29-03

1

$$f(E) = \frac{1}{e^{\beta(E-\mu)} + 1} = \frac{e^{-\frac{1}{2}\beta(E-\mu)}}{e^{\frac{1}{2}\beta(E-\mu)} + e^{-\frac{1}{2}\beta(E-\mu)}}$$

$$= \frac{1}{2}\left\{ \frac{e^{\frac{\beta}{2}(E-\mu)} + e^{-\frac{\beta}{2}(E-\mu)}}{e^{\frac{\beta}{2}(E-\mu)} + e^{-\frac{\beta}{2}(E-\mu)}} - \frac{e^{\frac{\beta}{2}(E-\mu)} - e^{-\frac{\beta}{2}(E-\mu)}}{e^{\frac{\beta}{2}(E-\mu)} + e^{-\frac{\beta}{2}(E-\mu)}} \right\}$$

$$= \frac{1}{2}\left\{ 1 - \tanh\left(\frac{\beta}{2}(E-\mu)\right) \right\}$$

$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$, $\cosh(x)$ er jafnstøtt um $x=0$

$\sinh(x)$ er oddstøtt um $x=0$

$\rightarrow$ $\tanh(x)$ er oddstøtt um $x=0$

~~Sda~~ betur sagt $\tanh(x)$ er andsamhvørft um $x=0$

$\rightarrow$ $f(E)$ er andsamhverft um $E=\mu$

$E \ll \mu \;\rightarrow\; f(E) \rightarrow 1$

$E \gg \mu \;\rightarrow\; f(E) \rightarrow 0$

$\cosh x \underset{x\to 0}{=} 1$

$\sinh x \underset{x\to 0}{\simeq} x$

Þegar $E \sim \mu \;\rightarrow\; \tanh\left(\frac{\beta}{2}(E-\mu)\right) \simeq \frac{1}{2}\beta(E-\mu)$

$$\rightarrow f(E) \simeq \frac{1}{2}\left\{1 - \frac{\beta}{2}(E-\mu)\right\}$$ þ. $E \sim \beta$

30-02

Sýna að þrýstingur p í fermigasi

við $T=0$ sé $p = \frac{2}{5} n E_F$

Munum eftir (30.30) $p = \frac{2U}{3V}$

og (30.31)

$$\langle E \rangle = \frac{\int_0^{E_F} dE\, E\, g(E)}{\int_0^{E_F} dE\, g(E)} = \frac{\int_0^{E_F} dE\, E\sqrt{E}}{\int_0^{E_F} dE\, \sqrt{E}}$$

$$= \frac{2 E_F^{5/2}/5}{2 E_F^{3/2}/3} = \frac{3}{5} E_F$$

$$p = \frac{2U}{3V} = \frac{2}{3}\frac{3}{5} E_F \cdot \frac{N}{V} = \underline{\frac{2}{5} n E_F}$$

30-03

Sýna að fyrir Fermigas með $g(E)$ fæst

$$\mu(T) = E_F - \frac{\pi^2}{6}(k_B T)^2 \frac{g'(E_F)}{g(E_F)} + \cdots$$

(30.38)

$$\hookrightarrow \quad N = \int_0^{\infty} dE\, g(E) f(E) = \int_0^{\mu} dE\, g(E) + \frac{\pi^2}{6}(k_B T)^2 \left(\frac{dg}{dE}\right)_{E=\mu} + \cdots$$

1. Stigs liðun gefur

$$\int_0^{\mu} dE\, g(E) = \int_0^{E_F} dE\, g(E) + (\mu - E_F)\, g(E_F)$$

Þegar $T=0$ gildir $\mu = E_F$, eða $\mu(0) = E_F$

$$\rightarrow N = \int_0^{E_F} dE\, g(E),$$ N er fasti óháður T

$$\rightarrow \quad (\mu - E_F)\, g(E_F) + \frac{\pi^2}{6}(k_B T)^2 g'(E_F) = 0$$

$$\rightarrow \quad \mu = E_F - \frac{\pi^2}{6}(k_B T)^2 \frac{g'(E_F)}{g(E_F)}$$

30-05 Sýna að B-E þétting sé ekki til í 2D

þurfum fyrst að finna ástandaþéttleikann í 2D

$$g^{2D}(k) = \frac{2\pi\, k\, dk\, (2S+1)}{\left(\frac{2\pi}{L}\right)^2} = (2S+1)\frac{A\, k\, dk}{2\pi}, \quad \text{því } A = L^2$$

$$E = \frac{\hbar^2 k^2}{2m} \rightarrow dE = \frac{\hbar^2 k\, dk}{m} \longrightarrow g(E)\, dE = (2S+1)\frac{A\, m}{2\pi \hbar^2}\, dE$$

því þarf ekki að hafa áhyggjur af að E=0 sé ekki með í heildi.....

ástandaþéttleikinn í 2D er fasti

$$N = \int_0^\infty \frac{dE\, g(E)}{e^{\beta(E-\mu)}-1} = \frac{(2S+1)Am}{2\pi\hbar^2} \underbrace{\int_0^\infty \frac{dE}{z^{-1}e^{\beta E}-1}}_{k_B T\, \Gamma(1)\, Li_1(z)}, \quad z = e^{\beta\mu}$$

$$\rightarrow N = \frac{2S+1}{2\pi}\frac{Am}{\hbar^2}(k_B T)\, Li_1(z)$$

höfum líka

$$\lambda_{th}^2 = \frac{h^2}{2\pi m k_B T} = \frac{2\pi\hbar^2}{m k_B T}$$

og því

$$N = \frac{(2S+1)A}{\lambda_{th}^2} Li_1(z), \qquad Li_1(z) \underset{z\to 1}{\longrightarrow} -\ln(1-z) \to \infty$$

$$n^{2D} = \frac{(2S+1)}{\lambda_{th}^2} Li_1(z)$$

því finnst ekker ósamræmi milli hægri og vinstri hliðar, báðar geta stefnt á $+\infty$, án þess að setja þarf til $E=0$ ástandið sérstaklega á stórsæjan hátt

30-06 (7)

Í 3D, er sama fyrsta örvaða ástandið líka stórsæ þegar eindir þéttast í lægsta ástandið?

Þurfum strjál ástönd, setjum kerfið í kassa

$$E(n_x, n_y, n_z) = \frac{\hbar^2 \pi^2}{2mL^2}\left(n_x^2 + n_y^2 + n_z^2\right), \quad n_i = 1, 2, 3, \ldots$$

Grunnástandið er $E(1,1,1)$, en fyrsta örvaða ástandið er

3-falt $\quad E(2,1,1)$, $E(1,2,1)$ og $E(1,1,2)$

$$\Delta E = E(2,1,1) - E(1,1,1) = \frac{\hbar^2\pi^2}{2mL^2}(6-3) = \frac{3\hbar^2\pi^2}{2mL^2}$$

Ef $z=1$ (30.45) $\quad N = \left(\frac{L}{\lambda_{th}}\right)^3 \cdot 2.6 \quad \rightarrow \quad \Delta E \simeq \frac{3\hbar^2\pi^2}{2m\lambda_{th}^2} N^{-2/3} (2.6)^{-2/3}$

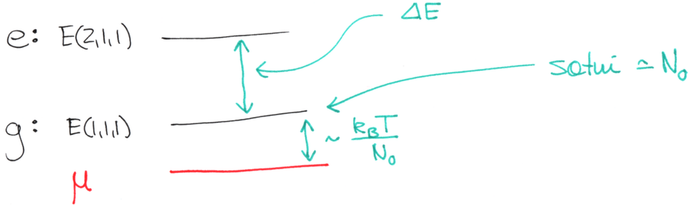
einföldum

$$\Delta E \simeq \frac{\hbar^2}{m \lambda_{th}^2} N^{-2/3}$$

$$N_0(T) \simeq \frac{1}{e^{-\mu\beta}-1} \simeq \frac{1}{1-1-\mu\beta} = -\frac{k_B T}{\mu}$$

$$\rightarrow \mu \simeq -\frac{k_B T}{N_0}$$ mjög nærri 0

skoðum setni



$$\frac{N_e}{N_0} = \frac{\dfrac{1}{e^{\beta\Delta E}e^{-\beta\mu}-1}}{\dfrac{1}{e^{-\mu\beta}-1}} = \frac{e^{-\mu\beta}-1}{e^{\beta\Delta E}e^{-\beta\mu}-1} \approx \frac{-\mu\cdot\beta}{\Delta E\cdot\beta}$$

$$\rightarrow \quad N_e \simeq N_0 \frac{|\mu|}{\Delta E}$$

setur örvaða ástandið e

fyrir rúmsentimetra He^4,

$$\mu \approx -\frac{k_B T}{N_0} \simeq -\frac{k_B T}{N_0}$$

$$\approx -\frac{k_B T}{4\cdot 10^{21}} \approx -2.5\cdot 10^{-22} k_B T$$

ef við tökum $N_0 \sim N$

$$N \approx \frac{(0{,}01)^3 m^3}{\lambda_{th}^3}\, 2.6 \simeq$$

$$\lambda_{th} = \left(\frac{2\pi\hbar^2}{m k_B T}\right)^{1/2} \sim 8.7\cdot 10^{-10} m \text{ við } 1K$$

$T \sim 1K$

$$\rightarrow \quad N \simeq \frac{(0.01)^3_{m^3}\cdot 2.6}{(8.7\cdot 10^{-10})^3_{m^3}} \approx 4\cdot 10^{21}$$

$$\frac{\Delta E}{k_B T} \simeq \frac{(1.05\cdot 10^{-34})^2\cdot N^{-2/3}}{4\cdot 1{,}6\cdot 10^{-27}(8.7\cdot 10^{-10})^2(1.38\cdot 10^{-23})} \simeq 7\cdot 10^{-16} \quad \text{fyrir } 1K$$

því er

$$N_e \simeq N_0 \cdot \frac{|\mu|}{\Delta E} \simeq N_0 \cdot \frac{2.5 \cdot 10^{-22}}{7 \cdot 10^{-16}} \approx N_0 \cdot 3.6 \cdot 10^{-7}$$

Þannig að við metum setni örvaða ástandsins miklu minni en grunnástandsins