}$$

En $\frac{dE}{dk} = \frac{\hbar^2 k}{m}$, bæði í 2D og í 3D

þess vegna er hægt að útbúa $g^d(E)dE$ og þá fæst

$g^{3D}(E) \sim \sqrt{E}$ $\qquad$ $g^{2D}(E) = $ fasti



6

21-02 Sackur-Tetrode

$$S = Nk_B\left\{\frac{5}{2} - \ln(n\lambda_{th}^3)\right\}$$

fyrir skammta kjörgas (óaðgreinanlegar eindir) er magnbundin. Athugum með skölun

$$\left.\begin{array}{l} V \to \alpha V \\ N \to \alpha N \end{array}\right\} \to n = \frac{N}{V} \to n$$

$$S' = \alpha N k_B\left\{\frac{5}{2} - \ln(n\lambda_{th}^3)\right\} = \alpha S$$

$\to$ S er magnbundin

7

En fyrir aðgreinanlegar eindir kjörgass fæst

$$S = Nk_B\left\{\frac{3}{2} - \ln\left(\lambda_{th}^3/V\right)\right\}$$

$$S' = \alpha N k_B\left\{\frac{3}{2} - \ln\left(\frac{\lambda_{th}^3}{\alpha V}\right)\right\} \neq \alpha S$$

fyrir aðgreinanlegar kjöreindir í gasi er S ekki magnbundin stærð!

1

21-03 Sýna að fjöldi ástanda gass með orku minni en E_{max} er

$$\int_0^{\sqrt{2mE_{max}/\hbar^2}} dk\, g(k) = \frac{V}{6\pi^2}\left(\frac{2mE_{max}}{\hbar^2}\right)^{3/2}$$

$$E_{max} = \frac{\hbar^2 k_{max}^2}{2m} \quad \to k_{max}^2 = \frac{2mE_{max}}{\hbar^2}$$

$$\hookrightarrow \int_0^{\sqrt{\frac{2mE_{max}}{\hbar^2}}} dk \frac{Vk^2}{2\pi^2} = \frac{V}{2\pi^2}\frac{k^3}{3}\Bigg|_0^{k_{max}} = \frac{V}{2\pi^2}\frac{k_{max}^3}{3} = \frac{V}{6\pi^2}\left\{\frac{2mE_{max}}{\hbar^2}\right\}^{3/2}$$

Setjum $E_{max} = \frac{3}{2}k_BT$ þá er fjöldi ástanda

$$\frac{V}{6\pi^2}\left\{\frac{2m\, 3k_BT}{2\hbar^2}\right\}^{3/2} \quad , \quad \text{og} \quad n_Q = \frac{1}{\hbar^3}\left\{\frac{mk_BT}{2\pi}\right\}^{3/2} = \left\{\frac{mk_BT}{2\pi\hbar^2}\right\}^{3/2}$$

$$\to \frac{6^{3/2}}{6\sqrt{\pi}} V n_Q = \boxed{\sqrt{\frac{6}{\pi}}} V n_Q$$

$\sim 1{,}38$

21-04 Atóm ~~með~~ tvö orkustig

g_2 ← margfeldni

g_1 $\quad \Big\} E = \Delta$

$$Z_{atóm} = \sum_i e^{-\beta E_i} = g_1 + g_2 e^{-\beta\Delta}$$

$$C = \frac{dU}{dT} = \frac{dU}{d\beta}\frac{d\beta}{dT} = \frac{d\left(\frac{1}{k_BT}\right)}{dT}\frac{dU}{d\beta} = -\frac{1}{k_BT^2}\frac{dU}{d\beta}$$

$$U = -\frac{d\ln Z_{atóm}}{d\beta} = -\frac{-\Delta g_2 e^{-\beta\Delta}}{g_1+g_2e^{-\beta\Delta}} = \frac{g_2\Delta e^{-\beta\Delta}}{g_1+g_2e^{-\beta\Delta}}$$

$$C = -\frac{1}{k_BT^2}\left\{\frac{-g_2\Delta^2 e^{-\beta\Delta}}{g_1+g_2e^{-\beta\Delta}} - \frac{g_2\Delta e^{-\beta\Delta}\cdot(-g_2\Delta e^{-\beta\Delta})}{(g_1+g_2e^{-\beta\Delta})^2}\right\}$$

$$C = -\frac{1}{k_BT^2}\left\{\frac{-g_2\Delta^2(g_1+g_2e^{-\beta\Delta})e^{-\beta\Delta}}{(g_1+g_2e^{-\beta\Delta})^2} + \frac{g_2^2\Delta^2 e^{-2\beta\Delta}}{(g_1+g_2e^{-\beta\Delta})^2}\right\}$$

$$= +\frac{1}{k_BT^2}\left\{\frac{g_1g_2\Delta^2e^{-\beta\Delta}}{(g_1+g_2e^{-\beta\Delta})^2}\right\}$$

ein atóma gas

$$Z_N = \frac{Z_1^N}{N!}$$

þ.s.

$$Z_1 = Z_{CM}\cdot Z_{atóm} = \frac{V}{\lambda_{th}^3}Z_{atóm}$$

$$= \frac{1}{N!}\left\{\frac{V}{\lambda_{th}^3}(g_1+g_2e^{-\beta\Delta})\right\}^N$$

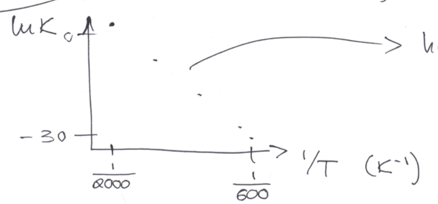
$$U = -\frac{d\ln Z_N}{d\beta} = -\frac{d}{d\beta}\left\{\ln\left(\frac{1}{N!}\left(\frac{V}{\lambda_{th}^3}\cdot Z_{atóm}\right)^N\right)\right\}$$

$$= -\frac{d}{d\beta}\left\{\ln\left[(VT^{3/2})^N\cdot Z_{atóm}^N\right]\right\} = -\frac{d}{d\beta}\left\{\frac{3N}{2}\ln T + N\ln Z_{atóm}\right\}$$

$$\to C = -\frac{1}{k_BT^2}\frac{dU}{d\beta} = N\left\{\frac{3}{2}k_B + \frac{g_1g_2\Delta^2e^{-\beta\Delta}}{k_BT^2(g_1+g_2e^{-\beta\Delta})^2}\right\}$$

þennan lið má bera saman við (21.31) ....

22-03 Bond enthalpy fyrir Br_2 gróflega metin af grafi



→ hallatala $-\frac{30}{\frac{1}{2000}-\frac{1}{600}} \sim -2.57\cdot 10^4$ K

$R = 8.314$ J/(mol K)

Hallatalan er $-\frac{\Delta H}{R}$

$$\to \Delta H \simeq 2.57\cdot 10^4\,\text{K}\cdot 8.314\,\frac{\text{J}}{\text{mol K}} \sim 2.14\cdot 10^5\,\frac{\text{J}}{\text{mol}}$$

$\sim 214\ \frac{\text{kJ}}{\text{mol}}$ innvermið

22-06 Jónun vetnis $H \rightleftharpoons p^+ + e^-$

skýra hvers vegna $\mu_H = \mu_p + \mu_e$

Í bókinni B-B er leidd út jafna (22.78)

$$\sum_{j=1}^{p+q}\nu_j\mu_j = 0 \quad\longrightarrow\quad -\mu_H + \mu_p + \mu_e = 0$$

eða $\mu_H = \mu_p + \mu_e$.

(6)

Gerum ráð fyrir að hér skipti aðeins máli grunnástandið og lægsta jónunarorka H: $-R$

$$Z_1^H = \frac{V}{\lambda_{th}^3} \cdot e^{\beta R}$$

fyrir eina orkustig H sem við tökum með

fyrir hreyfiorku H

$$Z = \frac{(Z_1^H)^{N_H}}{N_H!} \cdot \frac{(Z_1^p)^{N_p}}{N_p!} \cdot \frac{(Z_1^e)^{N_e}}{N_e!}$$

$$F = -k_B T \ln Z$$

$$\mu = \left(\frac{\partial F}{\partial N}\right)_{V,T}$$

$$F = -k_B T \left\{ N_H \ln(Z_1^H) - N_H \ln N_H + N_p \ln(Z_1^p) - N_p \ln N_p + N_e \ln(Z_1^e) - N_e \ln N_e \right\}$$

$$\rightarrow \mu_H = -k_B T \left\{ \ln(Z_1^H) - \ln N_H \right\} = -k_B T \ln\left(\frac{Z_1^H}{N_H}\right)$$

(7)

Á sama hátt

$$\mu_e = -k_B T \ln\left(\frac{Z_1^e}{N_e}\right) \quad \text{og} \quad \mu_p = -k_B T \ln\left(\frac{Z_1^p}{N_p}\right)$$

höfum $\mu_H = \mu_p + \mu_e$

$$\rightarrow -k_B T \ln\left(\frac{V e^{\beta R}}{\lambda_{th,H}^3 N_H}\right) = -k_B T \left\{ \ln\left(\frac{V}{\lambda_{th,e}^3 N_e}\right) + \ln\left(\frac{V}{\lambda_{th,p}^3 N_p}\right) \right\}$$

$$-k_B T \ln\left(\frac{e^{\beta R}}{\lambda_{th,H}^3 n_H}\right) = -k_B T \left\{ \ln\left(\frac{1}{n_e \lambda_{th,e}^3}\right) + \ln\left(\frac{1}{n_p \lambda_{th,p}^3}\right) \right\}$$

Munum að

$$\lambda_{th,x}^3 = \left\{ \frac{h}{\sqrt{2\pi m_x k_B T}} \right\}^3 = \frac{1}{n_Q^x}$$

$$\rightarrow -k_B T \ln\left(\frac{n_Q^H}{n_H} e^{\beta R}\right) = -k_B T \left\{ \ln\left(\frac{n_Q^e}{n_e}\right) + \ln\left(\frac{n_Q^p}{n_p}\right) \right\}$$

(8)

$$\ln\left(\frac{n_Q^H}{n_H} e^{\beta R}\right) = \ln\left(\frac{n_Q^e n_Q^p}{n_e n_p}\right)$$

$$\rightarrow \frac{n_e n_p}{n_H} = \frac{n_Q^e n_Q^p}{n_Q^H} e^{-\beta R}$$

Saha jafnan

$$m_x \sim m_p \rightarrow n_Q^H \sim n_Q^p \rightarrow \boxed{\frac{n_e n_p}{n_H} \simeq n_Q^e e^{-\beta R}}$$

b) $H \longleftrightarrow p + e^-$

$\rightarrow n_e = n_p$ því $N_e = N_p$ og V er fast.

Vinstra megin engin hleðsla

$n = n_H + n_p$ heildar þéttleiki ójónaðs og jónaðs vetnis

(9)

Ef $y = \frac{n_p}{n}$: hlutfall jónunar

Sýna að
$$\frac{y^2}{1-y} = \frac{e^{-\beta R}}{n \lambda_{th}^3} = \frac{n_Q^H e^{-\beta R}}{n}$$

$$n_p = ny \rightarrow n_H = n - n_p = n(1-y)$$

$$n_e = n_p = ny$$

höfðum
$$\frac{n_e n_p}{n_H} \simeq n_Q^e e^{-\beta R} \rightarrow \frac{n^2 y^2}{n(1-y)} = n_Q^e e^{-\beta R}$$

$$\rightarrow \frac{y^2}{1-y} = \frac{n_Q^e}{n} e^{-\beta R}$$

Finna jónun við $T = 1000\,K$, $n = 10^{20}\,m^{-3}$

$$n_Q^e \simeq 7.2 \cdot 10^{25}\,m^{-3} \rightarrow \frac{n_Q^e}{n} e^{-\beta R} \sim \frac{7.2 \cdot 10^{25}}{10^{20}} \exp\left\{-\frac{13.6}{8.617 \cdot 10^{-5} \cdot 1000}\right\}$$

$$\sim 2.1 \cdot 10^{-63}$$

mög smá jónun

$\to \frac{y^2}{1-y} \sim y^2 = 2.1\cdot 10^{-63}$ þá $y = 4.5\cdot 10^{-32}$

c) Hvers vegna eykst y þegar n lækkar

$$\frac{y^2}{1-y} = \frac{n_Q^e\, e^{-\beta R}}{n}$$

þá dregur úr sameiningu $p + e^- \to H$

22-07 NaCL í H_2O 0.9% af massa NaCl
er jafn þrýst blóði (isotonic)
sýna að osmósu þrýstingur blóðs
sé nálægt 8 atm

$$\Pi = \frac{n_B}{V} RT$$

Lausnin er með $n_B = 0{,}009\ \mathrm{kg/L} = 9\ \mathrm{kg/m^3}$ af NaCL

loft við 22.5 C $\sim$ 1,19 $\mathrm{kg/m^3}$

$$\to \frac{n_B^{NaCL}}{n_B^{air}} \sim 7{,}56$$

26-01 van der Waals gas. Sýna að κ_T fyrir $V = V_c$
uppfylli $\kappa_T = \frac{4b}{3R}(T-T_c)^{-1}$

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \qquad p = \frac{RT}{V-b} - \frac{a}{V^2}, \quad V_c = 3b$$

$$T_c = \frac{8a}{27Rb} \qquad p_c = \frac{a}{27b^2}$$

$$\left(\frac{\partial p}{\partial V}\right)_T = -\frac{RT}{(V-b)^2} + \frac{2a}{V^3}$$

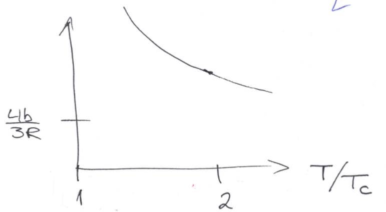
$$-\left(\frac{\partial V}{\partial p}\right)_T \frac{1}{V} = \frac{1}{\frac{RTV}{(V-b)^2} - \frac{2a}{V^2}} \xrightarrow{\text{þegar } V_c = 3b} \frac{1}{\frac{RT3b}{4b^2} - \frac{2a}{9b^2}}$$

$$\frac{1}{\frac{3RT}{4b} - \frac{2a}{9b^2}} = \frac{\frac{4b}{3R}}{T - \frac{8a}{27Rb}} = \frac{\frac{4b}{3R}}{T - T_c}$$

því fæst

$$\kappa_T = \frac{4b}{3R}\left(\frac{1}{T-T_c}\right) \quad \text{þegar } V = V_c$$

þegar $T \to T_c^+$



Ef $T \to T_c^-$
þá er $\kappa_T < 0$ og kerfið
er óstöðugt

26-02 Ef ástandsjafnan væri
$p(V-b) = RT$, b = fasti
jafnan er skrifuð fyrir 1 mole

Stærðargráða b?, sýna að $U = U(T)$:

3

Atóm gæti verið $1-3\ \mathring{A}$, $0{,}1-0{,}3$ nm

velju $0{,}2\cdot 10^{-9}$ m $\to$ rúmmál $\sim 8\cdot 10^{-30}\ m^3 = V_{atóm}$

Eitt mól tekur þá $N_A \cdot V_{atóm} = 6\cdot 10^{23}\cdot 8\cdot 10^{-30} \approx 4.8\cdot 10^{-6}\ m^3$

Fyrir 1 mól má því búast við að $b \lesssim 10^{-5}\ m^3$

$$p(V,T) = \frac{RT}{V-b}$$, tökum V,T sem breytur

$$p = -\left(\frac{\partial F}{\partial V}\right)_T \to F = -RT\ln(V-b) + f(T)$$

sem fremur vitum við að $dF = -SdT - pdV$, 1. lögmálið

$$S = -\left(\frac{\partial F}{\partial T}\right)_V = R\ln(V-b) - \frac{\partial f}{\partial T}$$

$$U = F + TS = -RT\ln(V-b) + f(T) + RT\ln(V-b) - T\frac{\partial f}{\partial T}$$

$$= f(T) - T\frac{\partial f}{\partial T}$$ aðeins háð T

4

26-03

Dieterici ástandsjafnan er

$$p(V-b) = RT\exp\left\{-\frac{a}{RTV}\right\}$$

umrita með $\tilde{p} = \frac{p}{p_c}$, $\tilde{T} = \frac{T}{T_c}$, $\tilde{V} = \frac{V}{V_c}$

Notum (26.38)

$$T_c = \frac{a}{4Rb},\quad p_c = \frac{a}{4e^2b^2},\quad V_c = 2b$$

$$p_c\tilde{p}(V_c\tilde{V} - b) = RT_c\tilde{T}\exp\left\{-\frac{a}{RT_c\tilde{T}\,V_c\tilde{V}}\right\}$$

$$\frac{a}{4e^2b^2}\tilde{p}(2b\tilde{V} - b) = R\frac{a}{4Rb}\tilde{T}\exp\left\{-\frac{a\,4Rb}{Ra\tilde{T}2b\tilde{V}}\right\}$$

$$\frac{\tilde{p}(2\tilde{V}-1)}{e^2} = \frac{R}{R}\tilde{T}\exp\left\{-2\frac{1}{\tilde{T}\tilde{V}}\right\}$$

5

$$\to \tilde{p}(2\tilde{V}-1) = \tilde{T}\exp\left\{+2 - \frac{2}{\tilde{T}\tilde{V}}\right\} = \tilde{T}\exp\left\{2\left(1 - \frac{1}{\tilde{T}\tilde{V}}\right)\right\}$$

26-04

Finna þenslustuðul van der Waals gass

$$\beta_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p,\quad p = \frac{RT}{V-b} - \frac{a}{V^2}$$

$$\beta_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T\left(\frac{\partial p}{\partial T}\right)_V$$

$$\left.\begin{aligned}\left(\frac{\partial p}{\partial V}\right)_T &= -\frac{RT}{(V-b)^2} + \frac{2a}{V^3}\\ \left(\frac{\partial p}{\partial T}\right)_V &= \frac{R}{V-b}\end{aligned}\right\} \to \beta_p = -\frac{\frac{1}{V}\cdot\frac{R}{V-b}}{-\frac{RT}{(V-b)^2} + \frac{2a}{V^3}}$$

6

$$= \frac{1}{\frac{V(V-b)}{R}\left\{\frac{RT}{(V-b)^2} - \frac{2a}{V^3}\right\}} = \frac{1}{T\left\{\frac{V}{V-b} - \frac{2a(V-b)}{V^2RT}\right\}}$$

$$= \frac{1}{T\left\{1 + \frac{b}{V-b} - \frac{2a}{pV+a^2}\right\}}$$

nota ástandsjöfnuna

$$pV^2 + a = \frac{RTV^2}{V-b}$$

Í markpunkti

$$\left.\begin{aligned}V_c &= 3b\\ T_c &= \frac{8a}{27Rb}\\ p_c &= \frac{a}{27b^2}\end{aligned}\right\} \to \left.\begin{aligned}\frac{V}{V-b} &\to \frac{3}{2}\\ \frac{2a(V-b)}{V^2RT} &\to \frac{3}{2}\end{aligned}\right\} \to \beta_p \to \infty$$

26-06

Heildar orka eins móls van der Waals gass er

$$U = \frac{f}{2}RT - \frac{a}{V}$$

þar sem f er fjöldi frelsisgráða

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{f}{2}R \,,$$

Munum (11.25)

$$C_p - C_V = \underbrace{\Big\{\underbrace{\left(\frac{\partial U}{\partial V}\right)_T}_{\frac{a}{V^2}} + p\Big\}}_{\frac{RT}{V-b}} \overbrace{\left(\frac{\partial V}{\partial T}\right)_p}^{V\beta_p}$$

$\frac{RT}{V-b}$ ← frá ástandsjöfnunni

$$V\beta_p = \frac{\left(\frac{V}{T}\right)}{\frac{V}{V-b} - \frac{2a(V-b)}{V^2RT}}$$

dæmi 26-04

$$\to C_p - C_V = \frac{R}{1 - \frac{2a}{VRT}\frac{(V-b)^2}{V^2}}$$

gerum ráð fyrir að $V >> b$ $\quad a << VTR$

$$\to C_p - C_V \simeq R\left\{1 + \frac{2a}{VRT}\right\} \simeq R + \frac{2a}{VT}$$

29-03

$$f(E) = \frac{1}{e^{\beta(E-\mu)}+1} = \frac{e^{-\frac{1}{2}\beta(E-\mu)}}{e^{\frac{1}{2}\beta(E-\mu)} + e^{-\frac{1}{2}\beta(E-\mu)}}$$

$$= \frac{1}{2}\left\{\frac{e^{\frac{\beta}{2}(E-\mu)} + e^{-\frac{\beta}{2}(E-\mu)}}{e^{\frac{\beta}{2}(E-\mu)} + e^{-\frac{\beta}{2}(E-\mu)}} - \frac{e^{\frac{\beta}{2}(E-\mu)} - e^{-\frac{\beta}{2}(E-\mu)}}{e^{\frac{\beta}{2}(E-\mu)} + e^{-\frac{\beta}{2}(E-\mu)}}\right\}$$

$$= \frac{1}{2}\left\{1 - \tanh\left(\frac{\beta}{2}(E-\mu)\right)\right\}$$

$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$, $\quad\cosh(x)$ er jafnstætt um $x=0$

$\sinh(x)$ er oddstætt um $x=0$

→ $\tanh(x)$ er oddstætt um $x=0$

Eða betur sagt $\tanh(x)$ er andsamhverft um $x=0$

→ $f(E)$ er andsamhverft um $E=\mu$

$E << \mu \to f(E) \to 1$

$E >> \mu \to f(E) \to 0$

$\cosh x = 1$ ($x \to 0$)

$\sinh x \simeq x$ ($x \to 0$)

Þegar $E \sim \mu \to \tanh\left(\frac{\beta}{2}(E-\mu)\right) \simeq \frac{1}{2}\beta(E-\mu)$

$$\to f(E) \simeq \frac{1}{2}\left\{1 - \frac{\beta}{2}(E-\mu)\right\} \quad \text{þ. } E \sim \beta$$

③

30-02 Sýna að þrýstingur p í fermigasi við $T=0$ sé $p=\frac{2}{5}nE_F$

Munum eftir (30.30) $p = \frac{2U}{3V}$

og (30.31)

$$\langle E\rangle = \frac{\int_0^{E_F} dE\, E\, g(E)}{\int_0^{E_F} dE\, g(E)} = \frac{\int_0^{E_F} dE\, E\sqrt{E}}{\int_0^{E_F} dE\, \sqrt{E}}$$

$$= \frac{2E_F^{5/2}/5}{2E_F^{3/2}/3} = \frac{3}{5}E_F$$

$$p = \frac{2U}{3V} = \frac{2}{3}\frac{3}{5}E_F\cdot\frac{N}{V} = \underline{\frac{2}{5}nE_F}$$

④

30-03 Sýna að fyrir Fermigas með $g(E)$ fæst

$$\mu(T) = E_F - \frac{\pi^2}{6}(k_BT)^2\frac{g'(E_F)}{g(E_F)} + \dots$$

(30.38)

$$N = \int_0^\infty dE\, g(E)f(E) = \int_0^\mu dE\, g(E) + \frac{\pi^2}{6}(k_BT)^2\left(\frac{dg}{dE}\right)_{E=\mu} + \dots$$

1. Stigs liðun gefur

$$\int_0^\mu dE\, g(E) = \int_0^{E_F} dE\, g(E) + (\mu - E_F)g(E_F)$$

Þegar $T=0$ gildir $\mu = E_F$, eða $\mu(0) = E_F$

$\to N = \int_0^{E_F} dE\, g(E)$, N er fasti óháður T

⑤

$$\to (\mu - E_F)g(E_F) + \frac{\pi^2}{6}(k_BT)^2 g'(E_F) = 0$$

$$\to \mu = E_F - \frac{\pi^2}{6}(k_BT)^2\frac{g'(E_F)}{g(E_F)}$$

30-05 Sýna að B-E þétting sé ekki til í 2D

Þurfum fyrst að finna ástandaþéttleikann í 2D

$$g^{2D}(k) = \frac{2\pi k\,dk\,(2S+1)}{\left(\frac{2\pi}{L}\right)^2} = (2S+1)\frac{A\,k\,dk}{2\pi}, \quad \text{því } A = L^2$$

$$E = \frac{\hbar^2k^2}{2m} \to dE = \frac{\hbar^2 k\,dk}{m} \qquad \to g(E)dE = (2S+1)\frac{Am}{2\pi\hbar^2}dE$$

ástandaþéttleikinn í 2D er fasti

Því þarf ekki að hafa áhyggjur af að $E=0$ sé ekki með í heildi....

⑥

$$N = \int_0^\infty \frac{dE\, g(E)}{e^{\beta(E-\mu)}-1} = \frac{(2S+1)Am}{2\pi\hbar^2}\underbrace{\int_0^\infty \frac{dE}{z^{-1}e^{\beta E}-1}}_{k_BT\,\Gamma(1)\,Li_1(z)}, \quad z = e^{\beta\mu}$$

$$\to N = \frac{2S+1}{2\pi}\frac{Am}{\hbar^2}(k_BT)\,Li_1(z)$$

höfum líka $\lambda_{th}^2 = \frac{h^2}{2\pi m k_BT} = \frac{2\pi\hbar^2}{mk_BT}$

og því

$$N = \frac{(2S+1)A}{\lambda_{th}^2}Li_1(z), \quad Li_1(z) \xrightarrow[z\to 1]{} -\ln(1-z) \to \infty$$

$$n^{2D} = \frac{(2S+1)}{\lambda_{th}^2}Li_1(z)$$

Því finnst ekker ósamræmi milli hægri og vinstri hliðar, báðir geta stefnt á $+\infty$, án þess að setja þurfi $E=0$ ástandið sérstaklega á stórsæjan hátt

30-06

Í 3D, er setni fyrsta örvaða ástandsins líka stórsæ þegar eindir þéttast í lægsta ástandið?

Þurfum strjál ástönd, setjum kerfið í kassa

$$E(n_x,n_y,n_z) = \frac{\hbar^2\pi^2}{2mL^2}\left(n_x^2+n_y^2+n_z^2\right), \quad n_i = 1,2,3,\dots$$

Grunnástandið er $E(1,1,1)$, en fyrsta örvaða ástandið er 3-falt $E(2,1,1)$, $E(1,2,1)$ og $E(1,1,2)$

$$\Delta E = E(2,1,1) - E(1,1,1) = \frac{\hbar^2\pi^2}{2mL^2}(6-3) = \frac{3\hbar^2\pi^2}{2mL^2}$$

Ef $z=1$ (30.45) $\quad N = \left(\frac{L}{\lambda_{th}}\right)^3 \cdot 2.6 \quad \rightarrow \quad \Delta E \simeq \frac{3\hbar^2\pi^2}{2m\lambda_{th}^2} N^{-2/3} (2.6)^{-2/3}$

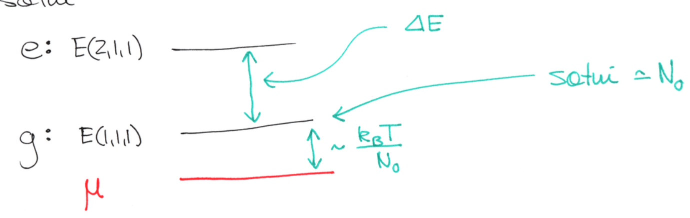
einföldum

$$\Delta E \approx \frac{\hbar^2}{m\lambda_{th}^2} N^{-2/3}$$

$$N_0(T) \simeq \frac{1}{e^{-\mu\beta}-1} \approx \frac{1}{1-1-\mu\beta} = -\frac{k_BT}{\mu}$$

$$\rightarrow \mu \simeq -\frac{k_BT}{N_0}$$ mjög nærri 0

skoðum setni



$$\frac{N_e}{N_0} = \frac{\frac{1}{e^{\beta\Delta E}e^{-\beta\mu}-1}}{\frac{1}{e^{-\mu\beta}-1}} = \frac{e^{-\mu\beta}-1}{e^{\beta\Delta E}e^{-\beta\mu}-1} \approx \frac{-\mu\beta}{\Delta E\cdot\beta}$$

$$\rightarrow N_e \simeq N_0 \frac{|\mu|}{\Delta E}$$ setni örvaða ástandsins e

Fyrir rúmsentimetra He^4,

$$\mu \approx -\frac{k_BT}{N_0} \simeq -\frac{k_BT}{N_0} \approx -\frac{k_BT}{4\cdot10^{21}} \approx -2.5\cdot10^{-22}k_BT$$

ef við tökum $N_0 \sim N$

$$N \approx \frac{(0.01)^3 m^3}{\lambda_{th}^3} 2.6 \simeq$$

$$\lambda_{th} = \left(\frac{2\pi\hbar^2}{mk_BT}\right)^{1/2} \sim 8.7\cdot10^{-10}\,m \text{ við } 1K$$

$T \sim 1K$

$$\rightarrow N \simeq \frac{(0.01)^3 m^3 \cdot 2.6}{(8.7\cdot10^{-10})^3 m^3} \approx 4\cdot10^{21}$$

$$\frac{\Delta E}{k_BT} \simeq \frac{(1.05\cdot10^{-34})^2\cdot N^{-2/3}}{4\cdot1.6\cdot10^{-27}(8.7\cdot10^{-10})^2(1.38\cdot10^{-23})} \simeq 7\cdot10^{-16}$$ fyrir 1K

Því er

$$N_e \simeq N_0\cdot\frac{|\mu|}{\Delta E} \simeq N_0\cdot\frac{2.5\cdot10^{-22}}{7\cdot10^{-16}} \approx \underbrace{N_0\cdot3.6\cdot10^{-7}}$$

Þannig að við metum setni örvaða ástandsins miklu minni en grunnástandsins

(24-03) Debye í d-vídd, sýna að $C \sim T^d$ ①

Í 3D $g^{3D}(q)dq = \frac{3Vq^2dq}{2\pi^2} \to g^d(q) \sim q^{d-1}$

nú gildir að $\omega = v_s q \to g^d(\omega) \sim \omega^{d-1}$

því fæst að

$$U = \text{fasti}\,1 + \text{fasti}\,2 \cdot \underbrace{\int_0^{\omega_D} \frac{d\omega\,\omega^d}{e^{\hbar\omega\beta}-1}}$$

$$\frac{1}{\beta^{d+1}}\int_0^{\omega_D\beta} \frac{d(\omega\beta)\,(\omega\beta)^d}{e^{\hbar\omega\beta}-1} = \frac{1}{\beta^{d+1}}\int_0^{x_D}\frac{dx\cdot x^d}{e^x-1}$$

$$= (k_BT)^{d+1}\cdot \text{fasti} \to C = \frac{\partial U}{\partial T} \sim T^d$$

(24-04) Finna ástandsþéttleika einvíðrar línul. keðju ②

$$\left\{ g(\omega) = \frac{2L}{\pi a}\frac{1}{\sqrt{\frac{4K}{m}-\omega^2}} \right\}$$

samkvæmt (24.33)

$$\omega = \sqrt{\frac{4K}{m}}\left|\sin\left(\frac{qa}{2}\right)\right|$$

fyrir 1D þá er $g(q)dq = \frac{2dq}{\left(\frac{2\pi}{L}\right)} = \frac{Ldq}{\pi}$

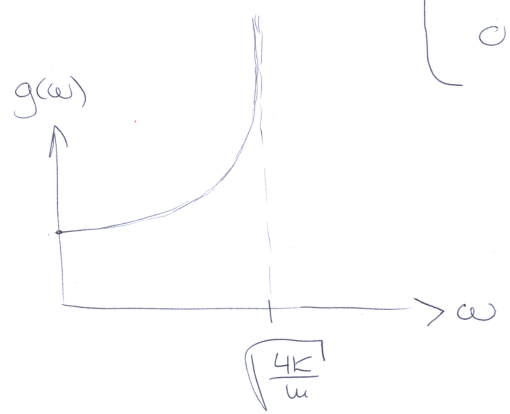
$$\to \frac{d\omega}{dq} = \sqrt{\frac{4K}{m}}\,\frac{a}{2}\cos\left(\frac{qa}{2}\right) \quad (\text{ef } q \neq 0)$$

$$\cos\left(\frac{qa}{2}\right) = \sqrt{1-\sin^2\left(\frac{qa}{2}\right)} = \sqrt{1-\frac{m\omega^2}{4K}}$$

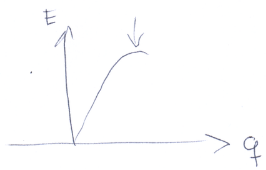
③

$$\to \frac{d\omega}{dq} = \frac{a}{2}\sqrt{\frac{4K}{m}-\omega^2}$$

$$g(\omega) = g(q)\frac{dq}{d\omega} = \begin{cases}\frac{2L}{\pi a}\left\{\frac{4K}{m}-\omega^2\right\}^{-1/2} & \text{ef } \omega < \sqrt{\frac{4K}{m}}\\ 0 & \text{ef } \omega > \sqrt{\frac{4K}{m}}\end{cases}$$



← ástæða sérstöðupunktsins sést best í tvíslurrófinu



(23-05) Varmageislun er hægt að lýsa sem ljóseindagasi ④

með ... $\left\{ U = u(T)V, \quad p = \frac{u(T)}{3} \right.$ | 1. lögmálið $\left\{ dU = TdS - pdV \right.$

sýna að

(a) óreiðuþéttnin sé $s = \frac{4p}{T}$

$$u = \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p = T\left(\frac{\partial S}{\partial V}\right)_T - \frac{u}{3}$$

$$\to \frac{4}{3}u = T\left(\frac{\partial S}{\partial V}\right)_T \to \left(\frac{\partial S}{\partial V}\right)_T = \frac{4u}{3T} = \frac{4p}{T}$$

ef $s = \left(\frac{\partial S}{\partial V}\right)_T$ fæst $s = \frac{4p}{T}$

5

b) $G = 0$

$$G = U + pV - TS = uV + \frac{u}{3}V - \frac{4u}{3}V = 0$$

c) $C_V = 3s$ á rúmmál

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V \qquad (16.65)$$

$$S = \frac{4u}{3T}V$$

og líka

Maxwell

$p = u(T)/3$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = V\left(\frac{\partial u}{\partial T}\right)_V = 3V\left(\frac{\partial p}{\partial T}\right)_V$$

$$= 3V\left(\frac{\partial S}{\partial V}\right)_T = 3Vs$$

6

(d) $C_p \to \infty$

$p = \frac{u}{3}$

$$H = U + pV = \frac{4uV}{3}$$

$$C_p = \left(\frac{\partial H}{\partial T}\right)_p$$

því í raun fast u

$$\left(\frac{\partial H}{\partial T}\right)_p = \frac{4u}{3}\left(\frac{\partial V}{\partial T}\right)_p = \frac{-\left(\frac{\partial p}{\partial T}\right)_V}{\left(\frac{\partial p}{\partial V}\right)_T}$$

og $\left(\frac{\partial p}{\partial V}\right)_T = \frac{1}{3}\left(\frac{\partial u}{\partial V}\right)_T = 0$

$$\to C_p \to \infty$$

p er óháð hvað $u(T)$ → óháð hvað T
→ fast p er fast T ← þá er ekki hægt að hnika T

7

23-06 Kássumma ljóseindagass
sleppum núllpunktsorku

Notfærum okkar (24.19) fyrir hljóðeindir

$$\to \ln Z = \int_0^\infty d\omega\, g(\omega) \ln\left\{\frac{1}{1 - e^{-\hbar\omega\beta}}\right\}$$

tvær skautanir aftur

$$\omega = cq \quad \to \quad g(p)\,dq = \frac{4\pi q^2 dq}{\left(\frac{2\pi}{L}\right)^3} \cdot 2$$

$$\to g(\omega)\,d\omega = \frac{V\omega^2 d\omega}{\pi^2 c^3}$$

$$\to \ln Z = -\frac{V}{\pi^2 c^3}\int_0^\infty d\omega\, \omega^2 \ln\left\{1 - e^{-\beta\hbar\omega}\right\}$$

8

$$\to \ln Z = \frac{V\pi^2}{45\hbar^3\beta^3 c^3} = \frac{V\pi^2 (k_B T)^3}{45\hbar^3 c^3}$$

$$F = -k_B T \ln Z = -\frac{V\pi^2 (k_B T)^4}{45\hbar^3 c^3} = -\frac{4\sigma V T^4}{3c}$$

því $\sigma = \frac{\pi^2 k_B^4}{60 c^2 \hbar^3}$

$$S = -\left(\frac{\partial F}{\partial T}\right)_V = \frac{16\sigma V T^3}{3c}$$

$$U = F + TS = -\frac{4\sigma V T^4}{3c} + \frac{16\sigma V T^4}{3c} = \frac{4\sigma V T^4}{c}$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_T = \frac{4\sigma T^4}{3c}$$

$\to U = -3F$

$pV = \frac{U}{3}$

$S = \frac{4U}{3T}$

23-7

$$N = \int_0^\infty \frac{d\omega\, g(\omega)}{e^{\beta\hbar\omega}-1}, \quad g(\omega) = \frac{V\omega^2 d\omega}{\pi^2 c^3}$$

$$N = \frac{V}{\pi^2 c^3}\int_0^\infty \frac{d\omega\, \omega^2}{e^{\beta\hbar\omega}-1} = \frac{V}{\pi^2 c^3}\frac{1}{(\beta\hbar)^3}\int_0^\infty \frac{dx\, x^2}{e^x-1}$$

$$= \frac{V}{\pi^2 c^3}\frac{1}{(\beta\hbar)^3}\cdot \zeta(3)\Gamma(3) = \frac{V}{\pi^2 c^3}\frac{1}{(\beta\hbar)^3} 2\zeta(3)$$

og $\zeta(3) = 1{,}20206$

$$U = \frac{4\sigma V T^4}{c}, \quad \sigma = \frac{\pi^2 k_B^4}{60 c^2 \hbar^3}$$

$$N = \frac{V}{\pi^2 c^3}\frac{(k_B T)^3}{\hbar^3} 2\zeta(3) \rightsquigarrow \frac{U}{N} = \frac{4\sigma V T^4 \cdot \pi^2 c^3 \hbar^3}{c \cdot V (k_B T)^3 2\zeta(3)}$$

$$\rightarrow \frac{U}{N} = \frac{4\pi^2 k_B^4}{60 c^2 \hbar^3}\frac{V T^4}{c}\frac{\pi^2 c^3 \hbar^3}{V (k_B T)^3 2\zeta(3)} = \frac{\pi^4}{30\zeta(3)}(k_B T)$$

$$\simeq 2.7012\, k_B T$$

$$\frac{S}{N} = \frac{16\sigma V T^3}{3cN}$$

$$= \frac{16 V T^3}{3c}\left(\frac{\pi^2 k_B^4}{60 c^2 \hbar^3}\right)\frac{\pi^2 c^3 \hbar^3}{(k_B T)^3\, 2\zeta(3)} = \frac{2\pi^4}{45\zeta(3)} k_B$$

$$\simeq 3.602\, k_B$$

fyrir kjörgas er $\frac{U}{N} = \frac{3}{2} k_B T$

og $\frac{S}{N} = k_B\left\{\frac{5}{2} - \ln(n\lambda_{th}^3)\right\}$