1 Frjálsar 2D-Fermi eindir (rafeindir) á L^2

$$\to \epsilon_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 \qquad n^2 = n_x^2 + n_y^2$$

a) finna $\epsilon_F = \tau_F$ sem fall af $n = \frac{N}{L^2}$, $T=0$

skammtatölurnar fyrir setnu ástöndin við $T=0$ eru fyrir ástöndin með orku lægri en ϵ_F. Þær mynda því skífu með geisla n_F í (n_x, n_y)-rúminu. Fjöldi þeirra er

$$N = 2 \times \frac{1}{4} \times \pi n_F^2 = \frac{\pi}{2} n_F^2 \to n_F = \left(\frac{2N}{\pi}\right)^{1/2}$$

spuni 1/2

flötur skífu

$n_x, n_y > 0$

$$\epsilon_F = \epsilon_{n_F} = \frac{\hbar^2}{2m}\left(\frac{n_F\pi}{L}\right)^2 = \frac{\hbar^2}{2m}\left(\frac{2N}{\pi}\,\frac{\pi^2}{L^2}\right) =$$

$$= \frac{\hbar^2\pi}{m}\left(\frac{N}{L^2}\right) = \tau_F$$

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b) finna út $\frac{U_0}{N}$, $T=0$

í tví vídd er heildisfrymið $d\varphi\, n\, dn$

Heildarorkan er

$$U_0 = 2\sum_{n\le n_F}\epsilon_n = 2\cdot\frac{1}{4}\cdot 2\pi\int_0^{n_F}\overbrace{dn\, n}\,\epsilon_n$$

$$= \pi\int_0^{n_F} dn\,\frac{\hbar^2\pi^2}{2mL^2}\,n^2\cdot n = \frac{\hbar^2\pi^3}{2mL^2}\int_0^{n_F} dn\, n^3$$

$$= \frac{\hbar^2 \pi^3}{2mL^2} \frac{n_F^4}{4} = \frac{\hbar^2 \pi^3}{2mL^2} \frac{4N^2}{4\pi^2} = \frac{\hbar^2 \pi}{2m} \left(\frac{N}{L^2}\right) \cdot N = \frac{1}{2} N \varepsilon_F$$

$$\rightarrow \frac{U_0}{N} = \frac{\varepsilon_F}{2} \quad \text{við } \tau = 0$$

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c)

$$\varepsilon_F = \frac{\hbar^2 2\pi}{2m} \left(\frac{N}{L^2}\right) \rightarrow \varepsilon(N) = \frac{\hbar^2 \pi}{m} \left(\frac{N(\varepsilon)}{L^2}\right)$$

$$\rightarrow N(\varepsilon) = \frac{L^2 \varepsilon m}{\hbar^2 \pi} \rightarrow \frac{dN(\varepsilon)}{d\varepsilon} = \frac{L^2 m}{\hbar^2 \pi}$$

$$\rightarrow \mathcal{D}_{2D}(\varepsilon) = \frac{dN(\varepsilon)}{d\varepsilon} = \frac{L^2 m}{\hbar^2 \pi} \quad \text{fasti}$$

d) Niðurstöðurnar falla saman, því ef ástandaþéttleikinn er fasti þá hlýtur meðalorkan að vera $\frac{\varepsilon_F}{2}$

2

$e + h = 0$

$m_e = 0.2\, m_0$

$m_h = 0.7\, m_0$

$$n_Q(m_0) = \left(\frac{m_0 \tau}{2\pi \hbar^2}\right)^{3/2} = 1.25 \cdot 10^{19}\ \text{cm}^{-3} \text{ við } T = 300\,\text{K}$$

Finna $[e] = [h]$

$$\prod_j n_j^{\nu_j} = \prod_j n_{Qj}^{\nu_j} \exp\left\{-\frac{\nu_j F_j^{int}}{\tau}\right\}$$

Minnsta orka sem losnar við hvar fið er $\frac{\Delta}{\tau} = 10$

Núllpunktur settur á ástand án eindar

$$\rightarrow F_e^{int} + F_h^{int} = \Delta \quad \Bigg| \quad \begin{aligned} \nu_e &= 1 \\ \nu_h &= 1 \end{aligned}$$

5

$$\rightarrow [e][h] = [e]^2 = n_{Qe} \cdot n_{Qh} \cdot \exp\left\{-\frac{\Delta}{\tau}\right\}$$

$$= \left(n_Q(m_0)\right)^2 \cdot \left(\frac{m_e \cdot m_h}{m_0 \cdot m_0}\right)^{3/2} \exp\left\{-\frac{\Delta}{\tau}\right\}$$

$$= \underbrace{\left[n_Q(m_0)\right]^2}_{\left\{1.25 \cdot 10^{19}\right\}^2} \left(0.2 \cdot 0.7\right)^{3/2} \exp\left\{-10\right\}$$

$$\Rightarrow \quad [e] = 1.93 \cdot 10^{16}\ \text{cm}^{-3}$$

3

$$-\frac{\mu}{\tau} = \left(\frac{\partial \sigma}{\partial N}\right)_{U,V} \qquad \mu = -0.2\ \text{eV}$$

Hversu mjög fjölgar smásæjum ástöndum þegar einni eind er bætt við (herbergishiti)

Munum frumjöfnu safneðlisfræðinnar

$$\sigma(N,U) = \ln\left[g(N,U)\right]$$

$$\rightarrow \quad \left(\frac{\partial \ln g}{\partial N}\right)_{U,V} = -\frac{\mu}{\tau}$$

$$\rightarrow \quad \Delta \ln g = -\frac{\mu}{\tau}\Delta N \quad \rightarrow \quad \frac{\Delta g}{g} = -\frac{\mu}{\tau}\Delta N$$

fyrir eina eind $\Delta N = 1$, $\tau \simeq 300K \cdot 8.617 \cdot 10^{-5} eV\, K^{-1}$

$\simeq 0.026\ eV$

$$\rightarrow \frac{\Delta g}{g} = -\frac{\mu}{\tau} = +\frac{0.2\ eV}{0.026\ eV}$$

$$\simeq 7.7$$

$$\rightarrow \Delta g \simeq 7.7 \cdot g$$

4

E	g
3ε	5
ε	3
0	1

Orkuróf

Körsumman er

$$Z = \sum_j g_j e^{-\frac{\varepsilon_j}{\tau}}$$

$$= 1 \cdot e^{-0} + 3e^{-\frac{\varepsilon}{\tau}} + 5e^{-\frac{3\varepsilon}{\tau}}$$

Setjum $\tau = \varepsilon$

$$\rightarrow Z(\varepsilon) = 1 + 3e^{-1} + 5e^{-3}$$

<u>Hlutfallsleg sókni</u>

$$P(\varepsilon_1) = \frac{1}{Z(\varepsilon)}, \quad P(\varepsilon_2) = \frac{3e^{-1}}{Z(\varepsilon)}, \quad P(\varepsilon_3) = \frac{5e^{-3}}{Z(\varepsilon)}$$

$\simeq 0.425$ $\qquad \simeq 0.469$ $\qquad \simeq 0.106$

Mean energy

$$Z(\tau) = 1 + 3e^{-\frac{\varepsilon}{\tau}} + 5e^{-\frac{3\varepsilon}{\tau}}$$

$$U = +\tau^2 \frac{\partial \ln Z}{\partial \tau}$$

$$U = \frac{\left\{3\varepsilon \cdot e^{-\frac{\varepsilon}{\tau}} + 5 \cdot 3 \cdot \varepsilon e^{-\frac{3\varepsilon}{\tau}}\right\}}{Z(\tau)}$$

$$\rightarrow U(\varepsilon) = \frac{\left\{3\varepsilon e^{-1} + 15 \cdot \varepsilon e^{-3}\right\}}{Z(\varepsilon)} = \underline{\underline{0{,}79\,\varepsilon}}$$

b) Hvenar er $P(\Sigma_2) = P(\Sigma_3)$

~~feg~~ gar

$$\frac{g_2 e^{-\frac{\Sigma}{\tau}}}{Z} = \frac{g_3 e^{-\frac{3\Sigma}{\tau}}}{Z}$$

~~Sola~~ ~~feg~~ gar

$$3e^{-\frac{\Sigma}{\tau}} = 5e^{-\frac{3\Sigma}{\tau}}$$

$$\rightarrow \frac{3}{5} = \exp\left\{-\frac{3\Sigma}{\tau} + \frac{\Sigma}{\tau}\right\} = e^{-\frac{2\Sigma}{\tau}}$$

$$\rightarrow \ln\left(\frac{3}{5}\right) = -\frac{2\Sigma}{\tau} \rightarrow \tau = -\frac{2\Sigma}{\ln\left(\frac{3}{5}\right)}$$

$$\simeq \underline{\underline{3.92\ \Sigma}}$$