Varmamjud Debye-kristalls

$$\ln Z = \int_0^{\omega_D} d\omega\, g(\omega) \ln\left\{\frac{e^{-\frac{1}{2}\hbar\omega\beta}}{1-e^{-\hbar\omega\beta}}\right\}$$

$$= -\int_0^{\omega_D} d\omega\, g(\omega)\, \frac{\hbar\omega\beta}{2} - \int_0^{\omega_D} d\omega\, g(\omega) \ln\left(1-e^{-\hbar\omega\beta}\right)$$

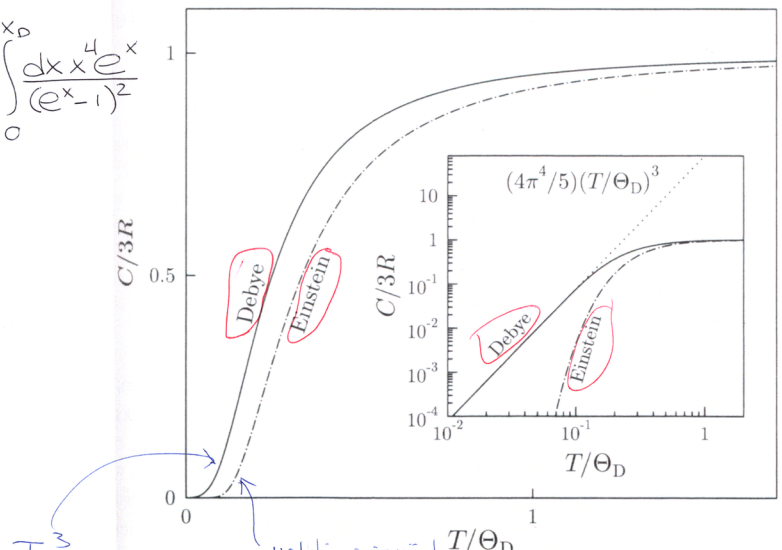
$$= -\frac{9}{8} N\hbar\omega_D\beta - \frac{9N}{\omega_D^3}\int_0^{\omega_D} d\omega\, \omega^2 \ln\left\{1-e^{-\hbar\omega\beta}\right\}$$

$$U = -\frac{\partial \ln Z}{\partial \beta} = \frac{9}{8} N\hbar\omega_D + \frac{9N\hbar}{\omega_D^3}\int_0^{\omega_D} \frac{d\omega\, \omega^3}{\left(e^{\hbar\omega\beta}-1\right)}$$

$$C = \left(\frac{\partial U}{\partial T}\right) = \frac{9Nt}{\omega_D^3}\int_0^{\omega_D} \frac{d\omega(-\omega^3)}{(e^{t\omega\beta}-1)^2}\, e^{t\omega\beta}\left(-\frac{t\omega}{k_B T^2}\right)$$

Setjum $x = t\beta\omega$, $x_D = t\beta\omega_D$

$$C = \frac{9R}{x_D^3}\int_0^{x_D} \frac{dx\, x^4 e^x}{(e^x-1)^2}$$



Stadum aðfellur

$T \to \infty$, $x \to 0 \Rightarrow e^x - 1 \to x$

$$C \to \frac{9R}{x_D^3} \int_0^{x_D} \frac{x^4}{x^2}\, dx = 3R$$

Dulong-Petit, eins og fyrir líkan Einsteins

$T \to 0$, $x \to \infty \Rightarrow e^x >> 1$

$$C \to \frac{9R}{x_D^3} \int_0^{\infty} \frac{dx\, x^4 e^x}{(e^x - 1)^2} = \frac{9R}{x_D^3} \Gamma(4)\zeta(4)$$

$$= \frac{9R}{x_D^3} \, 4 \frac{\pi^4}{90} \, 6 = \frac{12 R \pi^4}{5 x_D^3}$$

$$= 3R \cdot \frac{4\pi^4}{5} \left(\frac{T}{\Theta_D} \right)^3$$

Vex eins og T^3

Hljóðeinda tvístrun

Athugum einatóma línulega keðju

hreyfijafna

$$m\ddot{u}_n = K(u_{n+1} - u_n) - K(u_n - u_{n-1}) = K(u_{n+1} - 2u_n + u_{n-1})$$

Reynum lausn $u_n = \exp\{i(qna - \omega t)\}$

$$\rightarrow \quad -m\omega^2 = K\left(e^{iqa} - 2 + e^{-iqa}\right) = 2K(1 - \cos(qa))$$

$$\rightarrow \quad \omega^2 = \frac{4K}{m}\sin^2\left(\frac{qa}{2}\right)$$

tvístursamband

eða

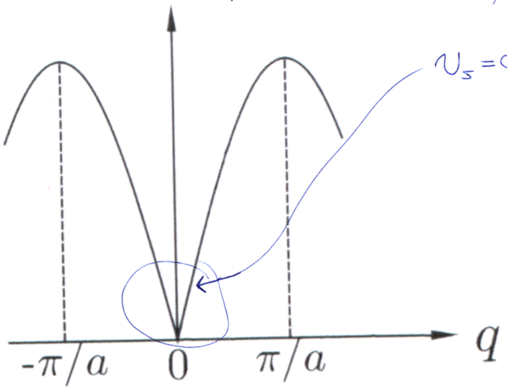
$$\omega = \sqrt{\frac{4K}{m}}\left|\sin\left(\frac{qa}{2}\right)\right|$$

langbylgjumálgun, $qa \to 0$

<u>tvísturróf</u>

$$\omega = \sqrt{\frac{4K}{m}}\,\left|\sin\left(\frac{qa}{2}\right)\right|$$

$$v_s = a\sqrt{\frac{K}{m}}$$



Langbylgjumálgunin fellur að líkani Debyes

3D - Tvísturróf

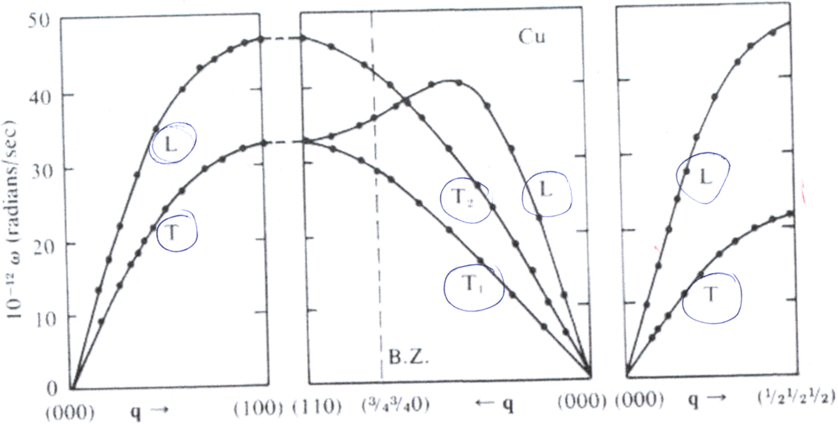
L: Langbylgjur

T: þverbylgjur

Efni	Θ_D (K)
Ne	63
Na	150
NaCl	321
Al	394
Si	625
C	1860

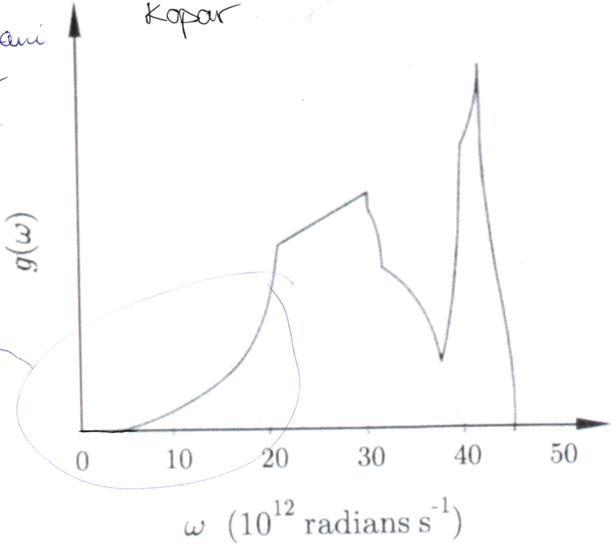
↑ demantur

Hljóðeindir í kopar



Orkulægstu sveifluhættirnir eru með línulegt tvístur

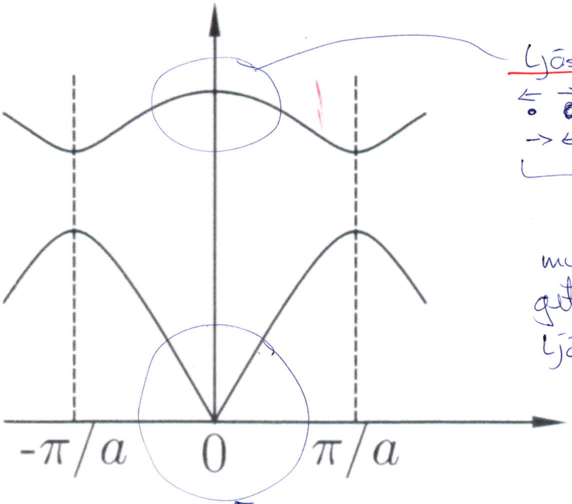
Samkvæmt líkani Debye fyrir lága orku



Ástandaþéttleikinn er hæstur þar sem tvístur rófið er með flata kafla

$$g(E) \sim \frac{1}{\left(\frac{\partial E}{\partial q}\right)}$$

Tvístur fyrir tvíatóma keðju



Ljósháttir

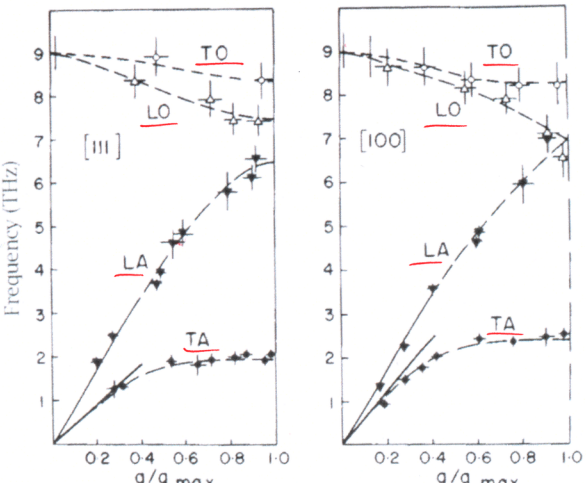
Jónir í andfasa

myndar raf tvístaut, sem geta víxlverkað við ljóseindir ⟷ Ljósháttir

hljóðháttir

nærri samfasa

Ge - kristallur



Ljóshaftirnir geta verið þver eða langsbylgjur

Hljóðeindir

Mjög mikil vægi t.a. skilja uppbyggingu og eiginleika efnis

Varmaleiðni

Geta leitt til ofurleiðni

Tengjast ljósvirkni....