10-1 van der Waals gas

a) Räkna σ [illegible]

$$F = -N\tau\left\{\ln\left[\frac{n_Q(V-Nb)}{N}\right]+1\right\} - \frac{N^2a}{V}$$

$$\sigma = -\left(\frac{\partial F}{\partial \tau}\right)_V$$

$$= N\left\{\ln\left[\frac{n_Q(V-Nb)}{N}\right]+1\right\} + N\tau\underbrace{\left(\frac{\partial \ln n_Q}{\partial \tau}\right)_V}$$

$$n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{3/2} \longrightarrow \qquad = \frac{3}{2}$$

$$\rightarrow \sigma = N\left\{\ln\left[\frac{n_Q(V-Nb)}{N}\right]+\frac{5}{2}\right\}$$

b) Finna U

$$F = U - \tau\sigma$$

$$\Rightarrow U = F + \tau\sigma = -N\tau\left\{\ln\left[\frac{n_Q(V-Nb)}{N}\right]+1\right\} + N\tau\left\{\ln\left[\frac{n_Q(V-Nb)}{N}\right]+\frac{5}{2}\right\} - \frac{N^2a}{V}$$

$$= -N\tau + \frac{5}{2}N\tau - \frac{N^2a}{V}$$

$$= \underline{\frac{3}{2}N\tau - \frac{N^2a}{V}} = \underline{\frac{3}{2}N\tau - Nan}$$

c) Reikna $H = U + pV$

ef $\frac{b}{V} \ll 1$

$$pV = \frac{N\tau V}{V-Nb} - \frac{N^2a}{V} = \frac{N\tau}{\left(1-\frac{Nb}{V}\right)} - \frac{N^2a}{V} \simeq N\tau\left(1+\frac{Nb}{V}\right) - \frac{N^2a}{V}$$

$$\to H \simeq \frac{3}{2}N\tau - \frac{N^2a}{V} + N\tau\left(1+\frac{Nb}{V}\right) - \frac{N^2a}{V}$$

$$= \frac{5}{2}N\tau + \frac{N^2\tau b}{V} - \frac{2N^2a}{V}$$

þurfum $H(N,\tau,p)$ en ekki $H(N,\tau,V)$

Reynum 1. stigs liðum í a og b á $\frac{1}{V}$

$$pV \simeq N\tau\left(1+\frac{Nb}{V}\right) - \frac{N^2a}{V}$$

þegar komið hér

$$\to pV \simeq N\tau$$

$$\to \frac{N}{V} \simeq \frac{p}{\tau}$$

$$H \simeq \frac{5}{2}N\tau + N\tau b\frac{p}{\tau} - 2Na\frac{p}{\tau} = \frac{5}{2}N\tau + Nbp - \frac{2Nap}{\tau}$$

Gas-fastefni

leyfa 3D-hreyfingu

Notum sömu táknum og í bók
hreintóna sveifill með róf

$$(n_x + n_y + n_z)\hbar\omega - \varepsilon_0$$

$$Z_s = \sum_{n_x n_y n_z} \exp\left\{-\frac{(n_x+n_y+n_z)\hbar\omega - \varepsilon_0}{\tau}\right\} = e^{\frac{\varepsilon_0}{\tau}}\left\{\sum_n e^{-\frac{n\hbar\omega}{\tau}}\right\}^3$$

$$= e^{\frac{\varepsilon_a}{\tau}}\left\{\frac{1}{1-e^{-\frac{\hbar\omega}{\tau}}}\right\}^3$$

a) finna $p(\tau)$, $\quad F_s = U_s - \tau\sigma_s = -\tau \ln Z_s$

$$G_s = F_s + pV_s = \mu_s$$

slepp pV

$$\lambda_s = e^{\frac{\mu_s}{\tau}} \simeq e^{\frac{F_s}{\tau}} = e^{-\ln Z_s}$$

$$= \frac{1}{Z_s} = e^{-\frac{\varepsilon_a}{\tau}} \left\{1 - e^{-\frac{\hbar\omega}{\tau}}\right\}^3$$

p. $\tau \gg \hbar\omega$

$$\lambda_s \simeq e^{-\frac{\varepsilon_a}{\tau}} \left\{1 - 1 + \left(\frac{\hbar\omega}{\tau}\right) + \cdots\right\}^3 \simeq e^{-\frac{\varepsilon_a}{\tau}} \left(\frac{\hbar\omega}{\tau}\right)^3$$

Áður var komið

$$\lambda_g = \frac{n}{n_Q} = \frac{p}{\tau}\left(\frac{2\pi\hbar^2}{M\tau}\right)^{3/2} \quad \text{fyrir kjörgas}$$

$\bar{I}$ japuvogi

$$\lambda_s = \lambda_g$$

$$\rightarrow \quad e^{-\frac{\varepsilon_0}{\tau}} \left(\frac{\hbar\omega}{\tau}\right)^3 = \frac{p}{\tau}\left(\frac{2\pi\hbar^2}{M\tau}\right)^{3/2}$$

$$\rightarrow \quad p \simeq \left(\frac{M}{2\pi}\right)^{3/2} \frac{\omega^3}{\sqrt{\tau}} e^{-\frac{\varepsilon_0}{\tau}}$$

b) ~~Broo~~ Shuvarmi

$$\ln p = -\frac{L_0}{\tau} + \text{fafi} \qquad \text{Clausius-Clapeyron}$$

$$\frac{d}{d\tau}\ln p = \frac{L_0}{\tau^2} \quad (*)$$

$$p \simeq \left(\frac{M}{2\pi}\right)^{3/2} \frac{\omega^3}{\sqrt{\tau}}\, e^{-\frac{\varepsilon_0}{\tau}}$$

$$\to \frac{d}{d\tau} \ln p \simeq \frac{\varepsilon_0}{\tau^2} - \frac{1}{2}\tau^{-1} = \frac{1}{\tau^2}\left(\varepsilon_0 - \frac{\tau}{2}\right)$$

samanburður við (*) gefur

$$L_0 \simeq \left(\varepsilon_0 - \frac{\tau}{2}\right)$$

heppilegra væri að taka orku sveiflunnar sem

$$\left\{(n_x + n_y + n_z) + \underbrace{\frac{3}{2}}_{\text{núllpunktsorka}}\right\}\hbar\omega - \varepsilon_0$$

Så fungist

$$\lambda_s \simeq e^{-\frac{\Sigma_0}{\tau}} \left\{ 1 - 1 + \left(\frac{\hbar\omega}{\tau}\right) \overbrace{- \frac{1}{2}\left(\frac{\hbar\omega}{\tau}\right)^2}^{\text{lidt længere}} + \cdots \right\}^3$$

$$= e^{-\frac{\Sigma_0}{\tau}} \left(\frac{\hbar\omega}{\tau}\right)^3 \left\{ 1 - \frac{1}{2}\left(\frac{\hbar\omega}{\tau}\right) \right\}^3$$

$$\simeq e^{-\frac{\Sigma_0}{\tau}} \left(\frac{\hbar\omega}{\tau}\right)^3 \left\{ 1 - \frac{3}{2}\left(\frac{\hbar\omega}{\tau}\right) \right\} \simeq \left(\frac{\hbar\omega}{\tau}\right)^3 e^{-\frac{\Sigma_0}{\tau}} e^{-\frac{3\hbar\omega}{2\tau}}$$

~~Sådan~~

$$\lambda_s \simeq \exp\left\{ - \frac{\left(\Sigma_0 - \frac{3\hbar\omega}{2}\right)}{\tau} \right\} \left(\frac{\hbar\omega}{\tau}\right) + \cdots$$

Så vai o-punkterne bedre setter

11-3

B er íbót í A með $x \ll 1$

Frjálsaorkan án blöndunar

$$f_0(x) = f_0(0) + x f_0'(0) \quad \text{f. vökva og fasta}$$

Gerum ráð fyrir að vökva blandan sé í jafnvægi við fasta blönduna

Reikna $k = \dfrac{x_S}{x_L}$

blöndunar óreiða

$$\sigma_M = -N\left\{\underbrace{(1-x)}_{\approx 1}\underbrace{\ln(1-x)}_{=(-x-\frac{x^2}{2}\cdots)} + x\ln x\right\} \simeq -N\{-x + x\ln x\}$$

$$= +N\{x(1-\ln x)\}$$

þ.v. er heildar frjálsa orkan á atóm

$$f(x) = f_o(x) + \frac{\tau \nabla_M}{N} = f_o(0) + x\left\{ f_o'(0) + \tau(1-\ln x) \right\}$$

$$\frac{d f(x)}{dx} = f_o'(0) + \tau(1-\ln x) - \tau = f_o'(0) - \tau \ln x$$

Gasið og fasta efnið eru í jafnvægi

þegar $f_L'(x_L) = f_S'(x_S)$

$$f_{Lo}'(0) - \tau \ln x_L = f_{so}'(0) - \tau \ln x_s$$

$$\rightarrow f_{Lo}'(0) - f_{so}'(0) = \tau \ln\left(\frac{x_s}{x_L}\right) = \tau \ln k$$

þ.s. $k = \frac{x_s}{x_L}$ og $\rightarrow$ $k = \exp\left\{ - \frac{(f_{so}'(0) - f_{Lo}'(0))}{\tau} \right\}$

fyrir $f'_{0S}(0) - f'_{0L}(0) = 1\,eV$, $T = 1000\,K$

$$1\,eV \simeq 1.16 \cdot 10^4\,K$$

$$\rightarrow k = \exp\left\{-\frac{1.16 \cdot 10^4\,K}{1000\,K}\right\} \simeq 9.2 \cdot 10^{-6}$$

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bera saman við mynd 11.5 í bók