# Kjörgas

Gas óvixlverkandi atóma eða sameinda í sigilda skilningi

Höfðum leitt út meðal sætni í FD og BE -gasi kjörgasið

$\hookrightarrow f \ll 1$

$$f(\varepsilon) = \frac{1}{e^{\frac{(\varepsilon-\mu)}{\tau}} \pm 1}$$

$$e^{\frac{(\varepsilon-\mu)}{\tau}} \gg 1$$

$$\rightarrow f(\varepsilon) \approx \lambda e^{-\frac{\varepsilon}{\tau}}$$

## Efnamætti

Heildarfjölda einda

$$N = \langle N \rangle = \sum_s f(\varepsilon_s)$$

$$\approx \lambda \sum_s e^{-\frac{\varepsilon_s}{\tau}} = \lambda Z_1$$

$Z_1$: Kerfisumma fyrir eitt atóm

$$= \lambda n_Q V$$

p.s.

$$n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{3/2}$$

en $V = \frac{N}{n} \rightarrow \frac{N}{V} = n$

$\rightarrow \frac{N}{V} = \lambda n_Q$

$\rightarrow \lambda = \frac{N}{V n_Q} = \frac{n}{n_Q}$

$= e^{\frac{\mu}{\tau}}$

og þar

$$\mu = \tau \ln\left(\frac{n}{n_Q}\right)$$

fyrir sígilt kjörgas

munum líka að $\frac{n}{n_Q} \ll 1$

þetta má umrita sem

$$\mu = \tau\left\{\ln N - \ln V - \frac{3}{2}\ln\tau + \frac{3}{2}\ln\left(\frac{2\pi\hbar^2}{\mu}\right)\right\}$$

til að spá hvaða áhrif einstakar breytur hafa á $\mu$

Frjálsa orkan

$$\mu = \left(\frac{\partial F}{\partial N}\right)_{\tau,V}$$

$$\rightarrow F(N,\tau,V) = \int_0^N dN'\,\mu(N',\tau,V)$$

$$= \tau\int_0^N dN'\left\{\ln N' + \cdots\right.$$

$$= N\tau\left\{\ln N - 1 - \ln V - \frac{3}{2}\ln\tau + \frac{3}{2}\ln\left(\frac{2\pi\hbar^2}{\mu}\right)\right\}$$

eda

$$F = N\tau\left\{\ln\left(\frac{n}{n_Q}\right) - 1\right\}$$

þrýstingur

$$p = -\left(\frac{\partial F}{\partial V}\right)_{\tau N} = \frac{N\tau}{V}$$

(innsetning $n = \frac{N}{V}$ í $F$)

$$\rightarrow \boxed{pV = N\tau}$$

Orka

$$F \equiv U - \tau S$$

$$\rightarrow U = F + \tau S$$

$$U = F - \tau\left(\frac{\partial F}{\partial \tau}\right)_{VN}$$

$$= -\tau^2\left(\frac{\partial}{\partial \tau}\frac{F}{\tau}\right)_{VN}$$

$$= -\tau^2\left\{-\frac{3N}{2\tau}\right\}$$

$$\rightarrow \boxed{U = \frac{3}{2}N\tau}$$

Óreiða

$$S = -\left(\frac{\partial F}{\partial \tau}\right)_{VN}$$

$$\rightarrow \boxed{S = N\left\{\ln\left(\frac{n_Q}{n}\right) + \frac{5}{2}\right\}} > 0$$

## Varmarýmd

$$C_V \equiv \tau \left(\frac{\partial \eta}{\partial \tau}\right)_V$$

liður úr $\eta_2$

$$= \tau \frac{\partial}{\partial \tau} \left\{ \frac{3}{2} N \ln \tau + \cdots \right\}$$

$$= \frac{3N}{2\tau}$$

$$\rightarrow \boxed{C_V = \frac{3}{2} N}$$

Eins er of sést eftir

$$C_p = \tau \left(\frac{\partial \eta}{\partial \tau}\right)_p$$

notum

$$\tau d\eta = dU + pdV$$

tíf ~~að~~ fá

$$\tau \left(\frac{\partial \eta}{\partial \tau}\right)_p = \left(\frac{\partial U}{\partial \tau}\right)_p + p \left(\frac{\partial V}{\partial \tau}\right)_p$$

U eróðeins háð $\tau$

$$C_p = C_V + N$$

því $p\left(\frac{\partial V}{\partial \tau}\right)_p = N$ eftirað $V = \frac{N\tau}{p}$

fyrir kjörgasið

$$C_p = \frac{3}{2} N + N = \frac{5}{2} N$$

## Kjörgas með innri frelsisgráður

Orka sameinder

$$\Sigma = \Sigma_u + \Sigma_{int}$$

↑ innri frelsisgráður, snúningur, titringur, ....

Kjörgas, sigilt stikasvæði, ekkert svigrúm er nýjög setzt

$$\rightarrow \mathcal{Z} \approx 1 + \lambda e^{-\frac{\Sigma_u}{\tau}}$$

hvar liðum (með hærri sætti) sleppt

Bætum við innri hreyfingum

$$\mathcal{Z} = 1 + \lambda \sum_{int} e^{-\frac{(\Sigma_u + \Sigma_{int})}{\tau}}$$

$$= 1 + \lambda e^{-\frac{\Sigma_u}{\tau}} \sum_{int} e^{-\frac{\Sigma_{int}}{\tau}}$$

en umrum að

$$\sum_{int} e^{-\frac{\Sigma_{int}}{\tau}} = Z_{int}$$

Kerfisumma innri hreyfinga

$$\rightarrow \mathcal{Z} = 1 + \lambda Z_{int} e^{-\frac{\Sigma_u}{\tau}}$$

Setui $n$ ókað innri frelsisgr.

$$f(\varepsilon_u) = \frac{\lambda Z_{int} e^{-\frac{\varepsilon_u}{\tau}}}{1+\lambda Z_{int} e^{-\frac{\varepsilon_u}{\tau}}} \simeq \lambda Z_{int} e^{-\frac{\varepsilon_u}{\tau}}$$

því á sígilda stóra svæðinu er $f(\varepsilon_u) \ll 1$

fyrir samsætt fast því

$$\lambda = \frac{n}{(n_Q Z_{int})}$$

$$\mu = \tau \left\{ \ln\left(\frac{n}{n_Q}\right) - \ln Z_{int} \right\}$$

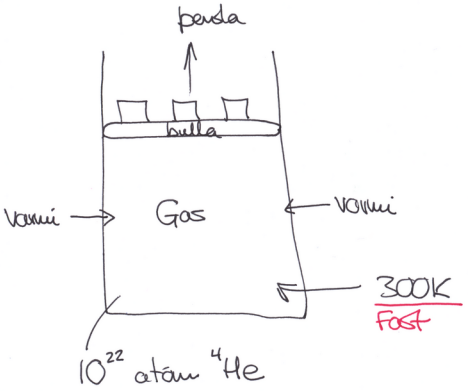
Við F bætist

$$F_{int} = -N\tau \ln Z_{int}$$

og við örsíðuna bætist

$$\tau_{int} = -\left(\frac{\partial F_{int}}{\partial \tau}\right)_N$$

# Jafngeng jafnkita þensla




**Breytum rúmmáli**

$10^3\ cm^3 \rightarrow 2 \cdot 10^3\ cm^3$

**Hver er þrýstingurinn eftir þenslu?**

$$p = \frac{N\tau}{V}$$

$$\rightarrow \boxed{p_f = \frac{1}{2} p_i}$$

**Breyting óreiðu**

$$\nabla = N\left\{\ln\left(\frac{n_Q}{n}\right) + \frac{5}{2}\right\} \qquad \frac{N}{V}$$

$$\rightarrow \nabla(V) = N \ln V + \text{fasti}$$

$$\rightarrow \nabla_f - \nabla_i = N \ln\left(\frac{V_f}{V_i}\right) = N \ln 2$$

Orkan vex því stærra V byður fleiri ástönd

## Jafngengt ferli

Ekki sama örkin!

En nógu hægt þ.a. fyrir hvert gildi á V sé p þekkt og jafnt fyrir allt kerfið

$$pV = N\tau$$

Hve mikla vinnu framkvæmir gasið

$$W = \int_{V_i}^{V_f} p\,dV = \int_{V_i}^{V_f} \frac{N\tau}{V}\,dV$$

$$= N\tau \ln\left(\frac{V_f}{V_i}\right) = N\tau \ln 2$$

Vinnan framkvæmd á gasinu með ytri kraft á belgnum er

$$W_p = -W$$

## Breyting orku U

$$U = \frac{3}{2} N\tau$$

Fasti hér, því $\tau$ er fasti

Frjálsa orkan minnkar um

$N\tau \ln 2$

Orkan í formi varma inn í gasið

$Q + W_p = 0$ ← vinna bullunnar á gasinu

Því V er fasti

$\rightarrow Q = -W_p = W$

Jafnvægt jafnvægisferli

Einangrað kerfi fær varma gegnum

$Q = 0$ adiabatic

Hvert er hitastigið eftir þensluna

$$T(\tau, V) = N\left\{\ln \tau^{3/2} + \ln V + \text{fasti}\right\}$$

$\Delta T = 0$

$\hookrightarrow \ln\left\{\tau^{3/2} V\right\} = \text{fasti}$

$\rightarrow \tau^{3/2} V =$ fast:

Eða

$$\tau_i^{3/2} V_i = \tau_f^{3/2} V_f$$

fyrir kjörgas

notum $V = \frac{N\tau}{P}$

$$\rightarrow \frac{\tau_i^{5/2}}{P_i} = \frac{\tau_f^{5/2}}{P_f}$$

Eða notum $\tau = \frac{PV}{N}$

$$\rightarrow P_i^{3/2} V_i^{5/2} = P_f^{3/2} V_f^{5/2}$$

Almennt fast

$$\tau_i V_i^{\gamma-1} = \tau_f V_f^{\gamma-1}$$

$$\tau_i^{\frac{\gamma}{1-\gamma}} P_i = \tau_f^{\frac{\gamma}{1-\gamma}} P_f$$

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

fyrir $\gamma \equiv \frac{C_P}{C_V}$

$T_i = 300\,K$ og $V_i/V_f = \frac{1}{2}$

$\rightarrow T_f = \left(\frac{1}{2}\right)^{2/3} (300\,K) = 189\,K$

gott hefur kólnað!

$U_f - U_i = \frac{3}{2} N(T_2 - T_1)$

$= -23\,J$

Vinnan framkvæmd af gasinu er 23 J sem er tekin úr gasinu

Engin varmaflutningur gasið kælnaði

↓

Kalíber

:

Snögg þensla út í tóm

$W = 0$, $Q = 0$

↑ valið hér

$\Delta U = 0$ sama hitastig

(kjörgas ⟷ raungas)

$\Delta T = T_f - T_i = N\ln 2$

ekki jafnvægisferli