(1)

2-1 Gerum ráð fyrir að $g(U) = CU^{3N/2}$, C er fasti
Kjörgas N: fjöldi einda

a) Sýna að $U = \frac{3}{2}N\tau$

Höfum $\frac{1}{\tau} = \left(\frac{\partial \sigma}{\partial U}\right)_N$ og $\sigma(N,U) = \ln\{g(N,U)\}$

$$\rightarrow \sigma = \ln\{CU^{3N/2}\} = \ln C + \frac{3N}{2}\ln U$$

$$\frac{1}{\tau} = \left(\frac{\partial \sigma}{\partial U}\right)_N = \frac{3N}{2}\cdot\frac{1}{U} \rightarrow U = \frac{3}{2}N\tau$$

Vex línulega með N og τ

b) Sýna að $\left(\frac{\partial^2 \sigma}{\partial U^2}\right)_N < 0$

$$\left(\frac{\partial^2 \sigma}{\partial U^2}\right)_N = \left(\frac{\partial}{\partial U}\left(\frac{\partial \sigma}{\partial U}\right)_N\right)_N = \left(\frac{\partial}{\partial U}\left\{\frac{3N}{2U}\right\}\right)_N =$$

$$= -\frac{3}{2}\frac{N}{U^2} < 0$$

Óreiðan er íhvolft fall af U

(3)

2-2

Módelið, N spunar með segulvægi m í segulsviði B. $N_\uparrow - N_\downarrow = 2s$

Finna jafnvægis gildi hlutfallsseglunarinnar

$$\boxed{\frac{M}{Nm} = 2\langle s\rangle\frac{1}{N}}$$

nota $g(N,s) \simeq g(N,0)\exp\left\{-\frac{2s^2}{N}\right\}$

$$\rightarrow \sigma(N,s) = \ln\{g(N,s)\} \simeq \ln(g(N,0)) - \frac{2s^2}{N}$$

því $|s| \ll N$ og $N \gg 1$

(4)

Hér gildir $U = -2smB \rightarrow s = -\frac{U}{2mB}$

$$\rightarrow \sigma(N,U) \approx \sigma_0 - \frac{2s^2}{N} = \sigma_0 - \frac{U^2}{2m^2B^2N}$$

með $\sigma_0 = \ln(g(N,0))$

$$\frac{1}{\tau} = \left(\frac{\partial \sigma}{\partial U}\right)_N = -\frac{U}{m^2B^2N}$$

$$\rightarrow U = -\frac{m^2B^2N}{\tau}$$

munum að τ hefur vídd orku hér

$$s = -\frac{U}{2mB} \quad \rightarrow \quad \langle s\rangle = -\frac{\langle U\rangle}{2mB}$$

og $\langle U\rangle = -\dfrac{m^2B^2N}{\tau}$

$$\rightarrow \langle s\rangle = \frac{m^2B^2N}{\tau\, 2mB} = \frac{mBN}{2\tau}$$

og því

$$\boxed{\frac{M}{Nm} = 2\langle s\rangle\frac{1}{N} = \frac{mB}{\tau}}$$

→ síðar fundum við að nákvæmlega gildi $= \tanh\left(\dfrac{mB}{\tau}\right)$

að þegar $\tau \gg mB$

Curie lögmálið fyrir meðseglum
gildir fyrir ekki of lágt τ

2-3 QHO, N - HO með tíðni ω
n - orkuskammtar

finna σ

jafna (1.55) gefur $g(N,n) = \dfrac{(N+n-1)!}{n!\,(N-1)!}$

$U = \hbar\omega n$ ← óháðir sveiflar

notum $\ln N! \simeq N\ln N - N$

$$\rightarrow \sigma(N,n) \approx \ln\left\{\frac{(N+n-1)!}{n!\,(N-1)!}\right\} = \ln(N+n-1)! - \ln n! - \ln(N-1)!$$

$$\approx (N+n)\ln(N+n) - (N+n) - n\ln n + n$$

$$- N\ln N + N = \underline{(N+n)\ln(N+n) - N\ln N - n\ln n}$$

b) $U = \hbar\omega\cdot n$ finna $\sigma(U,N)$

$$\boxed{n = \frac{U}{\hbar\omega}} \rightarrow \sigma(U,N) = \left(\frac{U}{\hbar\omega}+N\right)\ln\left(\frac{U}{\hbar\omega}+N\right) - N\ln N - \left(\frac{U}{\hbar\omega}\right)\ln\left(\frac{U}{\hbar\omega}\right)$$

$$\frac{1}{\tau} = \left(\frac{\partial\sigma}{\partial U}\right)_N = \frac{1}{\hbar\omega}\ln\left(\frac{U}{\hbar\omega}+N\right) + \left(\frac{U}{\hbar\omega}+N\right)\frac{1}{\left(\frac{U}{\hbar\omega}+N\right)}\frac{1}{\hbar\omega}$$

$$-\ln\left(\frac{U}{\hbar\omega}\right)\frac{1}{\hbar\omega} - \left(\frac{U}{\hbar\omega}\right)\frac{1}{\left(\frac{U}{\hbar\omega}\right)}\frac{1}{\hbar\omega}$$

$$= \frac{1}{\hbar\omega}\left\{\ln\left(\frac{U}{\hbar\omega}+N\right) + 1 - \ln\left(\frac{U}{\hbar\omega}\right) - 1\right\}$$

$$= \frac{1}{\hbar\omega}\left\{\ln\left(\frac{U}{\hbar\omega}+N\right) - \ln\left(\frac{U}{\hbar\omega}\right)\right\}$$

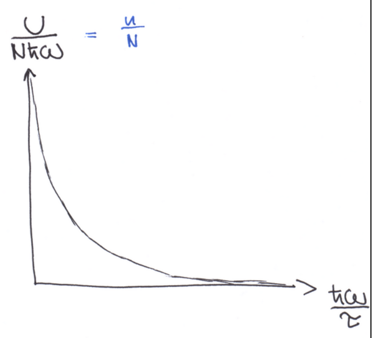
$$\rightarrow \frac{\hbar\omega}{\tau} = \ln\left\{\frac{\left(\frac{U}{\hbar\omega}+N\right)}{\frac{U}{\hbar\omega}}\right\} \rightarrow \exp\left(\frac{\hbar\omega}{\tau}\right) = \frac{\frac{U}{\hbar\omega}+N}{\frac{U}{\hbar\omega}}$$

$$\rightarrow \exp\left(\frac{\hbar\omega}{\tau}\right) = \frac{U + N\hbar\omega}{U} = 1 + \frac{N\hbar\omega}{U}$$

$$\rightarrow \exp\left(\frac{\hbar\omega}{\tau}\right) - 1 = \frac{N\hbar\omega}{U}$$

og

$$\boxed{U = \frac{N\hbar\omega}{\exp\left(\frac{\hbar\omega}{\tau}\right) - 1}}$$



(3-1) Two state system $0, \varepsilon$

a) finna F

$$Z = 1 + e^{-\frac{\varepsilon}{\tau}}, \quad F = -\tau \ln Z = -\tau \ln\left\{1 + e^{-\frac{\varepsilon}{\tau}}\right\}$$

b) Finnum σ frá

$$\left(\frac{\partial F}{\partial \tau}\right)_V = -\sigma \quad \rightarrow \quad \boxed{\sigma = \tau \frac{e^{-\frac{\varepsilon}{\tau}}\left(+\frac{\varepsilon}{\tau^2}\right)}{1 + e^{-\varepsilon/\tau}} + \ln\left\{1 + e^{-\varepsilon/\tau}\right\}}$$

og U frá

$$U = F + \tau\sigma$$

$$= -\tau \ln\left\{1 + e^{-\frac{\varepsilon}{\tau}}\right\} + \tau \ln\left\{1 + e^{-\frac{\varepsilon}{\tau}}\right\} + \frac{\varepsilon e^{-\frac{\varepsilon}{\tau}}}{1 + e^{-\varepsilon/\tau}}$$

$$= \frac{\varepsilon e^{-\frac{\varepsilon}{\tau}}}{1 + e^{-\varepsilon/\tau}}$$

Eðlilegra að nota

$$U = \frac{\sum_s \varepsilon_s e^{-\frac{\varepsilon}{\tau}}}{Z} = \tau^2\left(\frac{\partial \ln Z}{\partial \tau}\right)$$

$$= \frac{\varepsilon e^{-\frac{\varepsilon}{\tau}}}{1 + e^{-\varepsilon/\tau}}$$

eins og við sáum áður

(3-2) Nota Z til að finna M og χ sem föll af τ og B fyrir segulvogi í segulsviði | orka vogis $\pm mB$

n óháð vogi á rúmeiningu

Fyrir eittvogi fæst kórsumman

$$Z_1 = \exp\left(+\frac{mB}{\tau}\right) + \exp\left(-\frac{mB}{\tau}\right) = 2\cosh\left(\frac{mB}{\tau}\right)$$

N óháð vogi í heild, vegna formsins $F = -\tau \ln Z$ eða $Z = \exp\left(-\frac{F}{\tau}\right)$ sjáum við að N-vogis kórsumman er

$$Z = (Z_1)^N = 2^N \cosh^N\left(\frac{mB}{\tau}\right)$$

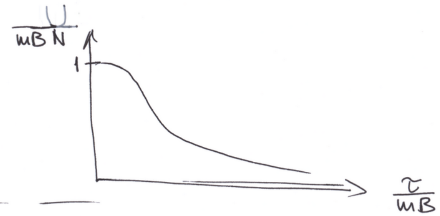
$$\ln Z = N \ln 2 + N \ln\left\{\cosh\left(\frac{mB}{\tau}\right)\right\}$$

$$U = +\tau^2\left(\frac{\partial \ln Z}{\partial \tau}\right) = +\tau^2 N \frac{\sinh\left(\frac{mB}{\tau}\right)}{\cosh\left(\frac{mB}{\tau}\right)}\left(+\frac{mB}{\tau^2}\right)$$

$$= + mB\boxed{N \tanh\left(\frac{mB}{\tau}\right)}$$

heildarfjöldi uppspuna

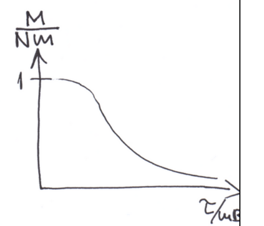
$$\frac{U}{V} = mBn\tanh\left(\frac{mB}{\tau}\right)$$



Á samskonar hátt og U er fundið frá ln Z má sjá að heildarsegulun er

$$M = \tau \frac{\partial \ln Z}{\partial B} = \tau N \frac{\sinh\left(\frac{mB}{\tau}\right)}{\cosh\left(\frac{mB}{\tau}\right)} m \frac{1}{\tau}$$

$$\rightarrow \frac{M}{V} = nm\tanh\left(\frac{mB}{\tau}\right)$$

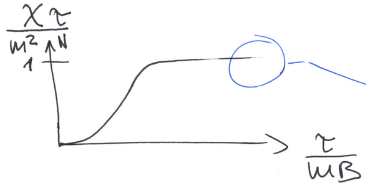


5

$$\chi = \frac{dM}{dB} \quad \text{Heildarviðtak}$$

$$= N\frac{m^2}{\tau}\,\mathrm{Sech}^2\left(\frac{mB}{\tau}\right) \;\rightarrow\; \frac{\chi}{V} = \frac{nm^2}{\tau}\,\mathrm{Sech}^2\left(\frac{mB}{\tau}\right)$$

$$= \frac{nm^2}{\tau}\,\frac{1}{\mathrm{Cosh}^2\left(\frac{mB}{\tau}\right)}$$



sjá c-lið

b) Finna F sem fall af τ og $x = \frac{M}{nm}$

$$F = -\tau \ln Z$$

$$= -\tau\left[N\ln 2 + N\ln\left\{\mathrm{Cosh}\left(\frac{mB}{\tau}\right)\right\}\right]$$

$$= -\tau N\left[\ln\left\{2\,\mathrm{Cosh}\left(\frac{mB}{\tau}\right)\right\}\right] = -\tau N \ln\left\{\frac{2\,\mathrm{Sinh}\left(\frac{mB}{\tau}\right)}{\tanh\left(\frac{mB}{\tau}\right)}\right\}$$

6

$$F = -\tau N \ln\left\{\frac{2\,\mathrm{Sinh}\left(\frac{mB}{\tau}\right)}{\frac{M}{nmV}}\right\}$$

$$= \tau N \ln\left(\frac{M}{nmV}\right) - \tau N \ln\left\{2\,\mathrm{Sinh}\left(\frac{mB}{\tau}\right)\right\}$$

c) Finna χ þegar $mB \ll \tau$

$$\frac{\chi}{V} = \frac{nm^2}{\tau}\,\frac{1}{\mathrm{Cosh}^2\left(\frac{mB}{\tau}\right)} \xrightarrow[mB \ll \tau]{} \frac{nm^2}{\tau}$$

7

(3-3) Einn HO $\quad \varepsilon_s = s\cdot\hbar\omega, \quad s = \{0,1,2,\ldots\}$

$$Z(\tau) = \sum_s \exp\left(-\frac{\varepsilon_s}{\tau}\right) = \sum_s \exp\left(-s\cdot\frac{\hbar\omega}{\tau}\right)$$

$$= \sum_s \left\{\exp\left(-\frac{\hbar\omega}{\tau}\right)\right\}^s = \frac{1}{1-\exp\left(-\frac{\hbar\omega}{\tau}\right)}$$

ef $e^{-\frac{\hbar\omega}{\tau}} < 1$

Sem er alltaf uppfyllt fyrir endanlegt hitastig

$$F = -\tau \ln Z$$

$$= \tau \ln\left\{1 - e^{-\frac{\hbar\omega}{\tau}}\right\}$$

Ef $\tau \gg \hbar\omega$

$$F \approx \tau \ln\left\{1 - 1 + \frac{\hbar\omega}{\tau} + \cdots\right\} = \tau \ln\left(\frac{\hbar\omega}{\tau}\right)$$

8

b) finna σ

$$\sigma = -\left(\frac{\partial F}{\partial \tau}\right)_V = -\left(\frac{\partial}{\partial \tau}\tau\ln\left\{1-e^{-\frac{\hbar\omega}{\tau}}\right\}\right)_V$$

$$= -\ln\left\{1-e^{-\frac{\hbar\omega}{\tau}}\right\} - \tau\frac{e^{-\frac{\hbar\omega}{\tau}}}{1-e^{-\frac{\hbar\omega}{\tau}}}\left(-\frac{\hbar\omega}{\tau^2}\right)$$

$$= -\ln\left\{1-e^{-\frac{\hbar\omega}{\tau}}\right\} + \left(\frac{\hbar\omega}{\tau}\right)\frac{e^{-\frac{\hbar\omega}{\tau}}}{1-e^{-\frac{\hbar\omega}{\tau}}}$$

$$= -\ln\left\{1-e^{-\frac{\hbar\omega}{\tau}}\right\} + \frac{\left(\frac{\hbar\omega}{\tau}\right)}{e^{\frac{\hbar\omega}{\tau}}-1}$$

og $\lim_{\tau\to 0} \sigma = 0$

36

Snúningur tvíatóma sameinda $\qquad \Sigma(j) = j(j+1)\Sigma_0 \;,\; j = 0, 1, \ldots$

Finna kórsummuna $Z_R(\tau)$ fyrir eina sameind

margfeldni hvers orkustigs er $g(j) = 2j+1$

$$Z_R(\tau) = \sum_j g(j)\, e^{-\Sigma(j)\Sigma_0/\tau}$$

$$= \sum_{j=0}^{\infty} (2j+1)\, e^{-\frac{j(j+1)\Sigma_0}{\tau}}$$

b) Reikna $Z_R(\tau)$ þ. $\tau \gg \Sigma_0$, þétt orkuróf miðað við τ

$\rightarrow$ breyta í heildi

$$Z_R(\tau) \xrightarrow[\frac{\Sigma_0}{\tau} \ll 1]{} \int_0^{\infty} (2x+1)\, e^{-x(x+1)\frac{\Sigma_0}{\tau}}\, dx$$

← gróf nálgun sem ég leyfi mér hér

sjá Thermodynamics and Statistical Mechanics, eftir Greiner, Neise og Stöcker á bls. 230 með betri umformun yfir í heildi

breyta $x(x+1) = y \;\rightarrow\; (2x+1)dx = dy$

$$\rightarrow Z_R(\tau) \xrightarrow[\frac{\Sigma_0}{\tau} \ll 1]{} \int_0^{\infty} dy\, e^{-y\frac{\Sigma_0}{\tau}} = \frac{\tau}{\Sigma_0}$$

c) Reikna $Z_R(\tau)$ þ. $\tau \ll \Sigma_0$

$$Z_R(\tau) \xrightarrow[\frac{\Sigma_0}{\tau} \gg 1]{} \left\{ 1 + 3e^{-\frac{2\Sigma_0}{\tau}} + \cdots \right\}$$

d) finna U og C sem föll af τ í báðum markgildum

$$U = +\tau^2 \frac{\partial \ln Z}{\partial \tau}$$

$\tau \ll \Sigma_0$: $\rightarrow$
$$U \simeq +\tau^2 \frac{\partial}{\partial \tau} \ln\left\{ 1 + 3e^{-\frac{2\Sigma_0}{\tau}} \right\}$$

$$= +\tau^2 \frac{3e^{-\frac{2\Sigma_0}{\tau}} \left(+\frac{2\Sigma_0}{\tau^2}\right)}{1 + 3e^{-2\Sigma_0/\tau}}$$

$$= \frac{6\Sigma_0 e^{-\frac{2\Sigma_0}{\tau}}}{1 + 3e^{-2\Sigma_0/\tau}} \simeq 6\Sigma_0 e^{-\frac{2\Sigma_0}{\tau}}$$

þegar $\tau \ll \Sigma_0$

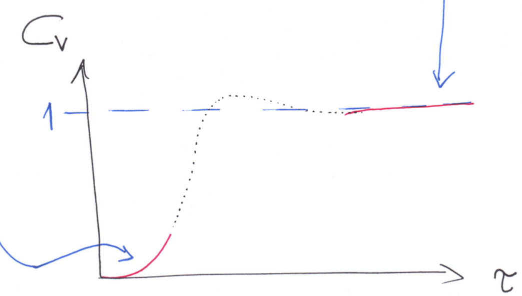
$$C_V = \left(\frac{\partial U}{\partial \tau}\right)_V = 12 \left(\frac{\Sigma_0}{\tau}\right)^2 e^{-\frac{2\Sigma_0}{\tau}}$$

skammtaáhrif fyrir lágt hitastig, lítil örvun fárra stiga

$\tau \gg \Sigma_0$:
$$U \simeq +\tau^2 \frac{\partial}{\partial \tau} \ln\left(\frac{\tau}{\Sigma_0}\right) = +\tau^2 \frac{1/\Sigma_0}{\tau/\Sigma_0} = +\tau$$

$$C_V = \left(\frac{\partial U}{\partial \tau}\right)_V = 1$$

← klassíska markgildið fyrir hátt hitastig mörg ástönd örvuð



(3-7) Rennilás N-tengi

Lokuð tengi orka: 0

opið tengi orka: ε

opnast aðeins frá vinstri

s getur opnast ef $(1, 2, \ldots, s-1)$ eru opin

Ástand p : p hlekkir opnir

a)

$$Z = \sum_{p=0}^{N} e^{-p\epsilon/\tau} = \sum_{p=0}^{N} \left(e^{-\epsilon/\tau}\right)^{p}$$

$$= 1 + \sum_{p=1}^{N} \left(e^{-\epsilon/\tau}\right)^{p} = 1 + \frac{\left\{\left(e^{-\epsilon/\tau}\right)^{N} - 1\right\}\left(e^{-\epsilon/\tau}\right)}{\left(e^{-\epsilon/\tau}\right) - 1}$$

(GR: 0.112)

$$= \frac{e^{-(N+1)\epsilon/\tau} - 1}{e^{-\epsilon/\tau} - 1}$$

(1)

b) Fyrir $\epsilon \gg \tau$ finna meðal fjölda opinna hlekkja

$$\langle p \rangle = \frac{\sum_{p=0}^{N} p e^{-p\epsilon/\tau}}{Z} = \frac{\sum_{p=0}^{N} p x^{p}}{\sum_{p=0}^{N} x^{p}} \quad \text{ef } x = e^{-\epsilon/\tau}$$

$$= x \frac{d}{dx} \ln Z = x \frac{d}{dx}\left\{\ln\left[\frac{1 - x^{N+1}}{1 - x}\right]\right\}$$

$$= x \frac{d}{dx}\left\{\ln(1 - x^{N+1}) - \ln(1 - x)\right\}$$

$$= x \left\{\frac{-(N+1)x^{N}}{1 - x^{N+1}} - \frac{(-1)}{1 - x}\right\} = \left\{\frac{x}{1-x} - \frac{(N+1)x^{N+1}}{1 - x^{N+1}}\right\}$$

$$= \left\{\frac{e^{-\epsilon/\tau}}{1 - e^{-\epsilon/\tau}} - \frac{(N+1)e^{-(N+1)\epsilon/\tau}}{1 - e^{-(N+1)\epsilon/\tau}}\right\}$$

(2)

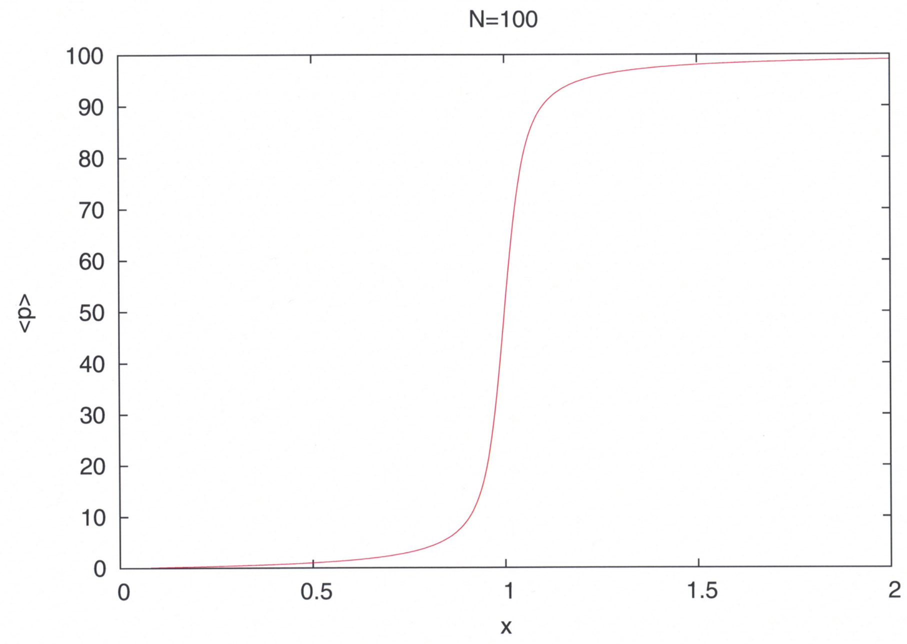
þegar $\epsilon \gg \tau$, $\epsilon > 0 \quad \rightarrow \quad x \ll 1$

$$\langle p \rangle = \left\{x + x^2 + x^3 + \cdots - (N+1)x^{N+1} \cdot \left(1 + x^{N+1} + x^{2(N+1)} + \cdots\right)\right\}$$

$$\simeq x = e^{-\epsilon/\tau}$$

(3)

(4)



3-8

5

Teningur með hlið L

Orka eins atóms

$$\varepsilon_n = \frac{\hbar^2}{2M}\left(\frac{\pi}{L}\right)^2 (n_x^2 + n_y^2 + n_z^2)$$

$$n_i \in \{1, 2, 3, \dots \infty\}$$

-> Grunnástand

$$\varepsilon_1 = \frac{\hbar^2}{2M}\left(\frac{\pi}{L}\right)^2 3$$

Finna L (velja n) þ.a.

$$\varepsilon_1 = \tau$$

$$\frac{\hbar^2 \pi^2 3}{2M\, V^{2/3}} = \tau$$

$$\to V^{2/3} = \frac{3\hbar^2\pi^2}{2M\tau}$$

$$\to V = \left(\frac{3\hbar^2\pi^2}{2M\tau}\right)^{3/2}$$

$$\to n = \frac{1}{V} = \left(\frac{2M\tau}{3\hbar^2\pi^2}\right)^{3/2}$$

$$= \left(\frac{M\tau}{2\pi\hbar^2}\right)^{3/2} \left(\frac{2\cdot 2}{3\pi}\right)^{3/2}$$

$$= n_Q \left(\frac{4}{3\pi}\right)^{3/2}$$

$$\sim n_Q \cdot 0{,}42$$

3-9

6

Sýna að körsumma tveggja óháðra kerfa í varmatengslum við sama hitastig sé

$$Z(1+2) = Z(1)Z(2)$$

Í hreinu ástandi (eiginástöndum)

$$Z_1(\tau) = \sum_s e^{-\varepsilon_s^1/\tau}, \qquad Z_2(\tau) = \sum_s e^{-\varepsilon_s^2/\tau}$$

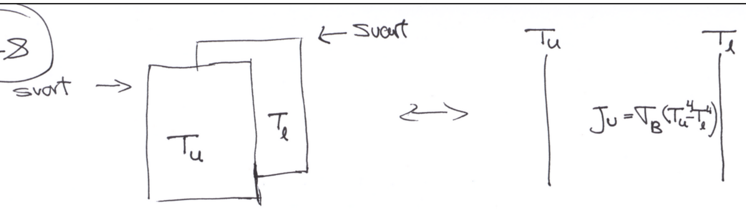
Orka þessara tveggja kerfa er $\varepsilon_s^1 + \varepsilon_r^2$ fyrir öll gildi á s og r

$$\to Z_{1+2} = \sum_{sr} e^{-(\varepsilon_s^1 + \varepsilon_r^2)/\tau} = \sum_{sr} e^{-\varepsilon_s^1/\tau} e^{-\varepsilon_r^2/\tau}$$

$$= \sum_s e^{-\varepsilon_s^1/\tau} \sum_r e^{-\varepsilon_r^2/\tau} = Z_1 \cdot Z_2$$

4-8

7



þriðja sléttan er sett á milli (líkasvört) fær hitastigið T_m eftir smátíma

Finna T_m og sýna að varmaflæðið sé helmingað

upphaflega

$$J_u = \sigma_B(T_u^4 - T_l^4)$$

Síðan

$$J_u' = \sigma_B(T_u^4 - T_m^4) \quad \text{og} \quad J_u'' = \sigma_B(T_m^4 - T_l)$$

8

Kerfið er í stöðugu ástandi með T_m fast

$$\to \quad J_u' = J_u'' \to T_u^4 - T_m^4 = T_m^4 - T_l^4$$

$$\to 2T_m^4 = T_u^4 + T_l^4$$

$$\to \boxed{T_m^4 = \frac{1}{2}(T_u^4 + T_l^4)}$$

og

$$J_u' = \sigma_B\Big(T_u^4 - \overbrace{\tfrac{1}{2}(T_u^4 + T_l^4)}^{T_m^4}\Big) = \sigma_B\left(\tfrac{1}{2}T_u^4 - \tfrac{1}{2}T_l^4\right)$$

$$= \frac{1}{2}\sigma_B(T_u^4 - T_l^4) = \frac{1}{2} J_u$$

Orka flæðið helmingast -> varmaskjöldur

4-1 (1)

Finna meðalheildarfjölda ljóseinda

$\sum \langle s_n \rangle$ í jafnvægi við τ í holrúmi með V

$$\sum_{n=1}^{\infty} \langle s_n \rangle = \sum_{n=1}^{\infty} \frac{1}{e^{\hbar\omega_n/\tau}-1} = N_{ph}$$

Gerum ráð fyrir stóru holi til þess að muna í heildi

$$\omega_n = n\pi \frac{c}{L}$$

2 skautanir

$$N_{ph} \sim \frac{2}{8} \int_0^{\infty} 4\pi n^2 dn \frac{1}{e^{\frac{\hbar c n\pi}{L\tau}}-1}$$

jákvæð n_x, n_y, n_z

notum $x = \frac{\hbar c n \pi}{L\tau}$

$$dx = \frac{\hbar c \pi}{L\tau} dn$$

$$n = x \frac{L\tau}{\hbar c\pi}$$

(2)

$$N_{ph} \sim \frac{2\cdot 4\pi}{8} \left(\frac{L\tau}{\hbar c\pi}\right)^3 \int_0^{\infty} x^2 dx \frac{1}{e^x - 1} = 2\cdot \frac{4\pi}{8}\left(\frac{L\tau}{\hbar c\pi}\right)^3 \Gamma(3)\,\zeta(3)$$

þ.s. (GR. 3.411.1) er notað

$$N_{ph} \sim 2\cdot \frac{V}{2\pi^2}\left(\frac{\tau}{\hbar c}\right)^3 \cdot 2 \cdot 1.20205 \cdot \simeq 2.404 \frac{V}{\pi^2}\left(\frac{\tau}{\hbar c}\right)^3$$

$$\zeta(z) \equiv \sum_{n=1}^{\infty} \frac{1}{n^z} \qquad (GR\ 9.522.1)$$

Óreiða alheimsins gæti að mestu leyti verið vegna ljóseinda

4-6 (3)

Ljóseindagas

a) Sýna að

$$p = -\left(\frac{\partial U}{\partial V}\right)_\tau = -\sum_j s_j \hbar \left(\frac{d\omega_j}{dV}\right)$$

s_j er fjöldi ljóseinda í hólfi j.

Höfum

$$U = \sum_j \langle \varepsilon_j \rangle = \sum_j \langle s_j \rangle \hbar\omega_j \ , \quad \omega_j = \frac{j\pi c}{V^{1/3}}$$

$$p = -\left(\frac{\partial}{\partial V} \sum_j \langle s_j \rangle \hbar\omega_j\right)_\tau = -\sum_j \left(\frac{\partial \langle s_j \rangle}{\partial V}\right)_\tau \hbar\omega_j - \sum_j \langle s_j \rangle \left(\frac{\partial \hbar\omega_j}{\partial V}\right)$$

(4)

fyrir ljóseindir gaf (23)

$$\sigma = \frac{4\pi^2 V}{45}\left(\frac{\tau}{\hbar c}\right)^3$$

og dæmi 4-1

$$N_{ph} = \text{fasti} \cdot V \left(\frac{\tau}{\hbar c}\right)^3$$

$$\rightarrow \left(\frac{\partial \langle s_j \rangle}{\partial V}\right)_\tau = 0$$

og því

$$p = -\sum_j \langle s_j \rangle \frac{\partial \hbar\omega_j}{\partial V}$$

b)

$$\frac{d\omega_j}{dV} = \frac{d}{dV}\left(\frac{j\pi c}{V^{1/3}}\right) = -\frac{j\pi c}{V^{4/3}} \cdot \frac{1}{3}$$

$$= -\frac{j\pi c}{V^{1/3}} \cdot \frac{1}{3V}$$

$$= -\frac{\omega_j}{3V}$$

c)

$$p = -\sum_j \langle s_j \rangle \frac{\partial \hbar\omega_j}{\partial V}$$

$$= \sum_j \langle s_j \rangle \frac{\hbar\omega_j}{3V}$$

$$= \frac{U}{3V}$$

d) H-atóm 1 mole/cm³

Við hvaða hitastig er þrýstingur ljóseinda-gasins jafn H-gasins

H-gas: $pV = N\tau = Nk_BT$
$= nRT$

ljóseindir: $pV = \frac{U}{3} = \frac{\pi^2 k_B^4 T^4}{45\hbar^3 c^3} V$

$\rightarrow \frac{nRT}{V} = \frac{\pi^2 k_B^4 T^4}{45\hbar^3 c^3}$

$$T = \left(\frac{45\hbar^3 c^3 nR}{\pi^2 k_B^4 V}\right)^{1/3}$$

$$= 3.2 \cdot 10^7 \text{ K}$$

$\hbar \approx 1.05 \cdot 10^{-27}$ erg·s

$c = 3 \cdot 10^{11}$ cm/s

$R \approx 8.31 \cdot 10^7 \frac{\text{erg}}{\text{mol} \cdot \text{K}}$

$k_B = 1.38 \cdot 10^{-16} \frac{\text{erg}}{\text{K}}$

$n = 1$ mole/cm³

$V = 1$ cm³

(5)

4-9 Ljóseindagas í 1D

finna $C_v(\tau)$

jaðarskilyrði $E(0), E(L) = 0$

L

$$v^2 \frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2} \rightarrow \text{Lausn } E(x) = \sin(\omega t) \cdot \sin\left(\frac{n\pi x}{L}\right) \cdot E_0, \quad n = \{1, \dots \infty\}$$

Innsetning gefur

$$\frac{v^2 \pi^2 n^2}{L^2} = \omega^2 \rightarrow \omega_n = \pi n \frac{v}{L}$$

$$U = \sum_{n=1}^{\infty} \langle \varepsilon_n \rangle = \sum_{n=1}^{\infty} \frac{\hbar\omega_n}{e^{\frac{\hbar\omega_n}{\tau}} - 1}$$

(6)

---

gerum ráð fyrir langri línu

$$\frac{\hbar\pi n v}{\tau L} = \frac{\hbar\omega_n}{\tau} = x$$

$$\rightarrow dn = dx \cdot \frac{\tau L}{\hbar\pi v}$$

$$U \simeq \int_0^\infty dn \cdot \frac{\hbar\omega_n}{e^{\frac{\hbar\omega_n}{\tau}} - 1}$$

$$= \frac{\tau^2 L}{\hbar\pi v} \int_0^\infty dx \frac{x}{e^x - 1}$$

$$= \frac{\tau^2 L}{\hbar\pi v} \Gamma(2)\zeta(2)$$

$$U = \frac{\tau^2 L}{\hbar\pi v} \frac{\pi^2}{6} = \frac{\tau^2 L\pi}{\hbar v 6}$$

$$\rightarrow \frac{U}{L} = \frac{\tau^2 \pi}{\hbar v 6}$$

$$C_v(\tau) = \left(\frac{\partial U}{\partial \tau}\right)_V = \frac{\tau L\pi}{3\hbar v}$$

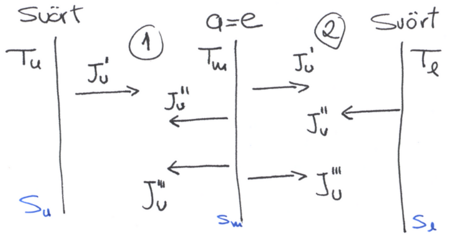
En í 3D fæst

$$C_v(\tau) = \left(\frac{\partial}{\partial\tau} \frac{\pi\tau^4 V}{15\hbar^3 c^3}\right)_V$$

$$= \frac{4\pi\tau^3 V}{15\hbar^3 c^2}$$

(7)

4-19



$r = 1 - a$

Á svæði (1)

$J_u' = \sigma_B T_u^4$ Geislun S_u svarthluts

$J_u'' = \sigma_B T_m^4 \cdot e$ Geislun S_m, ekki svarthluts

$J_u''' = \sigma_B T_u^4 \cdot r$ endurkast geislunar frá S_u af fleti S_m

$\rightarrow \sigma_B a (T_u^4 - T_m^4)$

$$J_u^{(1)} = \sigma_B (T_u^4 - eT_m^4 - rT_u^4) = \sigma_B (T_u^4(1-r) - aT_m^4)$$

(8)

9

Á svæði (2)

$J_U' = \sigma_B T_m^4 \cdot a$ Geislun sléttu S_m, ekki svört

$J_U'' = \sigma_B T_e^4$ Geislun sléttu S_e, svört

$J_U''' = \sigma_B T_e^4 \cdot r$ endurkast geislunar frá S_e af S_m

$$J_U^{(2)} = \sigma_B\left(T_m^4 \cdot a + T_e^4 \cdot r - T_e^4\right)$$
$$= \sigma_B\left(T_m^4 \cdot a - T_e^4 \cdot a\right)$$
$$= \sigma_B a\left(T_m^4 - T_e^4\right)$$

10

$$J_U^{(1)} = J_U^{(2)} \to \sigma_B a\left(T_u^4 - T_m^4\right) = \sigma_B a\left(T_m^4 - T_e^4\right)$$
$$\to 2T_m^4 = T_u^4 + T_e^4$$ sama svar og í dæmi 4.8

---

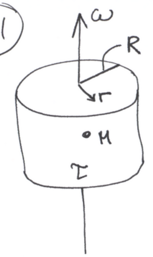
notum í $J_U^{(1)}$ til að finna orku flæði þéttunina

$$J_U^{(1)} = \sigma_B a\left(T_u^4 - T_m^4\right) = \sigma_B a\left(T_u^4 - \frac{T_u^4 + T_e^4}{2}\right)$$
$$= \sigma_B a \frac{1}{2}\left(T_u^4 - T_e^4\right) = \sigma_B (1-r)\frac{1}{2}\left(T_u^4 - T_e^4\right)$$

$(1-r)$ bætist við svarið úr 4-8. $r=1 \to$ ekkert flæði

1

5-1



Fastur hornhraði ω, finna $n(r)$ m.v. $n(0)$ þéttleika kjörgasins í skilvindunni

Inni í skilvindunni er gervi kraftur

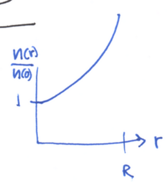
$+Mr\omega^2$ í átt að útvegg

Það er eins og bætist við mætti $V(r) = -\frac{1}{2}Mr^2\omega^2$

Notum $\mu = \mu_{int} + \mu_{ext} = \tau \ln\left(\frac{n}{n_Q}\right) + V(r)$

$$\to \tau\ln\left(\frac{n(0)}{n_Q}\right) = \tau\ln\left(\frac{n(r)}{n_Q}\right) - \frac{1}{2}Mr^2\omega^2$$

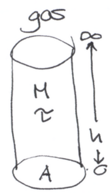
$$\ln\left(\frac{n(0)}{n(r)}\right) = -\frac{Mr^2\omega^2}{2\tau} \to n(r) = n(0)\,e^{+\frac{Mr^2\omega^2}{2\tau}}$$



Hér þyrfti að ákvarða $n(0)$ út frá heildar þéttleika kerfisins án snúnings ....

2

5-3



fast þyngdarsvið g. Finna meðal þyngdar og hreyfiorku á atóm

$$\mu = \tau\ln\left(\frac{n}{n_Q}\right) + Mgh$$
$$\to n(h) = n(0)\exp\left(-\frac{Mgh}{\tau}\right)$$

Fjöldi atóma á flöt A

$$\frac{N}{A} = \int_0^\infty dh\, n(h) = n(0)\frac{\tau}{Mg}$$
$$\to N = n(0)\frac{\tau A}{Mg} = n(0)\cdot A\cdot h_c$$
$$h_c = \frac{\tau}{Mg}$$

Heildar mættisorka á A

$$\frac{U_{pot}}{A} = \int_0^\infty dh\, n(h)\, Mgh$$
$$= Mg\, n(0)\int_0^\infty dh\, h\, e^{-\frac{Mgh}{\tau}}$$

notum $\frac{Mgh}{\tau} = x$

$$\frac{U_{pot}}{A} = n(0)\frac{\tau^2}{Mg}\int_0^\infty \left(\frac{Mg\,dh}{\tau}\right)\left(\frac{Mgh}{\tau}\right)e^{-\frac{Mgh}{\tau}}$$
$$= n(0)\frac{\tau^2}{Mg}\underbrace{\int_0^\infty dx\, x\, e^{-x}}_{1}$$

3

$$\rightarrow \frac{U_{pot}}{A} = n(0)\frac{\tau^2}{Mg}$$

mattis

$\rightarrow$ meðalorkan á atóm er

$$\frac{\left(\frac{U_{pot}}{A}\right)}{\frac{N}{A}} = \tau$$

Meðal hreyfiorkan fyrir kjörgas

$$U_{hre} = \frac{3}{2}\tau$$

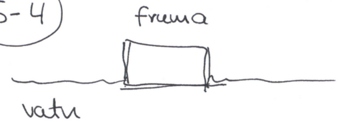
Heildar orkan er því

$$U = \left(\frac{3}{2}+1\right)\tau = \frac{5}{2}\tau$$

$$\rightarrow C_V = \left(\frac{\partial U}{\partial \tau}\right)_V = \frac{5}{2} \quad \text{fasti}$$

4

5-4



Finna $\Delta\mu(300K) \sim 0{,}24\,eV$

Lítum á jónirnar sem kjörgas

$$\frac{[K^+]_{fruma}}{[K^+]_{vatn}} \sim 10^4 \qquad \frac{n_f}{n_v} \sim 10^4$$

Kerfið er í ójafnvægi, en haldið stöðugu með $\Delta V q$

fruma: $\mu_f = \tau \ln\left(\frac{n_f}{n_Q}\right)$

Vatn $\mu_v = \tau \ln\left(\frac{n_v}{n_Q}\right)$

$$\Delta\mu = \mu_f - \mu_v > 0$$

$$= \tau \ln\left(\frac{n_f}{n_v}\right) = k_B T \ln\left(\frac{n_f}{n_v}\right)$$

$$= 8.617\cdot 10^{-5}\,\frac{eV}{K}\cdot 300K \cdot \ln(10^{+4})$$

$$\simeq 0{,}24\,eV$$

5

5-6 tveggja stiga kerfi

ε —— 2	ástönd	$(0,0)$	$E=0$
0 —— 1		$(1,0)$	$E=0$
		$(0,1)$	$E=\varepsilon$

a)

$$Z = \sum_{NS} \lambda^N \exp\left(-\frac{\varepsilon_s}{\tau}\right)$$

$$= 1 + \lambda + \lambda e^{-\frac{\varepsilon}{\tau}}$$

$$= 1 + \lambda\left(1 + e^{-\frac{\varepsilon}{\tau}}\right)$$

b)

$$\langle N \rangle = \lambda \frac{\partial}{\partial \lambda} \ln Z$$

$$= \lambda \frac{\partial}{\partial \lambda}\left\{\ln\left(1+\lambda\left[1+e^{-\frac{\varepsilon}{\tau}}\right]\right)\right\}$$

$$= \lambda \frac{1+e^{-\frac{\varepsilon}{\tau}}}{\ln\left(1+\lambda\left[1+e^{-\frac{\varepsilon}{\tau}}\right]\right)}$$

$$= \frac{\lambda\left(1+e^{-\frac{\varepsilon}{\tau}}\right)}{Z}$$

c) meðal setni ástands með orku ε

ástand $(0,1)$

$$\langle N_\varepsilon \rangle = \frac{\lambda e^{-\frac{\varepsilon}{\tau}}}{Z}$$

d) Finna U, meðal orku kerfisins

Hér er einfallt að nota

$$U = \frac{\sum_{NS} \varepsilon_s e^{\frac{N\mu - \varepsilon_s}{\tau}}}{Z}$$

$$= \frac{\varepsilon e^{\frac{\mu-\varepsilon}{\tau}}}{Z}$$

$$= \frac{\varepsilon\lambda e^{-\frac{\varepsilon}{\tau}}}{1+\lambda\left(1+e^{-\frac{\varepsilon}{\tau}}\right)}$$

6

e) Ef til viðbótar kemur ástandið $(1,1)$ með $E = \varepsilon$

$$\rightarrow Z = 1 + \lambda\left(1+e^{-\frac{\varepsilon}{\tau}}\right) + \lambda^2 e^{-\frac{\varepsilon}{\tau}}$$

$$= (1+\lambda)\left\{1 + \lambda e^{-\frac{\varepsilon}{\tau}}\right\}$$

tvö óháð kerfi því fyrir neðra stigið fast

$$Z_0 = 1 + \lambda$$

og efra stigið

$$Z_\varepsilon = 1 + \lambda e^{-\frac{\varepsilon}{\tau}}$$

undirkerfin voru ekki óháð þegar aðeins annað þeirra gat verið setið

8

CO eitrun

Allir viðtakar geta verið tómir eða setnir með $O_2 \leftrightarrow \varepsilon_A$

$$CO \leftrightarrow \varepsilon_B$$

N viðtakar í jafnvægi við

O_2 : $\lambda_A(O_2) = 1\cdot 10^{-5}$ $T = 37°C$

CO : $\lambda_B(CO) = 1\cdot 10^{-7}$

a) Án CO finna ε_A þ.a. 90% viðtaka hafi O_2, svar í eV á O_2

7

N viðtakar, O eða ε_A um hvern gildir

$$\mathcal{Z}_1 = 1 + \lambda_A e^{-\frac{\varepsilon_A}{\tau}}$$

og um N óháða

$$\mathcal{Z} = (\mathcal{Z}_1)^N = \left(1+\lambda_A e^{-\frac{\varepsilon_A}{\tau}}\right)^N$$

gefið að $\frac{\langle N_A\rangle}{N} = 0{,}9$

þurfum að finna ε_A

$$\langle N_A\rangle = \lambda_A \frac{\partial}{\partial \lambda_A} \ln\left(1+\lambda_A e^{-\frac{\varepsilon_A}{\tau}}\right)^N$$

$$= \lambda_A N \frac{\partial}{\partial\lambda_A} \ln\left(1+\lambda_A e^{-\frac{\varepsilon_A}{\tau}}\right)$$

$$= \lambda_A N \frac{e^{-\frac{\varepsilon_A}{\tau}}}{1+\lambda_A e^{-\frac{\varepsilon_A}{\tau}}}$$

8

$$\frac{\langle N_A\rangle}{N} = \frac{\lambda_A e^{-\frac{\varepsilon_A}{\tau}}}{1+\lambda_A e^{-\frac{\varepsilon_A}{\tau}}} = \frac{1}{\lambda_A^{-1} e^{\frac{\varepsilon_A}{\tau}} + 1}$$

$$\rightarrow \lambda_A^{-1} e^{\frac{\varepsilon_A}{\tau}} + 1 = \frac{N}{\langle N_A\rangle}$$

eða $\lambda_A^{-1} e^{\frac{\varepsilon_A}{\tau}} = \frac{N}{\langle N_A\rangle} - 1$

$$e^{\frac{\varepsilon_A}{\tau}} = \lambda_A\left(\frac{N}{\langle N_A\rangle} - 1\right)$$

$$\rightarrow \varepsilon_A = \tau \ln\left\{\lambda_A\left(\frac{N}{\langle N_A\rangle} - 1\right)\right\}$$

$$= \tau \ln\left\{10^{-5}\left(\frac{1}{0{,}9} - 1\right)\right\}$$

$$\simeq -13.71\cdot\tau = \simeq -13.71\cdot k_B T$$

$$= -13.71\cdot 8.617\cdot 10^{-5}\,\frac{eV}{K}\cdot(273+37) \simeq -0.366\ eV$$

9

b) Finna ε_B þ.a. aðeins 10% viðtaka séu setnir með O_2

Báðar gastegundir

Einn viðtaki

ekkert atóm O_2 CO

$$\mathcal{Z}_1 = 1 + \lambda_A e^{-\frac{\varepsilon_A}{\tau}} + \lambda_B e^{-\frac{\varepsilon_B}{\tau}}$$

N óháðir viðtakar

$$\mathcal{Z} = (\mathcal{Z}_1)^N = \left\{1+\lambda_A e^{-\frac{\varepsilon_A}{\tau}} + \lambda_B e^{-\frac{\varepsilon_B}{\tau}}\right\}^N$$

$$\langle N_A\rangle = \lambda_A \frac{\partial}{\partial\lambda_A} \ln\left\{1+\lambda_A e^{-\frac{\varepsilon_A}{\tau}} + \lambda_B e^{-\frac{\varepsilon_B}{\tau}}\right\}^N$$

$$= \lambda_A N \frac{e^{-\frac{\varepsilon_A}{\tau}}}{1+\lambda_A e^{-\frac{\varepsilon_A}{\tau}} + \lambda_B e^{-\frac{\varepsilon_B}{\tau}}}$$

10

$$\rightarrow \frac{\langle N_A\rangle}{N} = \frac{\lambda_A e^{-\frac{\varepsilon_A}{\tau}}}{1+\lambda_A e^{-\frac{\varepsilon_A}{\tau}} + \lambda_B e^{-\frac{\varepsilon_B}{\tau}}}$$

$$\frac{N}{\langle N_A\rangle} = \frac{1+\lambda_A e^{-\frac{\varepsilon_A}{\tau}} + \lambda_B e^{-\frac{\varepsilon_B}{\tau}}}{\lambda_A e^{-\frac{\varepsilon_A}{\tau}}} = \lambda_A^{-1} e^{\frac{\varepsilon_A}{\tau}} + \lambda_A + \frac{\lambda_B}{\lambda_A} e^{-\frac{\varepsilon_B - \varepsilon_A}{\tau}}$$

$$= \lambda_A^{-1} e^{\frac{\varepsilon_A}{\tau}} + \lambda_A + \frac{\lambda_B}{\lambda_A} e^{-\frac{\varepsilon_B}{\tau}} e^{+\frac{\varepsilon_A}{\tau}}$$

$$\rightarrow \frac{N}{\langle N_A\rangle} - \lambda_A^{-1} e^{\frac{\varepsilon_A}{\tau}} - \lambda_A = \frac{\lambda_B}{\lambda_A} e^{\frac{\varepsilon_A}{\tau}} e^{-\frac{\varepsilon_B}{\tau}}$$

$$\rightarrow \frac{\lambda_A}{\lambda_B} e^{-\frac{\varepsilon_A}{\tau}} \left\{\frac{N}{\langle N_A\rangle} - \lambda_A^{-1} e^{\frac{\varepsilon_A}{\tau}} - \lambda_A\right\} = e^{-\frac{\varepsilon_B}{\tau}}$$

(11)

$$\varepsilon_B = -\tau \ln\left[\frac{\lambda_A}{\lambda_B} e^{-\frac{\varepsilon_A}{\tau}}\left\{\frac{N}{\langle N_A\rangle} - \lambda_A^{-1} e^{\frac{\varepsilon_A}{\tau}} - \lambda_A\right\}\right]$$

$$= -\tau\ln\left[100\, e^{+\frac{0.366}{8.617\cdot 10^{-5}\cdot(273.15+37)}}\left\{\frac{1}{0.1} - 10^5 e^{-\cdots\cdots} - 10^{-5}\right\}\right]$$

$$\approx -\tau\ln\left[100\cdot 8.862\cdot 10^5\left\{10 - \frac{1}{8.862} - 10^{-5}\right\}\right]$$

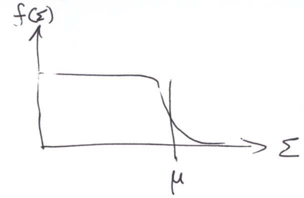
$$\approx -\tau\cdot 20.591 \approx -(273.15+37)\cdot 8.617\cdot 10^{-5}\cdot 20.591\,\mathrm{eV}$$

$$\approx -0.55\ \mathrm{eV}$$

Samanborið við -0.366 eV fyrir O_2 : $\varepsilon_A = -0.366$ eV, $\varepsilon_B = -0.55$ eV

ryður út O_2 !

(6-1) Reikna $-\frac{\partial f}{\partial \varepsilon}\Big|_{\varepsilon=\mu}$ (1)



$$f(\varepsilon) = \frac{1}{e^{\frac{\varepsilon-\mu}{\tau}}+1}$$

$$-\frac{\partial f}{\partial \varepsilon} = \frac{e^{\frac{\varepsilon-\mu}{\tau}}\cdot\frac{1}{\tau}}{\left(e^{\frac{\varepsilon-\mu}{\tau}}+1\right)^2}$$

$$-\frac{\partial f}{\partial \varepsilon}\Big|_{\varepsilon=\mu} = \frac{1\cdot\frac{1}{\tau}}{(1+1)^2} = \frac{1}{4\tau}$$

$$\rightarrow\ -\frac{\partial f}{\partial \varepsilon}\Big|_{\varepsilon=\mu} \xrightarrow[\tau\to 0]{} \infty$$

$f(\varepsilon)$ nálgast þrepafall $\theta(\mu-\varepsilon)$ fyrir $\tau = 0$

(6-2) Samhverfa $f(\varepsilon)$ um $\varepsilon=\mu$ (2)

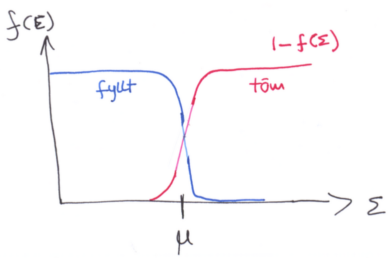
Setjum $\varepsilon = \mu+\delta$

$$f(\mu+\delta) = \frac{1}{e^{\frac{\delta}{\tau}}+1}$$

$$f(\mu-\delta) = \frac{1}{e^{-\frac{\delta}{\tau}}+1} = \frac{e^{\frac{\delta}{\tau}}}{1+e^{\frac{\delta}{\tau}}} = 1 - f(\mu+\delta)$$

$$\frac{1+e^{\frac{\delta}{\tau}}}{1+e^{\frac{\delta}{\tau}}} - \frac{1}{1+e^{\frac{\delta}{\tau}}} = \frac{e^{\frac{\delta}{\tau}}}{1+e^{\frac{\delta}{\tau}}}$$

$\rightarrow\ f(\mu+\delta) = 1 - f(\mu-\delta)$



(6-3) Svigrúmasetni 0, 1, 2 með orku 0, ε, 2ε (3)

a) Finna $\langle N\rangle$ fyrir kerfið í jafnvægi við einda og varmageymi með τ og μ

athugum ástönd (0,0) (0,1) (1,0) (1,1)

$0 \quad \varepsilon \quad \varepsilon \quad 2\varepsilon$ orka

$$\rightarrow\ \mathcal{Z} = 1 + 2\lambda e^{-\frac{\varepsilon}{\tau}} + \lambda^2 e^{-\frac{2\varepsilon}{\tau}}$$

$$\langle N\rangle = \frac{0\cdot 1 + 1\cdot 2\lambda e^{-\frac{\varepsilon}{\tau}} + 2\cdot\lambda^2 e^{-\frac{2\varepsilon}{\tau}}}{\mathcal{Z}} = \frac{2\left\{\lambda e^{-\frac{\varepsilon}{\tau}} + \lambda^2 e^{-\frac{2\varepsilon}{\tau}}\right\}}{1+2\lambda e^{-\frac{\varepsilon}{\tau}} + \lambda^2 e^{-\frac{2\varepsilon}{\tau}}}$$

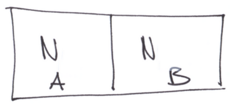
$$\rightarrow \langle N\rangle = \frac{2\lambda e^{-\varepsilon/\tau}\{1+\lambda e^{-\varepsilon/\tau}\}}{\{1+\lambda e^{-\varepsilon/\tau}\}^2} = \frac{2\lambda e^{-\varepsilon/\tau}}{\{1+\lambda e^{-\varepsilon/\tau}\}}$$

$$= \frac{2}{\lambda^{-1}e^{\varepsilon/\tau}+1} = \frac{2}{\exp\left\{\frac{\varepsilon-\mu}{\tau}\right\}+1}$$

b) orkustig ε er tvöfalt
er nákvæmlega a-liður

6-6

N-A og N-B í eindajafnvægi, τ, V sama



full komin blöndun

$\hookrightarrow \Delta\sigma = 2N\ln 2$

Sýna að ef A = B $\rightarrow \Delta\sigma = 0$ (Gibbs paradox)

fjöldaföllin g_A og g_B

$$g = g_A g_B$$
$$\downarrow$$
$$\sigma = \sigma_A + \sigma_B$$

fyrir og eftir blöndun

En við höfum

$$\sigma = N\left\{\ln\left(\frac{n_Q}{n}\right)+\frac{5}{2}\right\}$$

Fyrir

$$\sigma_i = N\left\{\ln\left(\frac{n_Q^A}{n_i}\right)+\ln\left(\frac{n_Q^B}{n_i}\right)+5\right\}$$

því N er það sama

Eftir $n_f = \frac{N}{2V} = \frac{n_i}{2}$

$$\sigma_f = N\left\{\ln\left(\frac{n_Q^A}{n_f}\right)+\ln\left(\frac{n_Q^B}{n_f}\right)+5\right\}$$

$$= N\left\{\ln\left(2\frac{n_Q^A}{n_i}\right)+\ln\left(2\frac{n_Q^B}{n_i}\right)+5\right\}$$

blöndunar óreiða

$$= \sigma_i + 2N\ln 2$$

Ef eindirnar þekkjast ekki í sundur þá gildir

$n_f = \frac{2N}{2V} = n_i$

$$\sigma_f = 2N\left\{\ln\left(\frac{n_Q}{n_i}\right)+\frac{5}{2}\right\}$$

engin munur á n_Q^A og n_Q^B sama fyrir og eftir

6-4

$\varepsilon = pc$ Kjörgas

fyrir klassíska kjörgasið með $\varepsilon = \frac{p^2}{2m}$

notuðum við

$$\varepsilon = \frac{\hbar^2}{2M}\left\{k_x^2+k_y^2+k_z^2\right\}, \quad k_i = \frac{\pi n_i}{L}$$

Nú höfum við

$$\varepsilon = pc = \hbar c\left\{k_x^2+k_y^2+k_z^2\right\}^{1/2} \text{ með } k_i = \frac{\pi n_i}{L}$$

$$= \frac{\hbar c\pi}{L}\left\{n_x^2+n_y^2+n_z^2\right\}^{1/2} = \frac{\pi\hbar c}{L}n$$

Körsumman fyrir eina eind í varmatengslum er

$$Z_1 = \frac{1}{8}4\pi\int_0^\infty dn\cdot n^2\cdot \exp\left\{-\frac{\pi\hbar c}{\tau L}n\right\} \quad \left\{\sum_n e^{-\frac{\pi\hbar c}{\tau L}n}\right\}$$

setjum $\frac{\pi\hbar c}{\tau L} n = x$

$$Z_1 = \frac{1}{8} 4\pi \left(\frac{L\tau}{\pi\hbar c}\right)^3 \int_0^\infty dx \; x^2 e^{-x} = \frac{\pi}{2}\left(\frac{L}{\pi\hbar c}\right)^3 \cdot 2 \cdot \tau^3$$

notum síðan

$$U = \tau^2\left(\frac{\partial \ln Z_1}{\partial \tau}\right) = \tau^2\left(\frac{\partial (3\ln\tau)}{\partial\tau}\right) = 3\tau$$

$\to \boxed{U = 3\tau}$ í stað $\frac{3}{2}\tau$ fyrir klassíska kjörgas

6-10 Jafnóreiðu ferli, kjörgas

$$\sigma = N\left\{\ln\left(\frac{n_Q}{n}\right) + \frac{5}{2}\right\} + \sigma_{int}, \quad n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{3/2}$$

$$\to \sigma = N\left\{\frac{3}{2}\ln\tau + \ln V + \text{fasti}\right\} + \sigma_{int}(\tau) \qquad (*)$$

Eins getum við sett $pV = N\tau \to V = \frac{N\tau}{p}$ til að fá

$$\sigma = N\left\{\frac{5}{2}\ln\tau - \ln p + \text{fasti}\right\} + \sigma_{int}(\tau) \qquad (**)$$

Notum

$$C_V = \tau\left(\frac{\partial\sigma}{\partial\tau}\right)_V$$

$$C_V = \frac{3}{2}N + \tau\left(\frac{\partial\sigma_{int}}{\partial\tau}\right)_V = \frac{3}{2}N + C_{int}$$

og

$$C_p = \tau\left(\frac{\partial\sigma}{\partial\tau}\right)_p$$

$$C_p = \frac{5}{2}N + \tau\left(\frac{\partial\sigma}{\partial\tau}\right)_p = \frac{5}{2}N + C_{int}$$

$$\to \quad C_p - C_V = \left(\frac{C_p}{C_V} - 1\right)C_V = (\gamma - 1)C_V = \left(\frac{\gamma-1}{\gamma}\right)C_p = N$$

með $\gamma = \left(\frac{C_p}{C_V}\right)$

Athugum (*)

$$d\sigma = \frac{3}{2}N\frac{d\tau}{\tau} + N\frac{dV}{V} + \left(\frac{\partial\sigma_{int}}{\partial\tau}\right)_V d\tau$$

$$= \frac{3}{2}N\frac{d\tau}{\tau} + N\frac{dV}{V} + C_{int}\frac{d\tau}{\tau}$$

$$= C_V\frac{d\tau}{\tau} + N\frac{dV}{V} = 0$$

$$\to \quad \frac{d\tau}{\tau} = -\frac{N}{C_V}\frac{dV}{V} = -(\gamma-1)\frac{dV}{V}$$

$$\to \quad \boxed{\frac{d\tau}{\tau} + (\gamma-1)\frac{dV}{V} = 0}$$

Athugum (**) (6)

$$d\mathcal{T} = \frac{5}{2}N\frac{d\tau}{\tau} - N\frac{dp}{p} + \left(\frac{\partial\mathcal{T}}{\partial\tau}\right)_p d\tau$$

$$= \frac{5}{2}N\frac{d\tau}{\tau} - N\frac{dp}{p} + C_{int}\frac{d\tau}{\tau}$$

$$= C_p\frac{d\tau}{\tau} - N\frac{dp}{p} = 0$$

nota $\frac{d\tau}{\tau} + (\gamma-1)\frac{dV}{V} = 0$

$$\rightarrow \frac{dp}{p} = \frac{C_p}{N}\frac{d\tau}{\tau} = \frac{\gamma}{\gamma-1}\frac{d\tau}{\tau} = -\gamma\frac{dV}{V}$$

$$\rightarrow \boxed{\frac{dp}{p} + \frac{\gamma}{\gamma-1}\frac{d\tau}{\tau} = 0} \quad \text{og} \quad \boxed{\frac{dp}{p} + \gamma\frac{dV}{V} = 0}$$

b) (7)

$$B_\sigma = -V\left(\frac{\partial p}{\partial V}\right)_\sigma \quad , \quad B_\tau = -V\left(\frac{\partial p}{\partial V}\right)_\tau$$

$$\frac{dp}{p} + \gamma\frac{dV}{V} = 0 \rightarrow \frac{dp}{dV} = \left(\frac{\partial p}{\partial V}\right)_\sigma = -\gamma\frac{p}{V}$$

$$\rightarrow \boxed{B_\sigma = +\gamma p}$$

fyrir <u>jafnhita</u>

$$p = \frac{N\tau}{V}$$

$$\left(\frac{\partial p}{\partial V}\right)_\tau = -\frac{N\tau}{V^2} = -\frac{p}{V}$$

$$\rightarrow \boxed{B_\tau = p}$$

(6-12) Kjörgas í 2D (8)

Sviggrunnur er $\quad \varepsilon_n = \frac{\hbar^2}{2M}\left(\frac{\pi}{L}\right)^2\{n_x^2 + n_y^2\} = \frac{(\hbar\pi)^2}{2MA}n^2$

þ.s. $n^2 = n_x^2 + n_y^2$ og $A = L^2$: flötur kerfisins

$$N = \sum_n f(\varepsilon_n) = \lambda\sum_n e^{-\frac{\varepsilon_n}{\tau}}$$

$$= \lambda\frac{1}{4}2\pi\int_0^\infty n\,dn\,e^{-\frac{\varepsilon_n}{\tau}} = \frac{\lambda\pi}{2}\int_0^\infty n\,dn\,e^{-\frac{\hbar^2\pi^2}{2MA\tau}n^2}$$

$n_x, n_y > 0$

setjum $\frac{\hbar\pi n}{\sqrt{2MA\tau}} = x$

$$= \lambda\pi\frac{2MA\tau}{\hbar^2\pi^2 2}\int_0^\infty x\,dx\,e^{-x^2}$$

(9)

$$N = \lambda\pi\frac{MA\tau}{\hbar^2\pi^2 2} = \lambda\frac{MA}{\hbar^2\pi 2} = \lambda A n_Q^{2D} \quad , \quad n_Q^{2D} = \frac{M\tau}{2\pi\hbar^2}$$

$$\lambda = e^{\frac{\mu}{\tau}} \rightarrow \mu = \tau\ln\lambda = \tau\ln\left(\frac{N}{A n_Q^{2D}}\right)$$

$$= \tau\ln\left(\frac{n^{2D}}{n_Q^{2D}}\right) \quad \text{þ.s.} \quad n^{2D} = \frac{N}{A}$$

---

b)

$$U = \sum_n \varepsilon_n f(\varepsilon_n) = \lambda\sum_n \varepsilon_n e^{-\frac{\varepsilon_n}{\tau}}$$

$$= \lambda\tau^2\frac{\partial}{\partial\tau}\sum_n e^{-\frac{\varepsilon_n}{\tau}} = \lambda\tau^2\frac{\partial}{\partial\tau}A n_Q^{2D}$$

$$= \lambda A\tau^2\left(\frac{\partial n_Q^{2D}}{\partial\tau}\right) = \lambda A n_Q^{2D}\tau = N\tau$$

$$\rightarrow \boxed{U = N\tau}$$

(10)

c) finna σ

Jafnan fyrir $\mu = \tau \ln\left(\frac{n^{2D}}{n_Q^{2D}}\right)$

er formlega eins og fyrir 3D, nema n^{2D} og n_Q^{2D} þýða annað

$$\rightarrow F = N\tau\left\{\ln\left(\frac{n^{2D}}{n_Q^{2D}}\right) - 1\right\}$$

með leiðum eins og í bók á bls. 163

Notum síðan

$$\sigma = -\left(\frac{\partial F}{\partial \tau}\right)_{A,N} = -N\left\{\ln\left(\frac{n^{2D}}{n_Q^{2D}}\right) - 1\right\} + N$$

$$= N\left\{\ln\left(\frac{n_Q^{2D}}{n^{2D}}\right) + 2\right\}$$

(1)

7-2 Afstæðilegt fermi gas

$\varepsilon \gg mc^2$

$\rightarrow \varepsilon \simeq pc$

$V = L^3$: teningur

Gerum ráð fyrir að

a)

$$p = \frac{\pi\hbar}{L}\sqrt{n_x^2 + n_y^2 + n_z^2}$$

$$\varepsilon_F = c p_F = c\frac{\pi\hbar}{L} n_F$$

$$N = 2 \times \frac{1}{8} \times \frac{4\pi}{3} n_F^3$$

$$n_F = \left(\frac{3N}{\pi}\right)^{1/3}$$

$$\rightarrow \varepsilon_F = \frac{c\pi\hbar}{L}\left(\frac{3N}{\pi}\right)^{1/3}$$

$$= \hbar\pi c\left(\frac{3N}{\pi L^3}\right)^{1/3}$$

$$= \hbar\pi c\left(\frac{3n}{\pi}\right)^{1/3}$$

b) finna U_0

$$U_0 = 2\sum_{n \le n_F} \varepsilon_n$$

$$= 2 \times \frac{1}{8} \times 4\pi \int_0^{n_F} dn\, n^2 \varepsilon_n$$

(2)

$$U_0 = 2 \times \frac{1}{8} \times 4\pi \int_0^{n_F} dn\, n^3 \frac{c\pi\hbar}{L}$$

$N = \frac{4\pi}{3} n_F^3 \cdot 4^{-1}$

$$= \frac{\pi^2 c\hbar}{L}\int_0^{n_F} dn\, n^3 = \frac{\pi^2 c\hbar}{L\,4} n_F^4 = \frac{\pi^2 c\hbar}{L4} \cdot \frac{3 \cdot 4}{4\pi} N \cdot n_F$$

En áður var komið

$$\varepsilon_F = c p_F = c\frac{\pi\hbar}{L} n_F \rightarrow n_F = \frac{\varepsilon_F L}{c\pi\hbar}$$

$$\rightarrow U_0 = \frac{\pi^2 c\hbar}{4L}\,\frac{3 \cdot 4}{4\pi} N\,\frac{\varepsilon_F L}{c\hbar\pi} = \frac{3}{4} N \varepsilon_F$$

(3)

7-3 a) sýna að fyrir fermigas í grunnástandi gildi

$$p = \frac{(3\pi^2)^{2/3}}{5} \cdot \frac{\hbar^2}{m}\left(\frac{N}{V}\right)^{5/3}$$

$$U_0 = \frac{3}{5} N \varepsilon_F \quad (\text{þýðir að } \tau = 0), \quad \varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

$n = \frac{N}{V}$

Vitum að

$$p = -\left(\frac{\partial U}{\partial V}\right)_{\sigma N} = -\left(\frac{\partial U_0}{\partial V}\right)_{\sigma N}$$

Áður var kynnt fyrir okkur að breyting á V þ.a. kerfið haldist í vissu ástandi er jafn óreiðubreyting

$$\rightarrow p = -\left(\frac{\partial U_0}{\partial V}\right)_N = -\frac{3}{5}N\left(\frac{\partial \varepsilon_F}{\partial V}\right)_N = \frac{3}{5}\cdot\frac{2}{3}\,\frac{N}{V}\varepsilon_F$$

$$= \frac{2}{5}\,\frac{N}{V}\varepsilon_F = \frac{2}{5} n \varepsilon_F$$

b) finna σ þegar $\tau \ll \varepsilon_F$

$$C_V = \tau \left(\frac{\partial \sigma}{\partial \tau}\right)_V$$

Vitum að $C_V \xrightarrow[\tau \to 0]{} \alpha \tau$

$$\Longrightarrow \left(\frac{\partial \sigma}{\partial \tau}\right)_V = \alpha$$

eða $\sigma = \alpha\tau = C_V$

Ef $\sigma \xrightarrow[\tau \to 0]{} 0$

þá kemur engin fasti til greina í $\sigma = \alpha \tau + \sigma_0$

$$\sigma = C_V = \frac{\pi^2 N \tau}{2 \varepsilon_F}$$

samkvæmt Eq. (37) í bók á bls. 193

(4)

(7-5) ^{3}He vökvi, $I = \frac{1}{2}$

(5)

finna v_F, ε_F, og T_F fyrir ^{3}He við $\tau = 0$

gefið $\rho = 0.081 \frac{g}{cm^3}$ $\quad n = \frac{\rho}{M}$, $M = 3$ amu

amu $= 1.66 \cdot 10^{-24}$ g

$$\varepsilon_F = \frac{\hbar^2}{2M}(3\pi^2 n)^{2/3}$$

$$= \frac{\hbar^2}{2M}\left(3\pi^2 \frac{\rho}{M}\right)^{2/3} = \frac{\hbar^2}{2M^{5/3}}(3\pi^2 \rho)^{2/3}$$

$$= \frac{(1.05 \cdot 10^{-27} \text{ erg s})^2}{2 \cdot (3 \cdot 1.66 \cdot 10^{-24} \text{ g})^{5/3}} \left(3\pi^2 \, 0.081 \frac{g}{cm^3}\right)^{2/3} = 6.8 \cdot 10^{-16} \text{ erg}$$

$$= 0.43 \text{ meV}$$

$$T_F = \frac{\varepsilon_F}{k_B} = 4.9 \text{ K}$$

(6)

$$\varepsilon_F = \frac{1}{2} M v_F^2 \rightarrow v_F = \sqrt{\frac{2\varepsilon_F}{M}}$$

$$v_F = \sqrt{\frac{2 \cdot 6.8 \cdot 10^{-16} \text{ erg}}{(3 \cdot 1.66 \cdot 10^{-24})}} = 1.65 \cdot 10^4 \frac{cm}{s}$$

---

b) C_V fyrir $\tau \ll \tau_F$ $\quad$ Mælt gildi $C_V = 2.89 N k_B T$

$C_V = \frac{\pi^2}{2} N \frac{\tau}{\tau_F}$ eða í venjul. ein. $\frac{\pi^2}{2} N \frac{k_B T}{T_F}$

$$\frac{C_V}{N} = \frac{\pi^2}{2} \frac{k_B T}{T_F} = \left(\frac{\pi^2}{2 T_F}\right) \cdot k_B T = 1.0 \, k_B T$$

Í þessum fermi vökva eru hreyfingar atómanna tengdar fylgni böndum, því er raun varmarýmdin hærri eins og atómin væru þyngri $\Longleftrightarrow$ vökva eiginleikar víxlverkan

(7-6) Hvíturdvergur: M, R

Rafeindir kulgas
Róteindir hlýgas

a) sjálforka (þyngdar)

Ef þéttleikinn er fastur

$$\frac{M}{V} = \rho$$

$$\rightarrow \rho = \frac{3M}{4\pi R^3}$$

því $V = \frac{4\pi}{3} R^3$

Sjálforkan er stöðuorkan fyrir allan massann (þyngdarstöðuorkan)

(7)

$$\Phi(r) = \frac{G\, m(r)}{r}$$

$$m(r) = \rho \cdot \frac{4\pi}{3} r^3 = M \frac{r^3}{R^3}$$

$$U_G = \int_0^R 4\pi r^2 dr \, \rho \, \Phi(r)$$

$$= \frac{GM}{R^3} 4\pi \left(\frac{3M}{4\pi R^3}\right) \int_0^R r^4 dr$$

$$= \frac{3GM^2}{R^6} \frac{R^5}{5} = \frac{3}{5} G M^2 \frac{1}{R}$$

8

Setjum stöðuorkuna 0 fyrir tvo massa þegar fjarlægð þeirra $\to \infty$

$$\to U_G = -\frac{3}{5}\frac{GM^2}{R}$$

ρ er í raun ekki fasti setjum sem staðargráðu-nálgun

$$U_G \approx -\frac{GM^2}{R}$$

b) Meta hreyfiorku

Rafeindamassi: m

Róteindarmassi: M_H

$$N = N_e = N_H \approx \frac{M}{M_H}$$

$$U_0 = \frac{3}{5}N\varepsilon_F$$

$$\varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

$$\to U_0 = \frac{3\hbar^2}{2m}\left(\frac{3\pi^2}{V}\right)^{2/3}\frac{3}{5}N^{5/3}$$

$$= \frac{3\hbar^2}{2m}\left(\frac{3\pi^2\cdot 3}{4\pi R^3}\right)^{2/3}\frac{3}{5}N^{5/3}$$

$$= \frac{\hbar^2 N^{5/3}}{mR^2}\cdot\underbrace{\frac{9}{10}\left(\frac{3\pi}{4}\right)^{2/3}}_{\sim 1{,}6} \sim \frac{\hbar^2 N^{5/3}}{mR^2}$$

9

$$U_0 \sim \frac{\hbar^2 N^{5/3}}{mR^2} = \frac{\hbar^2}{mR^2}\left(\frac{M}{M_H}\right)^{5/3}$$

c) Ef $U_0 \sim |U_G|$

$$\to \frac{\hbar^2}{mR^2}\left(\frac{M}{M_H}\right)^{5/3} \sim \frac{GM^2}{R}$$

$$\to \frac{\hbar^2}{mR}\left(\frac{M}{M_H}\right)^{5/3}M^{-2} \sim G$$

$$\frac{\hbar^2}{m}\left(\frac{1}{M_H}\right)^{5/3}\frac{1}{RM^{1/3}} \sim G$$

$$RM^{1/3} \sim \frac{\hbar^2}{m}\left(\frac{1}{M_H}\right)^{5/3}\frac{1}{G}$$

$$RM^{1/3} \sim 8\cdot 10^{19}\ \mathrm{g^{1/3}\,cm} \sim 10^{20}\ \mathrm{g^{1/3}\,cm}$$

d) Ef $M = M_\odot = 2\cdot 10^{33}$ g hver er þá þvermálsdvergsins

$$RM^{1/3} \sim 10^{20}$$

$$\to R = \frac{10^{20}\ \mathrm{g^{1/3}cm}}{M^{1/3}}$$

$$\approx 8\cdot 10^{8}\ \mathrm{cm}$$

$$\approx 8\cdot 10^{3}\ \mathrm{km}$$

10

e) Nifteindastjarna

$$U_0 = \frac{\hbar^2}{m\cdot 1837 R^2}\left(\frac{M}{M_H}\right)^{5/3}$$

$$RM^{1/3} \sim \frac{\hbar^2}{1837\cdot m}\left(\frac{1}{M_H}\right)^{5/3}\frac{1}{G}$$

$$\sim 4\cdot 10^{16}$$

$$\to R \sim 4\ \mathrm{Km}$$

1

7-7

SF - alheimur með N_γ = fasti

$$N = 10^{20}\ \frac{1}{\mathrm{cm}^3}$$

Getum ekki notað $\tau_E = \frac{2\pi\hbar^2}{M}\left(\frac{n}{2.612}\right)^{2/3}$

þ.s. $M = 0$

Í dæmi 4-1 fékkst $N_e = \frac{2.404\, V\tau^3}{\pi^2\hbar^3 c^3}$

Finna τ_c þ.a. fyrir $\tau < \tau_c$ $N_e < N$

$$n_e = \frac{2.404\,\tau^3}{\pi^2\hbar^3 c^3} \to n = \frac{2.404\,\tau_c^3}{\pi^2\hbar^3 c^3}$$

$$\to \tau_c^3 = \frac{n\cdot\pi^2\hbar^3 c^3}{2.404} \to \tau_c = \hbar c\sqrt[3]{\frac{n\pi^2}{2.404}}$$

(2)

$$\tau_c = 1.05\cdot 10^{-27}\,\text{erg s}\; 3\cdot 10^{10}\,\frac{\text{cm}}{\text{s}}\sqrt[3]{\frac{10^{20}\frac{1}{\text{cm}^3}\pi^2}{2.404}}$$

$$= 2.34\cdot 10^{-10}\,\text{erg} \quad = 146\text{ eV}$$

$$\rightarrow T_c = \tau_c/k_B = 1.7\cdot 10^{6}\,\text{K}$$

(3)

7-9 Bóse-eindir í einni vídd

Rekna $N_e(\tau)$

Ein vídd

Eins og sést í 7-1 er

$$\mathcal{D}_1(\varepsilon) = \frac{L}{2\pi}\sqrt{\frac{2m}{\hbar^2\varepsilon}}$$

$$N \simeq N_0 + \int_0^\infty d\varepsilon\, \mathcal{D}_1(\varepsilon)\, f(\varepsilon,\tau)$$

$$= N_0 + N_\varepsilon$$

$$N_\varepsilon = \frac{L}{2\pi}\sqrt{\frac{2m}{\hbar^2}}\int_0^\infty d\varepsilon\,\frac{f(\varepsilon,\tau)}{\sqrt{\varepsilon}}$$

$$= \frac{L}{2\pi}\sqrt{\frac{2m}{\hbar^2}}\int_0^\infty \frac{d\varepsilon}{\sqrt{\varepsilon}\{\lambda^{-1}e^{\varepsilon/\tau}-1\}}$$

ef $\lambda \sim 1$

$$N_\varepsilon = \frac{L\tau}{2\pi}\sqrt{\frac{2m}{\hbar^2\tau}}\int_0^\infty \frac{dx}{\sqrt{x}(e^x-1)}$$

$$= \frac{L}{2\pi}\sqrt{\frac{2m\tau}{\hbar^2}}\int_0^\infty \frac{dx}{\sqrt{x}\{e^x-1\}}$$

(4)

fallið $\frac{1}{\sqrt{x}(e^x-1)} \xrightarrow[x\to 0]{} \frac{1}{\sqrt{x}(1-x+\cdots-1)} \sim x^{-3/2}$

er ósamfellt og ekki heildanlegt

$\rightarrow$ lagt er til að heildis að framan sé ekki notað.

(5)

7-11 fyrir fermi-eindir (eitt svigrúm)

$$\langle(\Delta N)^2\rangle = \langle N\rangle\{1-\langle N\rangle\}$$

Eitt svigrúm með setu 0 eða 1

$$\langle(\Delta N)^2\rangle = \langle(N-\langle N\rangle)^2\rangle = \langle N^2\rangle - \langle N\rangle^2$$

fyrir eittsvigrúm gildir $N^2 = N \rightarrow \langle N^2\rangle = \langle N\rangle$

$$\rightarrow \langle(\Delta N)^2\rangle = \langle N\rangle - \langle N\rangle^2 = \langle N\rangle\{1-\langle N\rangle\}$$

(6)

7-12 fyrir Bóse-eindir með eitt svigrúm

$$\langle(\Delta N)^2\rangle = \langle N\rangle\left(1+\langle N\rangle\right)$$

Höfum (5-59)

$$\langle N\rangle = \frac{\tau}{\mathfrak{z}}\frac{\partial \mathfrak{z}}{\partial \mu}, \quad \mathfrak{z} = \sum_{N=0}^{\infty} \exp\left\{\frac{(N\mu-\varepsilon)}{\tau}\right\}$$

$$\langle N^2\rangle = \frac{1}{\mathfrak{z}}\sum_{N=0}^{\infty} N^2 \exp\left\{\frac{(N\mu-\varepsilon)}{\tau}\right\} = \frac{\tau^2}{\mathfrak{z}}\frac{\partial^2 \mathfrak{z}}{\partial \mu^2}$$

$$\rightarrow \langle(\Delta N^2)\rangle = \langle N^2\rangle - \langle N\rangle^2 = \tau^2\left\{\frac{1}{\mathfrak{z}}\frac{\partial^2 \mathfrak{z}}{\partial\mu^2} - \frac{1}{\mathfrak{z}^2}\left(\frac{\partial \mathfrak{z}}{\partial \mu}\right)^2\right\}$$

$$= \tau\frac{\partial\langle N\rangle}{\partial\mu}$$

(7)

$$\rightarrow \langle(\Delta N^2)\rangle = \tau\frac{\partial\langle N\rangle}{\partial\mu} = \tau\frac{\partial}{\partial\mu}\left\{\frac{1}{e^{\frac{\varepsilon-\mu}{\tau}}-1}\right\}$$

$$= \frac{e^{\frac{\varepsilon-\mu}{\tau}}}{\left\{e^{\frac{\varepsilon-\mu}{\tau}}-1\right\}^2} = \left\{\frac{1}{e^{\frac{\varepsilon-\mu}{\tau}}-1}\right\}\left\{\frac{e^{\frac{\varepsilon-\mu}{\tau}}}{e^{\frac{\varepsilon-\mu}{\tau}}-1}\right\}$$

$$= \langle N\rangle\left\{\langle N\rangle+1\right\}$$

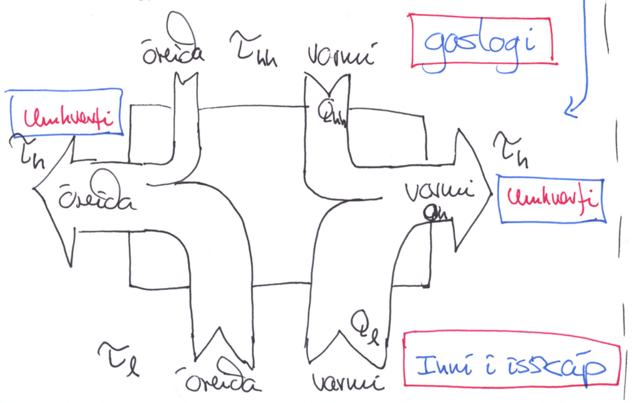
(1)

8.2 Gasísskápur

Varmi frá gasloga notaður til að knýja kerfið

$\tau_{hh} > \tau_h$

τ_l



b) Reikna $\dfrac{Q_l}{Q_{hh}}$ fyrir jafngengt ferli

$$Q_h = Q_{hh} + Q_l \quad (1)$$

$$\sigma_h = \sigma_{hh} + \sigma_l = \frac{Q_{hh}}{\tau_{hh}} + \frac{Q_l}{\tau_l}$$

og $\sigma_h = \dfrac{Q_h}{\tau_h}$

(2)

$$\sigma_h\tau_h = Q_{hh}\frac{\tau_h}{\tau_{hh}} + Q_l\frac{\tau_h}{\tau_l} = Q_h \quad (2)$$

$$(1)-(2) \rightarrow Q_{hh} + Q_l - Q_{hh}\frac{\tau_h}{\tau_{hh}} - Q_l\frac{\tau_h}{\tau_l} = 0$$

$$\rightarrow Q_{hh}\left\{1-\frac{\tau_h}{\tau_{hh}}\right\} + Q_l\left\{1-\frac{\tau_h}{\tau_l}\right\} = 0$$

$$\rightarrow \frac{Q_l}{Q_{hh}} = -\frac{\left\{1-\frac{\tau_h}{\tau_{hh}}\right\}}{\left\{1-\frac{\tau_h}{\tau_l}\right\}} = \frac{\left\{1-\frac{\tau_h}{\tau_{hh}}\right\}}{\left\{\frac{\tau_h}{\tau_l}-1\right\}}$$

$$= \left(\frac{\tau_l}{\tau_{hh}}\right)\frac{(\tau_{hh}-\tau_h)}{(\tau_h-\tau_l)} = \eta_c(\tau_{hh},\tau_h)\cdot\gamma_c(\tau_h,\tau_l)$$

8-3 Ljóseinda Carnot-vél

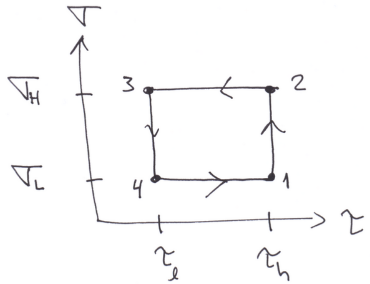
Ljóseindagas, notum

$$\frac{U}{V} = \frac{\pi^2}{15\hbar^3 c^3}\tau^4 \quad \text{og} \quad \sigma = \frac{4\pi^2 V}{45}\left(\frac{\tau}{\hbar c}\right)^3$$

sem

$$U = \alpha V \tau^4 \quad \text{og} \quad \sigma = \frac{4\alpha}{3} V\tau^3$$

a) τ_h, τ_l, V_1 og V_2 gefin
reikna V_3 og V_4



frá 2-3 er jafnóreiðuferill

$$\rightarrow \frac{4\alpha}{3}V_2\tau_h^3 = \frac{4\alpha}{3}V_3\tau_l^3$$

$$\boxed{V_3 = V_2\left(\frac{\tau_h}{\tau_l}\right)^3}$$

Líka frá 4-1:

$$\frac{4\alpha}{3}V_4\tau_l^3 = \frac{4\alpha}{3}V_1\tau_h^3$$

$$\boxed{V_4 = V_1\left(\frac{\tau_h}{\tau_l}\right)^3}$$

b) Hver er Q_h tekinn inn og vinnan í fyrstu jafnhitaþenslunni?

$$đQ = \tau\, d\sigma$$

$$Q_h = Q_{12} = \int_{\sigma_1}^{\sigma_2}\tau_h\, d\sigma$$

$$= \tau_h(\sigma_2-\sigma_1)$$

$$= \tau_h \frac{4\alpha}{3}\tau_h^3(V_2-V_1)$$

$$= \frac{4\alpha}{3}\tau_h^4(V_2-V_1)$$

$$đW = dU - đQ$$

$$W_{12} = U_2 - U_1 - Q_{12} = \alpha\tau_h^4(V_2-V_1) - \frac{4\alpha}{3}\tau_h^4(V_2-V_1)$$

$$= -\frac{\alpha}{3}\tau_h^4(V_2-V_1) = -\frac{Q_h}{4} \quad , \quad Q_h = Q_{12}$$

$$\rightarrow W_{12} \neq Q_h$$

skoðum líka (3-4) jafnhitaferli þar fæst

$$W_{34} = -\frac{\alpha}{3}\tau_l^4(V_4-V_3) = -\frac{\alpha}{3}\tau_l\,\tau_l^3(V_4-V_3)$$

nota nú að $\tau_l^3 V_4 = \tau_H^3 V_1$ og $\tau_l^3 V_3 = \tau_h^3 V_2$ ← jafnóreiðu tengsl

$$\rightarrow W_{34} = \frac{\alpha}{3}\tau_l\tau_h^3(V_2-V_1)$$

Heildar vinnan á kerfið er

$$W_{12} + W_{34} = -\frac{\alpha}{3}\tau_h^3(\tau_h-\tau_l)(V_2-V_1)$$

c) Styttast jafnóreiðuferlin (2->3) og (4->1) út?

$$W_{23} = U_3 - U_2 = \alpha V_3\tau_l^4 - \alpha V_2\tau_h^4$$

$$= -\alpha V_2\tau_h^3(\tau_h-\tau_l)$$

$$W_{41} = U_1 - U_4 = \alpha V_1\tau_h^4 - \alpha V_2\tau_l^4$$

$$= \alpha V_1\tau_h^3(\tau_h-\tau_l)$$

$$\rightarrow W_{23}+W_{41} = -\alpha\tau_h^3(\tau_h-\tau_l)(V_2-V_1) \neq 0$$

7

d) Heildar vinnan framkvæmd af kerfinu

$$W = -\left(W_{12} + W_{23} + W_{34} + W_{41}\right)$$

$$= \underbrace{\alpha\tau_h^3(\tau_h-\tau_l)(V_2-V_1)}_{-W_{23}-W_{41}} + \underbrace{\frac{\alpha}{3}\tau_h^3(\tau_h-\tau_l)(V_2-V_1)}_{W_{12}+W_{34}}$$

$$= \frac{4\alpha}{3}\tau_h^3(\tau_h-\tau_l)(V_2-V_1)$$

$$\rightarrow \eta = \frac{W}{Q_h} = \frac{\frac{4\alpha}{3}\tau_h^3(\tau_h-\tau_l)(V_2-V_1)}{\frac{4\alpha}{3}\tau_h^4(V_2-V_1)} = \frac{\tau_h-\tau_l}{\tau_h} = \eta_c$$

8-6 8

Carnot loftkælir milli T_h úti og T_l inni

Inn flæðir vegna lélegrar einangrunar $A(T_h-T_l) = \frac{dQ_l}{dt}$

Afl kælis er P, finna T_l stöðugt hitastig inni

$$W = \frac{\tau_h-\tau_l}{\tau_l} Q_l$$

$$\rightarrow \frac{dW}{dt} = \frac{\tau_h-\tau_l}{\tau_l}\frac{dQ_l}{dt} = \frac{T_h-T_l}{T_l} A(T_h-T_l)$$

$$= A\frac{(T_h-T_l)^2}{T_l} = P$$

$$\rightarrow A(T_h-T_l)^2 = PT_l$$

$$T_l^2 - \left(2T_h + \frac{P}{A}\right)T_l + T_h^2 = 0$$

9

finnum rót með $T_l < T_h$ eins og krafist var

$$T_l = \frac{\left(2T_h + \frac{P}{A}\right) \pm \sqrt{\left(2T_h+\frac{P}{A}\right)^2 - 4T_h^4}}{2}$$

$$= \left(T_h + \frac{P}{2A}\right) \pm \sqrt{\left(T_h + \frac{P}{2A}\right)^2 - T_h^4}$$

b) $T_h = 37°C$

$T_l = 17°C$

$P = 2kW$

finna A

$$A = \frac{PT_l}{(T_h-T_l)^2}$$

$$= \frac{2kW \cdot 290K}{(20K)^2} = 1.45\ kW/K$$

7-1 1

Sýna Maxwell tengslin

① $\left(\frac{\partial V}{\partial \tau}\right)_{pN} = -\left(\frac{\partial \sigma}{\partial p}\right)_{\tau N}$

② $\left(\frac{\partial V}{\partial N}\right)_{p\tau} = +\left(\frac{\partial \mu}{\partial p}\right)_{N\tau}$

③ $\left(\frac{\partial \mu}{\partial \tau}\right)_{Np} = -\left(\frac{\partial \sigma}{\partial N}\right)_{\tau p}$

① Notum $V = \left(\frac{\partial G}{\partial p}\right)_{N\tau}$

$$\rightarrow \left(\frac{\partial V}{\partial \tau}\right)_{pN} = \left(\frac{\partial}{\partial \tau}\frac{\partial G}{\partial p}\right)_N = \left(\frac{\partial}{\partial p}\frac{\partial G}{\partial \tau}\right)_N$$

$$= -\left(\frac{\partial \sigma}{\partial p}\right)_{\tau N} \quad \text{því} \quad \left(\frac{\partial G}{\partial \tau}\right)_{Np} = -\sigma$$

②

$$\left(\frac{\partial V}{\partial N}\right)_{p\tau} = \left(\frac{\partial}{\partial N}\frac{\partial G}{\partial p}\right)_\tau = \left(\frac{\partial}{\partial p}\frac{\partial G}{\partial N}\right)_\tau$$

$$= \left(\frac{\partial \mu}{\partial p}\right)_{N\tau} \quad \text{því} \quad \mu = \left(\frac{\partial G}{\partial N}\right)_{\tau p}$$

③

$$\left(\frac{\partial \mu}{\partial \tau}\right)_{Np} = \left(\frac{\partial}{\partial \tau}\frac{\partial G}{\partial N}\right)_p = \left(\frac{\partial}{\partial N}\frac{\partial G}{\partial \tau}\right)_p$$

$$= -\left(\frac{\partial \sigma}{\partial N}\right)_{\tau p} \quad \text{því} \quad -\sigma = \left(\frac{\partial G}{\partial \tau}\right)_{Np}$$

②

b) Sýna með ① og 3. lögmálinu að

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial \tau}\right)_p \xrightarrow[\tau\to 0]{} 0$$

Margfeldni grunnástands g er venjulega fasti ~ 1.....
óháður p

$$\rightarrow \left(\frac{\partial \sigma}{\partial p}\right)_{\tau N} = 0$$

því $\sigma = \ln g$

$$\rightarrow \left(\frac{\partial V}{\partial \tau}\right)_{pN} \rightarrow 0$$

og þar með

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial \tau}\right)_p \rightarrow 0$$

þegar $\tau \rightarrow 0$

③

9-2 Ferlið

$$e + H^+ \longleftrightarrow H$$

Vitum $\prod_j n_j^{\nu_j} = K(\tau)$ (*)

og $K(\tau) = \prod_j n_{Qj}^{\nu_j} \exp\left\{-\nu_j F_j^{int} \frac{1}{\tau}\right\}$ (**)

$$e + H^+ - H = 0$$

$\rightarrow \nu(e) = 1$
$\nu(H^+) = 1$
$\nu(H) = -1$

Sleppum spuna

$\hookrightarrow F_e^{int} = 0$, E_I : jónunarorka H

$F_{H^+}^{int} = 0$ $\hookrightarrow F_H^{int} = -E_I$

④

(*) $\rightarrow$

$$n_e \cdot n_{H^+} \cdot n_H^{-1} = K(\tau)$$

(**)

$$K(\tau) = (n_Q)_e \cdot (n_Q)_{H^+} \cdot (n_Q)_H^{-1} \exp\left\{-\left(-\frac{F_H^{int}}{\tau}\right)\right\}$$

~1 $\quad m_{H^+} \sim m_H$

eða

$$n_e \cdot n_{H^+} \cdot n_H^{-1} \simeq (n_Q)_e \exp\left\{-\frac{E_I}{\tau}\right\}$$

eða

$$\frac{[e][H^+]}{[H]} \simeq (n_Q)_e \exp\left\{-\frac{E_I}{\tau}\right\}$$

⑤

Ef allar rafeindir og róteindir koma frá jónum H

$\rightarrow [e] = [H^+]$ og kvaðratrót gefur

$$[e] = [H]^{1/2} (n_Q)_e^{1/2} \exp\left\{-\frac{E_I}{2\tau}\right\}$$

b) $[H_{exc}]$ er þéttleiki H-atóma í fyrsta örvaða ástandinu

$E_{exc} = \frac{3}{4}E_I$ og ástandið er fjórfallt

$$\frac{P(H_{exc})}{P(H)} = \frac{4\exp\left(-\frac{E_{exc}}{\tau}\right)}{\exp\left(-\frac{E_I}{\tau}\right)} = \frac{4\exp\left(-\frac{3}{4\tau}E_I\right)}{\exp\left(-\frac{E_I}{\tau}\right)} = 4\exp\left(\frac{E_I}{4\tau}\right)$$

6

9-3 Si + rafgjafar $\epsilon = 11.7$, $m^* = 0.3 m_e$

Ef $n_d = 10^{17}\ \mathrm{cm}^{-3}$ finna n_e, $T = 100\mathrm{K}$

Notum úrlausnina á undan

$$[e][n_d^+][n_d^0]^{-1} = n_Q(m^*)\exp\left(-\frac{E_I}{\tau}\right)$$

$$E_I = \frac{e^4 m^*}{2\hbar^2\epsilon^2} = R_y^* = \frac{0.3}{(11.7)^2}R_y = 0.0022\,R_y$$

$$= 0.03\ \mathrm{eV} = 30\ \mathrm{meV}$$

$$n_Q(m^*) \simeq 4\cdot 10^{17}\ \mathrm{cm}^{-3}$$

$$\rightarrow n_Q(m^*)\exp\left(-\frac{E_I}{\tau}\right) = 4\cdot 10^{17}\ \mathrm{cm}^{-3}\exp\left\{-\frac{30\ \mathrm{meV}\cdot \mathrm{K}}{8.62\cdot 10^{-2}\ \mathrm{meV}\ 100\mathrm{K}}\right\}$$

$$\simeq 1.2\cdot 10^{16}\ \mathrm{cm}^{-3}$$

7

Hér er því ekki víst að það sé góð nálgun að setja $[n_d^+][n_d^0]^{-1} \sim 1$ eða $[n_d^0] \sim$

betra væri

$$[n_d^0] = [n_d] - [n_d^+] = [n_d] - [e]$$

$$\rightarrow \frac{[e][e]}{[n_d]-[e]} = K, \quad [e]^2 - K\{[n_d]-[e]\} = 0$$

eða

$$[e]^2 + K[e] - K[n_d] = 0$$

leysa annars stigs jöfnu

$$[e] = -\frac{K}{2} + \sqrt{\left(\frac{K}{2}\right)^2 + K[n_d]}$$

jákvæða rótin

$$\approx 2.9\cdot 10^{16}\ \mathrm{cm}^{-3}$$

1

10-1 van der Waals gas

a) Reikna óreiðu

$$F = -N\tau\left\{\ln\left[\frac{n_Q(V-Nb)}{N}\right] + 1\right\} - \frac{N^2 a}{V}$$

$$\sigma = -\left(\frac{\partial F}{\partial \tau}\right)_V$$

$$= N\left\{\ln\left[\frac{n_Q(V-Nb)}{N}\right] + 1\right\} + N\tau\underbrace{\left(\frac{\partial \ln n_Q}{\partial \tau}\right)_V}$$

$$n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{3/2} \longrightarrow \quad = \frac{3}{2}$$

$$\rightarrow \sigma = N\left\{\ln\left[\frac{n_Q(V-Nb)}{N}\right] + \frac{5}{2}\right\}$$

2

b) finna U

$$F = U - \tau\sigma$$

$$\Rightarrow U = F + \tau\sigma = -N\tau\left\{\ln\left[\frac{n_Q(V-Nb)}{N}\right] + 1\right\} + N\tau\left\{\ln\left[\frac{n_Q(V-Nb)}{N}\right] + \frac{5}{2}\right\} - \frac{N^2 a}{V}$$

$$= -N\tau + \frac{5}{2}N\tau - \frac{N^2 a}{V}$$

$$= \frac{3}{2}N\tau - \frac{N^2 a}{V} = \frac{3}{2}N\tau - Nan$$

c) Reikna $H = U + pV$

ef $\frac{b}{V} << 1$

$$pV = \frac{N\tau V}{V - Nb} - \frac{N^2 a}{V} = \frac{N\tau}{\left(1 - \frac{Nb}{V}\right)} - \frac{N^2 a}{V} \simeq N\tau\left(1 + \frac{Nb}{V}\right) - \frac{N^2 a}{V}$$

(3)

$$\to H \simeq \frac{3}{2}N\tau - \frac{N^2a}{V} + N\tau\left(1+\frac{Nb}{V}\right) - \frac{N^2a}{V}$$

$$= \frac{5}{2}N\tau + \frac{N^2\tau b}{V} - \frac{2N^2a}{V}$$

þurfum $H(N,\tau,p)$ en ekki $H(N,\tau,V)$

Reynum 1. stigs liðum í a og b á $\frac{1}{V}$

$$pV \simeq N\tau\left(1+\frac{Nb}{V}\right) - \frac{N^2a}{V}$$

þegar komið hér

$$\to pV \simeq N\tau$$

$$\to \frac{N}{V} = \frac{p}{\tau}$$

$$H \simeq \frac{5}{2}N\tau + N\tau b\frac{p}{\tau} - 2Na\frac{p}{\tau} = \frac{5}{2}N\tau + Nbp - \frac{2Nap}{\tau}$$

(4)

(10-4) Gas-fastefni

leyfa 3D-hreyfingu

Notum sömu táknun og í bók

Þrívíða sveifill með róf

$$(n_x+n_y+n_z)\hbar\omega - \Sigma_0$$

$$Z_s = \sum_{n_x n_y n_z} \exp\left\{-\frac{(n_x+n_y+n_z)\hbar\omega - \Sigma_0}{\tau}\right\} = e^{\frac{\Sigma_0}{\tau}}\left[\sum_n e^{-\frac{n\hbar\omega}{\tau}}\right]^3$$

$$= e^{\frac{\Sigma_0}{\tau}}\left\{\frac{1}{1-e^{-\frac{\hbar\omega}{\tau}}}\right\}^3$$

(5)

a) finna $p(\tau)$, $F_s = U_s - \tau\sigma_s = -\tau\ln Z_s$

$$G_s = F_s + pV_s = \mu_s$$

slepp pV

$$\lambda_s = e^{\frac{\mu_s}{\tau}} \simeq e^{\frac{F_s}{\tau}} = e^{-\ln Z_s}$$

$$= \frac{1}{Z_s} = e^{-\frac{\Sigma_0}{\tau}}\left\{1-e^{-\frac{\hbar\omega}{\tau}}\right\}^3$$

þ. $\tau >> \hbar\omega$

$$\lambda_s \simeq e^{-\frac{\Sigma_0}{\tau}}\left\{1-1+\left(\frac{\hbar\omega}{\tau}\right)+\cdots\right\}^3 \simeq e^{-\frac{\Sigma_0}{\tau}}\left(\frac{\hbar\omega}{\tau}\right)^3$$

Áður var komið

$$\lambda_g = \frac{n}{n_Q} = \frac{p}{\tau}\left(\frac{2\pi\hbar^2}{M\tau}\right)^{3/2}$$ fyrir kjörgas

(6)

Í jafnvægi

$$\lambda_s = \lambda_g$$

$$\to e^{-\frac{\Sigma_0}{\tau}}\left(\frac{\hbar\omega}{\tau}\right)^3 = \frac{p}{\tau}\left(\frac{2\pi\hbar^2}{M\tau}\right)^{3/2}$$

$$\to p \simeq \left(\frac{M}{2\pi}\right)^{3/2}\frac{\omega^3}{\sqrt{\tau}}e^{-\frac{\Sigma_0}{\tau}}$$

b) Brotthvarmi

$$\ln p = -\frac{L_0}{\tau} + \text{fasti}$$ Clausius-Clapeyron

$$\frac{d}{d\tau}\ln p = \frac{L_0}{\tau^2} \quad (*)$$

7

$$p \simeq \left(\frac{M}{2\pi}\right)^{3/2} \frac{\omega^3}{\sqrt{\tau}}\, e^{-\frac{\varepsilon_0}{\tau}}$$

$$\to \frac{d}{d\tau}\ln p \simeq \frac{\varepsilon_0}{\tau^2} - \frac{1}{2}\tau^{-1} = \frac{1}{\tau^2}\left(\varepsilon_0 - \frac{\tau}{2}\right)$$

samanburður við (*) gefur

$$L_0 \simeq \left(\varepsilon_0 - \frac{\tau}{2}\right)$$

heppilegra væri að taka orku sveiflunnar sem

$$\left\{(n_x+n_y+n_z) + \frac{3}{2}\right\}\hbar\omega - \varepsilon_0$$

← núllpunkts orka

7b

þá fengist

líta lengra

$$\lambda_s \simeq e^{-\frac{\varepsilon_0}{\tau}}\left\{1 - 1 + \left(\frac{\hbar\omega}{\tau}\right) - \frac{1}{2}\left(\frac{\hbar\omega}{\tau}\right)^2 + \dots\right\}^3$$

$$= e^{-\frac{\varepsilon_0}{\tau}}\left(\frac{\hbar\omega}{\tau}\right)^3\left\{1 - \frac{1}{2}\left(\frac{\hbar\omega}{\tau}\right)\right\}^3$$

$$\simeq e^{-\frac{\varepsilon_0}{\tau}}\left(\frac{\hbar\omega}{\tau}\right)^3\left\{1 - \frac{3}{2}\left(\frac{\hbar\omega}{\tau}\right)\right\} \simeq \left(\frac{\hbar\omega}{\tau}\right)^3 e^{-\frac{\varepsilon_0}{\tau}} e^{-\frac{3\hbar\omega}{2\tau}}$$

eða

$$\lambda_s \simeq \exp\left\{-\frac{(\varepsilon_0 - \frac{3\hbar\omega}{2})}{\tau}\right\}\left(\frac{\hbar\omega}{\tau}\right) + \dots$$

þá væri 0-punkturinn betur settur

8

11-3

B er íbót í A með $x \ll 1$

Frjálsa orkan án blöndunar

$$f_0(x) = f_0(0) + x f_0'(0) \qquad \text{f. vökva og fasta}$$

Gerum ráð fyrir að vökva blandan sé í jafnvægi við fasta blönduna

Reikna $k = \frac{x_s}{x_L}$

blöndunar óreiða

$$\sigma_M = -N\left\{\underbrace{(1-x)}_{\simeq 1}\underbrace{\ln(1-x)}_{=(-x-\frac{x^2}{2}\dots)} + x\ln x\right\} \simeq -N\{-x + x\ln x\}$$

$$= +N\{x(1-\ln x)\}$$

9

þ.v. er heildar frjálsa orkan á atóm

$$f(x) = f_0(x) + \frac{\tau\sigma_M}{N} = f_0(0) + x\left\{f_0'(0) + \tau(1-\ln x)\right\}$$

$$\frac{df(x)}{dx} = f_0'(0) + \tau(1-\ln x) - \tau = f_0'(0) - \tau\ln x$$

Gasið og fasta efnið eru í jafnvægi

þegar $f_L'(x_L) = f_S'(x_S)$

$$f_{L0}'(0) - \tau\ln x_L = f_{S0}'(0) - \tau\ln x_s$$

$$\to f_{L0}'(0) - f_{S0}'(0) = \tau\ln\left(\frac{x_s}{x_L}\right) = \tau\ln k$$

þ.s. $k = \frac{x_s}{x_L}$ og $\to$ $k = \exp\left\{-\frac{(f_{S0}'(0) - f_{L0}'(0))}{\tau}\right\}$

fyrir $f'_{os}(0) - f'_{ol}(0) = 1\,eV$, $T = 1000\,K$

$$1\,eV \simeq 1.16 \cdot 10^4\,K$$

$$\rightarrow k = \exp\left\{-\frac{1.16 \cdot 10^4\,K}{1000\,K}\right\} \simeq 9.2 \cdot 10^{-6}$$

Bera saman við mynd 11.5 í bók