Lághitanálgun fyrir Fermieindagas

Munum sjá að hún gildir fyrir $k_BT << E_F$
(t.d. í málmum)

Reiknum heildið

$$I = \int_0^\infty dE\, \phi(E)\, f(E)$$

sem veldisröð í k_BT.

Byrjum með fallið

$$\Psi(E) = \int_0^E dE'\, \phi(E')$$

$$\frac{d\Psi(E)}{dE} = \phi(E)$$

Notum nú að

$$I = \int_0^\infty dE\, \frac{d\Psi}{dE}\, f(E)$$

hlutheildun
stofnheildis verður miklu minni

$$= \Big[\Psi(E)\, f(E)\Big|_0^\infty\Big] - \int_0^\infty dE\, \Psi(E)\, \frac{df}{dE}$$

Setjum $x = \frac{E-\mu}{k_BT}$, þá fæst

$$\frac{df}{dE} = -\frac{1}{k_BT}\,\frac{e^x}{(e^x+1)^2}$$

Liðum $\Psi(E)$ sem

$$\Psi(E) = \sum_{s=0}^\infty \frac{x^s}{s!}\left(\frac{d^s\Psi}{dx^s}\right)_{x=0}$$

og setjum saman

$$I = \sum_{s=0}^{\infty} \frac{1}{s!}\left(\frac{d^s\psi}{dx^s}\right)_{x=0} \int_{-\mu/k_BT}^{\infty} \frac{dx\, x^s e^x}{(e^x+1)^2}$$

notum

$$\frac{1}{(1+z)^2} = \sum_{m=0}^{\infty} (-1)^m (m+1) z^m$$

$\frac{e^x}{(e^x+1)^2}$ er jafnstætt fall

Ef $k_BT << E_F$ þá $\longrightarrow -\infty$, og

$$\int_{-\infty}^{\infty} \frac{dx\, x^s e^x}{(e^x+1)^2} = 2\int_0^{\infty} \frac{dx\, x^s e^x}{(e^x+1)^2} =$$

fyrir jöfn s

$$2\int_0^{\infty} \frac{dx\, x^s e^{-x}}{(e^{-x}+1)^2} = 2\int_0^{\infty} dx\, x^s e^{-x}\left\{1 - 2(e^{-x}) + 3(e^{-x})^2 - 4(e^{-x})^3 + \cdots\right\}$$

$$= 2\int_0^{\infty} dx\, x^2 e^{-x} \sum_{m=0}^{\infty} (-1)^m (m+1) e^{-mx}$$

$$= 2\sum_{n=1}^{\infty} (-1)^{n+1} n \int_0^{\infty} dx\, x^s e^{-nx}$$

skiptum $n = m+1$

③

$$\rightarrow \quad 2\sum_{n=1}^{\infty}(-1)^{n+1}\int_0^{\infty} d(nx)\,(nx)^s e^{-nx}\,\frac{1}{n^s}$$

Riemann z-fallið

$$= 2(s!)\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^s} = 2(s!)(1-2^{1-s})\zeta(s)$$

fyrir jöfn s

því fæst

$$I = \sum_{s=0,2,4,\dots} 2\left(\frac{d^s\psi}{dx^s}\right)_{x=0}(1-2^{1-s})\zeta(s)$$

x = 0

$$= \psi(0) + \frac{\pi^2}{6}\,2\psi''(0) + \frac{7\pi^4}{360}\,2\psi''''(0) + \dots$$

$$= \int_0^{\mu} dE\,\phi(E) + \frac{\pi^2}{6}(k_BT)^2\left(\frac{d\phi}{dE}\right)_{E=\mu} + \frac{7\pi^4}{360}(k_BT)^4\left(\frac{d^3\phi}{dE^3}\right)_{E=\mu} + \dots$$

Sommerfeld jafnan

veljum $S=\frac{1}{2}$ og finnum

$$N = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty dE\sqrt{E}\, f(E)$$

þ.a. hér er $\phi(E)=\sqrt{E}$

$$\left(\phi'(E)\right)_{E=\mu} = \frac{1}{2}\frac{1}{\sqrt{\mu}}$$

$$\int_0^\mu dE\sqrt{E} = \frac{2}{3}\mu^{3/2}$$

$$N = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left\{\frac{2}{3}\mu^{3/2} + \frac{\pi^2}{6}(k_BT)^2\frac{1}{2}\frac{1}{\sqrt{\mu}} + \cdots\right\}$$

$$= \frac{V}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\mu^{3/2}\left\{1 + \frac{\pi^2}{8}\left(\frac{k_BT}{\mu}\right)^2 + \cdots\right\}$$

Túlkum sem

$$N(T) = \frac{V}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left(\mu(T)\right)^{3/2}\left\{1 + \frac{\pi^2}{8}\left(\frac{k_BT}{\mu(T)}\right)^2 + \cdots\right\}$$

og

$$N(0) = \frac{V}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left(\mu(0)\right)^{3/2}$$

$$\rightarrow \frac{N(T)}{N(0)} = 1 = \left(\frac{\mu(T)}{\mu(0)}\right)^{3/2}\left[1 + \frac{\pi^2}{8}\left(\frac{k_B T}{\mu(T)}\right)^2 + \cdots\right]$$

$$\rightarrow \mu(T) \simeq \mu(0)\left[1 + \frac{\pi^2}{8}\left(\frac{k_B T}{\mu(T)}\right)^2 + \cdots\right]^{-2/3}$$

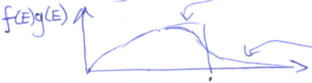
$$\simeq \mu(0)\left[1 - \frac{\pi^2}{12}\left(\frac{k_B T}{\mu(0)}\right)^2 + \cdots\right]$$

Gildir vel fyrir venjulega málma við herbergishita því $k_BT << \mu(0)$

fjöldi eindar ~~verð~~ veittur, t.d. rafeinda

$\mu(T) < \mu(0)$

vegna þess að $N(T) = N(0)$ og ástönd með $E > \mu$ bætast við og forni með $E < \mu$ dregast frá



Reikner $U(T)$ og $C_V(T)$ med valgen Sommerfeld

$$U(T) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty dE\, E^{3/2} f(E) \,, \qquad \int_0^\mu dE\, E^{3/2} = \frac{5}{2}\mu^{5/2}$$

$$= \frac{V}{5\pi^2}\left(\frac{2m}{\hbar^2}\right)\left(\mu(T)\right)^{5/2}\left\{1 + \frac{5\pi^2}{8}\left(\frac{k_BT}{\mu(T)}\right)^2 + \cdots\right\}$$

$$= \frac{3}{5}\,\overbrace{N(T)}^{N}\,\mu(T)\left\{1 + \frac{\pi^2}{2}\left(\frac{k_BT}{\mu(T)}\right)^2 + \cdots\right\}$$

$$\simeq \frac{3}{5}\,N\,\mu(0)\left\{1 + \frac{5\pi^2}{12}\left(\frac{k_BT}{\mu(0)}\right)^2 + \cdots\right\}$$

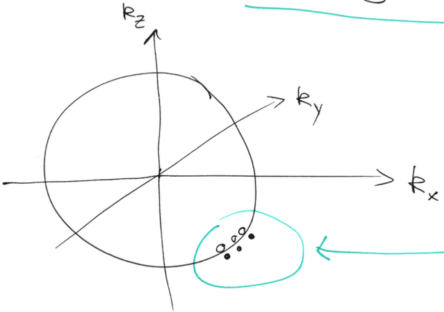
$$\rightarrow C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{3}{2}Nk_B\left(\frac{\pi^2 k_BT}{3\mu(0)}\right) + \cdots$$

lineært i T

Fermi yfirborð

Fermi yfirborðið í k-rúminu (nykurrúminu) er yfirborðið með orku jafna E_F eða $\mu(0)$

Í málmum eru einu ástöndin sem geta breytt setni sinni nærri þessu yfirborði því $k_BT \ll \mu(0)$



Rafeindakerfið er því mjög stíft í málmum og öll ferli tengd k_BT gerast aðeins um Fermi-yfirborðið

Leiðni, örvanir . . . . . .