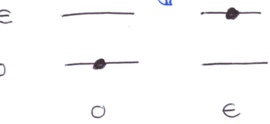
Aðgreinanleiki

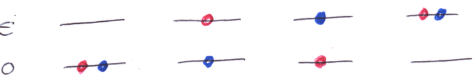
Byrjum með einfalt dæmi — tveir möguleikar

Eind í tvístiga kerfi



$$\rightarrow \quad Z_1 = e^{0} + e^{-\beta\epsilon} = 1 + e^{-\beta\epsilon}$$

Tvær eindir aðgreinanlegar — mögulegar umraðanir

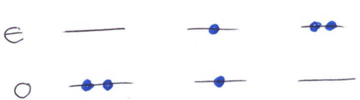


$$Z_2 = e^{0} + e^{-\beta\epsilon} + e^{-\beta\epsilon} + e^{-2\beta\epsilon}$$

$$\rightarrow \quad Z_2 = (Z_1)^2$$

og áður notuðum við $Z_N = (Z_1)^N$

En tvær óaðgreinanlegar eindir ?



$$\} \to Z_2 = e^{0} + e^{-\beta\epsilon} + e^{-2\beta\epsilon} \neq (Z_1)^2$$

Ef allar eindirnar væru í mismunandi ástöndum
þá værum við að of-telja um N!

$\to$ einföld nálgun $$Z_N = \frac{(Z_1)^N}{N!}$$

fyrir kjörgas verður þá fjöldi setjanlegra ástanda
við T að vera miklu hærri en fjöldi einda, eða

$$n \ll n_Q$$

þéttleiki einda ($\to n$)

Ef þetta heldur þá er

$$Z_N = \frac{1}{N!}\left(\frac{V}{\lambda_{th}^3}\right)^N$$

Sjáum á næstu síðu að nálgunin er í lagi fyrir N_2 við herbergishita og hærri, en ekki fyrir rafeindir í málmi

$$n \ll n_Q$$

Sjáum síðar að þessi gös eru mjög ólík

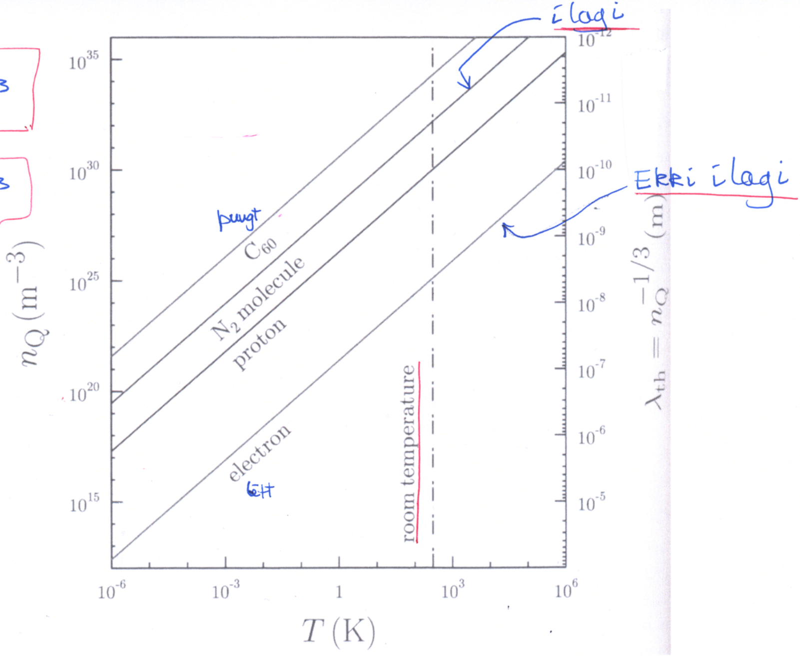
hlýgas : N_2

kulgas : e í málmum

$n_{N_2} \sim 10^{25}\ m^{-3}$

$n_e \sim 10^{29}\ m^{-3}$

↑ i malmi



Ástandsbreytur fyrir kjörgas

$$Z_N = \frac{1}{N!}\left(\frac{V}{\lambda_{th}^3}\right)^N \sim \left\{V T^{3/2}\right\}^N \quad \leftarrow \lambda_{th} \sim T^{-1/2}$$

$$\rightarrow \ln Z_N = N \ln V + \underbrace{\frac{3N}{2}\ln T}_{\frac{3N}{2}\ln\left(\frac{1}{\beta}\right)} + \text{fasti}$$

$$\rightarrow U = -\frac{d\ln Z_N}{d\beta} = \frac{3}{2} N k_B T$$

$$\rightarrow C_V = \frac{3}{2} N k_B \quad \leftarrow \text{eins og áður sást}$$

$$F = -k_B T \ln Z_N = -k_B T N \ln V - \frac{3N}{2} k_B T \ln T - k_B T \cdot \text{fasti}$$

$$\rightarrow p = -\left(\frac{\partial F}{\partial V}\right)_T = \frac{N k_B T}{V} = n k_B T \quad \Leftarrow \left\{\begin{array}{l}\text{ástandsjafna} \\ \text{kjörgass}\end{array}\right.$$

$$H = U + pV = \frac{5}{2} N k_B T$$

$$Z_N = \frac{1}{N!}\left(\frac{V}{\lambda_{th}^3}\right)^N \quad \text{og} \quad \ln N! \approx N\ln N - N$$

$$\rightarrow \ln Z_N \approx N\ln V - 3N\ln\lambda_{th} - N\ln N - N$$

$$= N\ln\left\{\frac{Ve}{N\lambda_{th}^3}\right\}$$

the number e

$\ln e = 1$

$$\rightarrow F = -Nk_BT\ln\left\{\frac{Ve}{N\lambda_{th}^3}\right\} = Nk_BT\left\{\ln(n\lambda_{th}^3) - 1\right\}$$

$$S = \frac{U-F}{T} = \frac{3}{2}Nk_B + Nk_B\ln\left\{\frac{Ve}{N\lambda_{th}^3}\right\}$$

$$= Nk_B\ln\left\{\frac{Ne^{5/2}}{N\lambda_{th}^3}\right\} = Nk_B\left\{\frac{5}{2} - \ln(n\lambda_{th}^3)\right\}$$

$$G = H - TS = \frac{5}{2} N k_B T - N k_B T \ln\left\{\frac{V e^{5/2}}{N \lambda_{th}^3}\right\}$$

$$= N k_B T \ln\left\{n \lambda_{th}^3\right\}$$