7-1 Sýna Maxwell tengslin

① $\left(\frac{\partial V}{\partial \tau}\right)_{pN} = -\left(\frac{\partial \sigma}{\partial p}\right)_{\tau N}$

② $\left(\frac{\partial V}{\partial N}\right)_{p\tau} = +\left(\frac{\partial \mu}{\partial p}\right)_{N\tau}$

③ $\left(\frac{\partial \mu}{\partial \tau}\right)_{Np} = -\left(\frac{\partial \sigma}{\partial N}\right)_{\tau p}$

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① Notum $V = \left(\frac{\partial G}{\partial p}\right)_{N\tau}$

$\rightarrow \left(\frac{\partial V}{\partial \tau}\right)_{pN} = \left(\frac{\partial}{\partial \tau}\frac{\partial G}{\partial p}\right)_{N} = \left(\frac{\partial}{\partial p}\frac{\partial G}{\partial \tau}\right)_{N}$

$= -\left(\frac{\partial \sigma}{\partial p}\right)_{\tau N}$ því $\left(\frac{\partial G}{\partial \tau}\right)_{Np} = -\sigma$

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②

$\left(\frac{\partial V}{\partial N}\right)_{p\tau} = \left(\frac{\partial}{\partial N}\frac{\partial G}{\partial p}\right)_{\tau} = \left(\frac{\partial}{\partial p}\frac{\partial G}{\partial N}\right)_{\tau}$

$= \left(\frac{\partial \mu}{\partial p}\right)_{N\tau}$ því $\mu = \left(\frac{\partial G}{\partial N}\right)_{\tau p}$

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③

$\left(\frac{\partial \mu}{\partial \tau}\right)_{Np} = \left(\frac{\partial}{\partial \tau}\frac{\partial G}{\partial N}\right)_{p} = \left(\frac{\partial}{\partial N}\frac{\partial G}{\partial \tau}\right)_{p}$

$= -\left(\frac{\partial \sigma}{\partial N}\right)_{\tau p}$ því $-\sigma = \left(\frac{\partial G}{\partial \tau}\right)_{Np}$

b) Sýna með 1 og 3. lögmálinu að

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial \tau}\right)_p \xrightarrow[\tau \to 0]{} 0$$

Margfeldni grunnástands g er venjulega á fasti ~ 1.....

óháður p

$$\rightarrow \left(\frac{\partial \sigma}{\partial p}\right)_{\tau N} = 0$$

því $\sigma = \ln g$

$$\rightarrow \left(\frac{\partial V}{\partial \tau}\right)_{pN} \rightarrow 0$$

og þar með

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial \tau}\right)_p \rightarrow 0$$

þegar $\tau \rightarrow 0$

9-2 ferlið

$$e + H^+ \longleftrightarrow H$$

Vitum $\prod_j n_j^{\nu_j} = K(\tau)$ (*)

og $K(\tau) = \prod_j n_{Qj}^{\nu_j} \exp\left\{-\nu_j F_j^{int} \frac{1}{\tau}\right\}$ (**)

$$e + H^+ - H = 0$$

$\rightarrow \nu(e) = 1$

$\nu(H^+) = 1$

$\nu(H) = -1$

Sleppum spuna

$\hookrightarrow F_e^{int} = 0$, E_I : jónunarorka H

$F_{H^+}^{int} = 0$ $\hookrightarrow F_H^{int} = -E_I$

(*) $\longrightarrow$

$$n_e \cdot n_{H^+} \cdot n_H^{-1} = K(\tau)$$

$\sim 1 \qquad m_{H^+} \sim m_H$

(**)

$$K(\tau) = (n_Q)_e \cdot (n_Q)_{H^+} \cdot (n_Q)_H^{-1} \exp\left\{-\left[-\left(-\frac{F_H^{int}}{\tau}\right)\right]\right\}$$

Seda

$$n_e \cdot n_{H^+} \cdot n_H^{-1} \simeq (n_Q)_e \exp\left\{-\frac{E_I}{\tau}\right\}$$

Seda

$$\frac{[e][H^+]}{[H]} \simeq (n_Q)_e \exp\left\{-\frac{E_I}{\tau}\right\}$$

Ef allar rafeindir og róteindir koma frá jónun H

$\rightarrow [e] = [H^+]$ og kvaðratrót gefur

$$[e] = [H]^{1/2}\sqrt{(n_Q)^e}\,\exp\left\{-\frac{E_I}{2\tau}\right\}$$

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b) $[H_{exc}]$ er þéttleiki H-atóma í fyrsta örvaða ástandinu

$E_{exc} = \frac{3}{4}E_I$ og ástandið er fjórfallt

$$\frac{P(H_{exc})}{P(H)} = \frac{4\cdot\exp\left(-\frac{E_{exc}}{\tau}\right)}{\exp\left(-\frac{E_I}{\tau}\right)} = \frac{4\cdot\exp\left(-\frac{3}{4\tau}E_I\right)}{\exp\left(-\frac{E_I}{\tau}\right)} = 4\exp\left(\frac{E_I}{4\tau}\right)$$

9-3 Si + rafgjafar $\epsilon = 11.7$, $m^* = 0.3\, m_e$

Ef $n_d = 10^{17}\ \mathrm{cm}^{-3}$ finna n_e, $T = 100\mathrm{K}$

Notum úrdómiun á undan

$$[e][n_d^+][n_d^0]^{-1} = n_Q(m^*) \exp\left(-\frac{E_I}{\tau}\right)$$

$$E_I = \frac{e^4 m^*}{2\hbar^2 \epsilon^2} = R_y^* = \frac{0.3}{(11.7)^2} R_y = 0.0022\, R_y$$

$$= 0.03\ \mathrm{eV} = 30\ \mathrm{meV}$$

$$n_Q(m^*) \simeq 4 \cdot 10^{17}\ \mathrm{cm}^{-3}$$

$$\rightarrow n_Q(m^*) \exp\left(-\frac{E_I}{\tau}\right) = 4 \cdot 10^{17}\ \mathrm{cm}^{-3} \exp\left\{-\frac{30\ \mathrm{meV} \cdot \mathrm{K}}{8.62 \cdot 10^{-2}\ \mathrm{meV}\ 100\mathrm{K}}\right\}$$

$$\simeq 1.2 \cdot 10^{16}\ \mathrm{cm}^{-3}$$

Hér er því ekki víst að það sé góð nálgun

að setja $[n_d^+][n_d^0]^{-1} \sim 1$ eða $[n_d^0] \sim$

Betra væri

$$[n_d^0] = [n_d] - [n_d^+] = [n_d] - [e]$$

$$\rightarrow \quad \frac{[e][e]}{[n_d]-[e]} = K \quad , \quad [e]^2 - K\{[n_d]-[e]\} = 0$$

eða

$$[e]^2 + K[e] - K[n_d] = 0$$

leysa sem annars stigs jöfnu

$$[e] = -\frac{K}{2} + \sqrt{\left(\frac{K}{2}\right)^2 + K[n_d]}$$

jákvæða rótin

$$\approx 2{,}9 \cdot 10^{16}\ \mathrm{cm}^{-3}$$