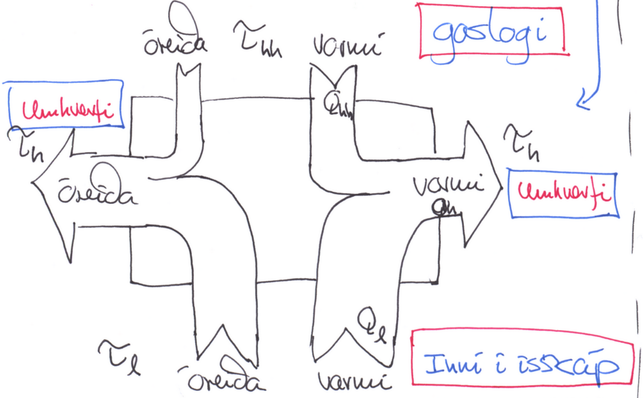
8.2

Gasísskápur

Varmi frá gasloga notaður til að knýja kerfið

$\tau_{hh} > \tau_h$

τ_l



b) Reikna $\frac{Q_l}{Q_{hh}}$ fyrir jafngengt ferli

$$Q_h = Q_{hh} + Q_l \quad (1)$$

$$T_h = T_{hh} + T_l = \frac{Q_{hh}}{\tau_{hh}} + \frac{Q_l}{\tau_l}$$

og $T_h = \frac{Q_h}{\tau_h}$

$$\nabla_h \tau_h = Q_{hh}\frac{\tau_h}{\tau_{hh}} + Q_\ell \frac{\tau_h}{\tau_\ell} = Q_h \qquad (2)$$

$$(1)-(2) \rightarrow \quad Q_{hh} + Q_\ell - Q_{hh}\frac{\tau_h}{\tau_{hh}} - Q_\ell \frac{\tau_h}{\tau_\ell} = 0$$

$$\rightarrow \quad Q_{hh}\left\{1 - \frac{\tau_h}{\tau_{hh}}\right\} + Q_\ell \left\{1 - \frac{\tau_h}{\tau_\ell}\right\} = 0$$

$$\rightarrow \quad \frac{Q_\ell}{Q_{hh}} = -\frac{\left\{1 - \frac{\tau_h}{\tau_{hh}}\right\}}{\left\{1 - \frac{\tau_h}{\tau_\ell}\right\}} = \frac{\left\{1 - \frac{\tau_h}{\tau_{hh}}\right\}}{\left\{\frac{\tau_h}{\tau_\ell} - 1\right\}}$$

$$= \left(\frac{\tau_\ell}{\tau_{hh}}\right)\frac{(\tau_{hh} - \tau_h)}{(\tau_h - \tau_\ell)} = \eta_c(\tau_{hh}, \tau_h)\cdot \eta_c(\tau_h, \tau_\ell)$$

(8-3) Ljóseinda Carnot-vél

Ljóseindagas, notum

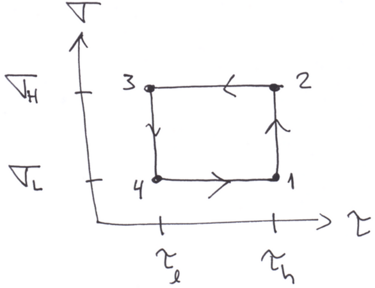
$$\frac{U}{V} = \frac{\pi^2}{15\hbar^3 c^3}\tau^4 \quad \text{og} \quad \sigma = \frac{4\pi^2 V}{45}\left(\frac{\tau}{\hbar c}\right)^3$$

sem

$$U = \alpha V \tau^4 \quad \text{og} \quad \sigma = \frac{4\alpha}{3} V \tau^3$$

a) τ_h, τ_l, V_1 og V_2 gefin

reikna V_3 og V_4



frå 2-3 er jafnøreida prill

$$\rightarrow \frac{4\alpha}{3} V_2 \tau_h^3 = \frac{4\alpha}{3} V_3 \tau_l^3$$

$$V_3 = V_2 \left(\frac{\tau_h}{\tau_l}\right)^3$$

Lika frå 4-1:

$$\frac{4\alpha}{3} V_4 \tau_l^3 = \frac{4\alpha}{3} V_1 \tau_h^3$$

$$V_4 = V_1 \left(\frac{\tau_h}{\tau_l}\right)^3$$

b) Hver er Q_h tekinn inn og Vinnan i fyrstu jafnhita ~~pe~~ansunni?

$$dQ = \tau d\sigma$$

$$Q_h = Q_{12} = \int_{\sigma_1}^{\sigma_2} \tau_h d\sigma$$

$$= \tau_h (\sigma_2 - \sigma_1)$$

$$= \tau_h \frac{4\alpha}{3} \tau_h^3 (V_2 - V_1)$$

$$= \frac{4\alpha}{3} \tau_h^4 (V_2 - V_1)$$

$$đW = dU - đQ$$

$$W_{12} = U_2 - U_1 - Q_{12} = \alpha \tau_h^4 (V_2 - V_1) - \frac{4\alpha}{3} \tau_h^4 (V_2 - V_1)$$

$$= -\frac{\alpha}{3} \tau_h^4 (V_2 - V_1) = -\frac{Q_h}{4} \quad , \quad Q_h = Q_{12}$$

$$\rightarrow \quad W_{12} \neq Q_h$$

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skoðum líka (3-4) jafnhitaferli þar fæst

$$W_{34} = -\frac{\alpha}{3} \tau_l^4 (V_4 - V_3) = -\frac{\alpha}{3} \tau_l \, \tau_l^3 (V_4 - V_3)$$

nota nú að $\tau_l^3 V_4 = \tau_H^3 V_1$ og $\tau_l^3 V_3 = \tau_h^3 V_2$ ← jafnóreiðu tengsl

$$\rightarrow W_{34} = \frac{\alpha}{3} \tau_l \, \tau_h^3 (V_2 - V_1)$$

Heildar vinnan á ker{f}i{ð} er

$$W_{12} + W_{34} = -\frac{\alpha}{3}\tau_h^3(\tau_h - \tau_l)(V_2 - V_1)$$

c) Styttast jafnörðu ferlin (2->3) og (4->1) út?

$$W_{23} = U_3 - U_2 = \alpha V_3 \tau_l^4 - \alpha V_2 \tau_h^4$$

$$= -\alpha V_2 \tau_h^3 (\tau_h - \tau_l)$$

$$W_{41} = U_1 - U_4 = \alpha V_1 \tau_h^4 - \alpha V_2 \tau_l^4$$

$$= \alpha V_1 \tau_h^3 (\tau_h - \tau_l)$$

$$\rightarrow W_{23} + W_{41} = -\alpha \tau_h^3 (\tau_h - \tau_l)(V_2 - V_1) \neq 0$$

d) Heildar vinnan framkvæmd af kerfinu

$$W = -\left(W_{12} + W_{23} + W_{34} + W_{41}\right)$$

$$= \underbrace{\alpha \tau_h^3 (\tau_h - \tau_\ell)(V_2 - V_1)}_{-W_{23} - W_{41}} + \underbrace{\frac{\alpha}{3} \tau_h^3 (\tau_h - \tau_\ell)(V_2 - V_1)}_{W_{12} + W_{34}}$$

$$= \frac{4\alpha}{3} \tau_h^3 (\tau_h - \tau_\ell)(V_2 - V_1)$$

$$\rightarrow \eta = \frac{W}{Q_h} = \frac{\frac{4\alpha}{3} \tau_h^3 (\tau_h - \tau_\ell)(V_2 - V_1)}{\frac{4\alpha}{3} \tau_h^4 (V_2 - V_1)} = \frac{\tau_h - \tau_\ell}{\tau_h} = \eta_c$$

Carnot loftkælir milli T_h úti og T_ℓ inni

Innflæðir vegna lélegrar einangrunar $A(T_h - T_\ell) = \frac{dQ_\ell}{dt}$

Afl kælis er P, finna T_ℓ stöðugt hitastig inni

$$W = \frac{\tau_h - \tau_\ell}{\tau_\ell} Q_\ell$$

$$\rightarrow \frac{dW}{dt} = \frac{\tau_h - \tau_\ell}{\tau_\ell} \frac{dQ_\ell}{dt} = \frac{T_h - T_\ell}{T_\ell} A (T_h - T_\ell)$$

$$= A \frac{(T_h - T_\ell)^2}{T_\ell} = P$$

$$\rightarrow A (T_h - T_\ell)^2 = P T_\ell$$

$$T_\ell^2 - \left(2T_h + \frac{P}{A}\right) T_\ell + T_h^2 = 0$$

finnum rōt med $T_l < T_h$ eins og krafist var

$$T_l = \frac{\left(2T_h + \frac{P}{A}\right) \pm \sqrt{\left(2T_h + \frac{P}{A}\right)^2 - 4T_h^4}}{2}$$

$$= \left(T_h + \frac{P}{2A}\right) \pm \sqrt{\left(T_h + \frac{P}{2A}\right)^2 - T_h^4}$$

b) $T_h = 37°C$

$T_l = 17°C$

$P = 2kW$

finna A

$$A = \frac{P\,T_l}{(T_h - T_l)^2}$$

$$= \frac{2kW \cdot 290K}{(20K)^2} = 1.45\ kW/K$$