24-03

Debye í d-vídd, sýna að $C \sim T^d$

Í 3D $\quad g^{3D}(q)\,dq = \dfrac{3Vq^2\,dq}{2\pi^2} \rightarrow g^d(q) \sim q^{d-1}$

nú gildir að $\quad \omega = v_s q \rightarrow g^d(\omega) \sim \omega^{d-1}$

því fæst að

$$U = \text{fasti } 1 + \text{fasti } 2 \cdot \underbrace{\int_0^{\omega_D} \frac{d\omega\,\omega^d}{e^{\hbar\omega\beta}-1}}$$

$$\frac{1}{\beta^{d+1}} \int_0^{\omega_D\beta} \frac{d(\omega\beta)\,(\omega\beta)^d}{e^{\hbar\omega\beta}-1} = \frac{1}{\beta^{d+1}} \int_0^{x_D} \frac{dx\cdot x^d}{e^x-1}$$

$$= (k_B T)^{d+1} \cdot \text{fasti} \rightarrow C = \frac{\partial U}{\partial T} \sim T^d$$

24-04 Finna ástand þéttleika einvídd línul. ~~keð~~ keðju

$$\left\{ g(\omega) = \frac{2L}{\pi a} \frac{1}{\sqrt{\frac{4K}{m} - \omega^2}} \right\}$$

samkvæmt (24.33)

$$\omega = \sqrt{\frac{4K}{m}} \left| \sin\left(\frac{qa}{2}\right) \right|$$

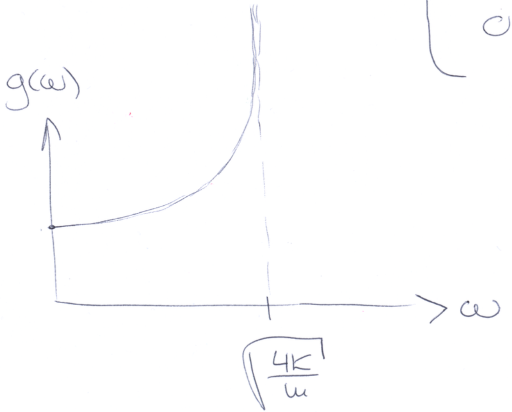
fyrir 1D þá er $g(q)dq = \dfrac{2dq}{\left(\frac{2\pi}{L}\right)} = \dfrac{L\,dq}{\pi}$

$$\frac{d\omega}{dq} = \sqrt{\frac{4K}{m}} \; \frac{a}{2} \cos\left(\frac{qa}{2}\right) \qquad (\text{ef } q \neq 0)$$

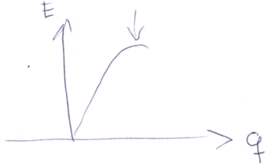
$$\cos\left(\frac{qa}{2}\right) = \sqrt{1 - \sin^2\left(\frac{qa}{2}\right)} = \sqrt{1 - \frac{m\omega^2}{4K}}$$

$$\rightarrow \frac{d\omega}{dq} = \frac{a}{2}\sqrt{\frac{4K}{m} - \omega^2}$$

$$g(\omega) = g(q)\frac{dq}{d\omega} = \begin{cases} \dfrac{2L}{\pi a}\left\{\dfrac{4K}{m} - \omega^2\right\}^{-1/2} & \text{ef } \omega \leq \sqrt{\dfrac{4K}{m}} \\ 0 & \text{ef } \omega > \sqrt{\dfrac{4K}{m}} \end{cases}$$



← ástæða sérstöðupunktsins sést best í tvístrunarrófinu



Varmageislun er hægt að lýsa sem ljóseindagasi með [illegible]

$$\begin{cases} U = u(T)V \\ p = \dfrac{u(T)}{3} \end{cases}$$

1. Lögmálið

$$dU = TdS - pdV$$

Sýna að

(a) óreiðuþéttnin sé

$$s = \frac{4p}{T}$$

$$u = \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p = T\left(\frac{\partial S}{\partial V}\right)_T - \frac{u}{3}$$

$$\rightarrow \frac{4}{3}u = T\left(\frac{\partial S}{\partial V}\right)_T \rightarrow \left(\frac{\partial S}{\partial V}\right)_T = \frac{4u}{3T} = \frac{4p}{T}$$

ef $s = \left(\frac{\partial S}{\partial V}\right)_T$ fæst

$$s = \frac{4p}{T}$$

b) $G = 0$

$$G = U + pV - TS = uV + \frac{u}{3}V - \frac{4u}{3}V = 0$$

c) $C_V = 3s$ á rúmmál

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V \qquad (16.65)$$

$$S = \frac{4u}{3T}V$$

Maxwell

$p = u(T)/3$

og líka

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = V\left(\frac{\partial u}{\partial T}\right)_V = 3V\left(\frac{\partial p}{\partial T}\right)_V$$

$$= 3V\left(\frac{\partial S}{\partial V}\right)_T = 3Vs$$

(d) $C_p \to \infty$

$P = \frac{u}{3}$

$$H = U + pV = \frac{4uV}{3}$$

$$C_p = \left(\frac{\partial H}{\partial T}\right)_p$$

pui i rum fast u

$$\left(\frac{\partial H}{\partial T}\right)_p = \frac{4u}{3}\left(\frac{\partial V}{\partial T}\right)_p = \frac{-\left(\frac{\partial p}{\partial T}\right)_V}{\left(\frac{\partial p}{\partial V}\right)_T}$$

og $\left(\frac{\partial p}{\partial V}\right)_T = \frac{1}{3}\left(\frac{\partial u}{\partial V}\right)_T = 0$

$\to C_p \to \infty$

p er alene hvd u(T) $\to$ alene hvd. T
$\to$ fast p er fast T $\leftarrow$ så er ikke hogt at ænka T

23-06 Kórsumma ljóseindagass

sleppum núllpunktsorku

Notfærum okkur (24.19) fyrir ljóseindir

$$\rightarrow \ln Z = \int_0^\infty d\omega\, g(\omega) \ln\left\{\frac{1}{1-e^{-\hbar\omega\beta}}\right\}$$

tvær skautunar

áttir

$$\omega = cq \quad \rightarrow \quad g(p)\,dq = \frac{4\pi q^2\, dq}{\left(\frac{2\pi}{L}\right)^3}\cdot 2$$

$$\rightarrow g(\omega)\,d\omega = \frac{V\omega^2\, d\omega}{\pi^2 c^3}$$

$$\rightarrow \ln Z = -\frac{V}{\pi^2 c^3}\int_0^\infty d\omega\, \omega^2 \ln\left\{1-e^{-\beta\hbar\omega}\right\}$$

$$\rightarrow \ln Z = \frac{V\pi^2}{45\hbar^3\beta^3 c^3} = \frac{V\pi^2 (k_B T)^3}{45\hbar^3 c^3}$$

$$F = -k_B T \ln Z = -\frac{V\pi^2 (k_B T)^4}{45\hbar^3 c^3} = -\frac{4\sigma V T^4}{3c}$$

pui $\sigma = \frac{\pi^2 k_B^4}{60 c^2 \hbar^3}$

$$S = -\left(\frac{\partial F}{\partial T}\right)_V = \frac{16\sigma V T^3}{3c}$$

$$U = F + TS = -\frac{4\sigma V T^4}{3c} + \frac{16\sigma V T^4}{3c} = \frac{4\sigma V T^4}{c}$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_T = \frac{4\sigma T^4}{3c}$$

$\rightarrow U = -3F$

$pV = \frac{U}{3}$

$S = \frac{4U}{3T}$

23-7

$$N = \int_0^\infty \frac{d\omega\, g(\omega)}{e^{\beta\hbar\omega} - 1}, \quad g(\omega) = \frac{V\omega^2 d\omega}{\pi^2 c^3}$$

$$N = \frac{V}{\pi^2 c^3} \int_0^\infty \frac{d\omega\, \omega^2}{e^{\beta\hbar\omega} - 1} = \frac{V}{\pi^2 c^3} \frac{1}{(\beta\hbar)^3} \int_0^\infty \frac{dx\, x^2}{e^x - 1}$$

$$= \frac{V}{\pi^2 c^3} \frac{1}{(\beta\hbar)^3} \cdot \zeta(3)\Gamma(3) = \frac{V}{\pi^2 c^3} \frac{1}{(\beta\hbar)^3} 2\zeta(3)$$

og $\zeta(3) = 1{,}20206$

$$U = \frac{4\sigma V T^4}{c}, \quad \sigma = \frac{\pi^2 k_B^4}{60 c^2 \hbar^3}$$

$$N = \frac{V}{\pi^2 c^3} \frac{(k_B T)^3}{\hbar^3} 2\zeta(3) \rightsquigarrow \frac{U}{N} = \frac{4\sigma V T^4 \cdot \pi^2 c^3 \hbar^3}{c \cdot V (k_B T)^3 2\zeta(3)}$$

$$\rightarrow \frac{U}{N} = \frac{4\pi^2 k_B^4}{60c^2\hbar^3}\frac{VT^4}{c}\frac{\pi^2 c^3\hbar^3}{V(k_BT)^3 2\zeta(3)} = \frac{\pi^4}{30\zeta(3)}(k_BT)$$

$$\simeq \underbrace{2.7012\, k_BT}$$

$$\frac{S}{N} = \frac{16\sigma VT^3}{3cN}$$

$$= \frac{16VT^3}{3c}\left(\frac{\pi^2 k_B^4}{60c^2\hbar^3}\right)\frac{\pi^2 c^3\hbar^3}{(k_BT)^3\, 2\zeta(3)} = \frac{2\pi^4}{45\zeta(3)}k_B$$

$$\simeq 3.602\, k_B$$

fyrir kjörgas er $\frac{U}{N} = \frac{3}{2}k_BT$

og $\frac{S}{N} = k_B\left\{\frac{5}{2} - \ln(n\lambda_{th}^3)\right\}$