Sýna að fjöldi ástanda gass með orku minni en E_{max} er

$$\int_0^{\sqrt{2mE_{max}/\hbar^2}} dk\, g(k) = \frac{V}{6\pi^2}\left(\frac{2mE_{max}}{\hbar^2}\right)^{3/2}$$

$$E_{max} = \frac{\hbar^2 k_{max}^2}{2m}$$

$$\to k_{max}^2 = \frac{2mE_{max}}{\hbar^2}$$

$$\to \int_0^{\sqrt{\frac{2mE_{max}}{\hbar^2}}} dk\, \frac{Vk^2}{2\pi^2} = \frac{V}{2\pi^2}\left.\frac{k^3}{3}\right|_0^{k_{max}} = \frac{V}{2\pi^2}\frac{k_{max}^3}{3} = \frac{V}{6\pi^2}\left\{\frac{2mE_{max}}{\hbar^2}\right\}^{3/2}$$

Setjum $E_{max} = \frac{3}{2}k_BT$ þá er fjöldi ástanda

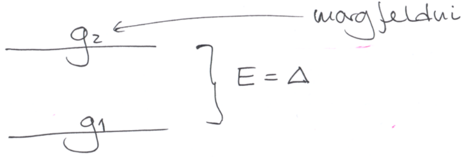
$$\frac{V}{6\pi^2}\left\{\frac{2m\,3k_BT}{2\hbar^2}\right\}^{3/2}, \quad \text{og} \quad n_Q = \frac{1}{\hbar^3}\left\{\frac{mk_BT}{2\pi}\right\}^{3/2} = \left\{\frac{mk_BT}{2\pi\hbar^2}\right\}^{3/2}$$

$$\to \frac{6^{3/2}}{6\sqrt{\pi}} V n_Q = \sqrt{\frac{6}{\pi}}\, V n_Q$$

$\sim 1{,}38$

21-04

Atóm ~~með~~ tvö orkustig



$$Z_{atom} = \sum_i e^{-\beta E_i} = g_1 + g_2 e^{-\beta\Delta}$$

$$C = \frac{dU}{dT} = \frac{dU}{d\beta}\frac{d\beta}{dT} = \frac{d\left(\frac{1}{k_BT}\right)}{dT}\frac{dU}{d\beta} = -\frac{1}{k_BT^2}\frac{dU}{d\beta}$$

$$U = -\frac{d\ln Z_{atom}}{d\beta} = -\frac{-\Delta g_2 e^{-\beta\Delta}}{g_1 + g_2 e^{-\beta\Delta}} = \frac{g_2\Delta e^{-\beta\Delta}}{g_1 + g_2 e^{-\beta\Delta}}$$

$$C = -\frac{1}{k_BT^2}\left\{\frac{-g_2\Delta^2 e^{-\beta\Delta}}{g_1 + g_2 e^{-\beta\Delta}} - \frac{g_2\Delta e^{-\beta\Delta}\cdot\left(-g_2\Delta e^{-\beta\Delta}\right)}{\left(g_1 + g_2 e^{-\beta\Delta}\right)^2}\right\}$$

$$C = -\frac{1}{k_B T^2}\left\{\frac{-g_2\Delta^2(g_1+g_2e^{-\beta\Delta})e^{-\beta\Delta}}{(g_1+g_2e^{-\beta\Delta})^2} + \frac{g_2^2\Delta^2 e^{-2\beta\Delta}}{(g_1+g_2e^{-\beta\Delta})^2}\right\}$$

$$= +\frac{1}{k_B T^2}\left\{\frac{g_1g_2\Delta^2 e^{-\beta\Delta}}{(g_1+g_2e^{-\beta\Delta})^2}\right\}$$

---

ein atóma gas

$$Z_N = \frac{Z_1^N}{N!}$$

p.s.

$$Z_1 = Z_{CM}\cdot Z_{atóm} = \frac{V}{\lambda_{th}^3} Z_{atóm}$$

$$= \frac{1}{N!}\left\{\frac{V}{\lambda_{th}^3}(g_1+g_2e^{-\beta\Delta})\right\}^N.$$

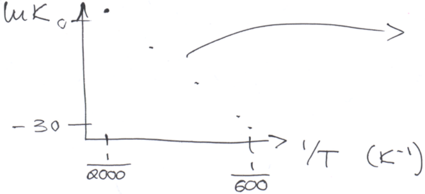
$$U = -\frac{d \ln Z_N}{d\beta} = -\frac{d}{d\beta}\left\{ \ln\left(\frac{1}{N!}\left(\frac{V}{\lambda_{th}^3}\cdot Z_{at\acute{o}m}\right)^N\right)\right\}$$

$$= -\frac{d}{d\beta}\left\{ \ln\left[\left(V T^{3/2}\right)^N \cdot Z_{at\acute{o}m}^N\right]\right\} = -\frac{d}{d\beta}\left\{\frac{3N}{2}\ln T + N \ln Z_{at\acute{o}m}\right\}$$

$$\rightarrow C = -\frac{1}{k_B T^2}\frac{dU}{d\beta} = N\left\{\frac{3}{2}k_B + \frac{g_1 g_2 \Delta^2 e^{-\beta\Delta}}{k_B T^2 (g_1 + g_2 e^{-\beta\Delta})^2}\right\}$$

þennan lið má bera saman við (21.31) . . . .

22-03 Bond enthalpy fyrir Br_2 gróflega metin af grafi



hallatala $-\dfrac{30}{\frac{1}{2000}-\frac{1}{600}} \sim -2.57\cdot 10^4\ \mathrm{K}$

$R = 8.314\ \mathrm{J/(mol\,K)}$

Hallatalan er $-\frac{\Delta H}{R}$

$\rightarrow \Delta H \simeq 2.57 \cdot 10^4 K \cdot 8.314 \frac{J}{mol\,K} \sim 2.14 \cdot 10^5 \frac{J}{mol}$

$\sim 214 \frac{KJ}{mol}$ innvermið

22-06 Jónun vetnis $H \rightleftharpoons p^+ + e^-$

skýra hvers vegna $\mu_H = \mu_p + \mu_e$

Í bókinni B-B er leidd út jafna (22.78)

$$\sum_{j=1}^{p+q} \nu_j \mu_j = 0 \quad \rightarrow \quad -\mu_H + \mu_p + \mu_e = 0$$

eða $\mu_H = \mu_p + \mu_e$.

Gerum ráð fyrir að hér skipti aðeins máli grunnástandið og lægsta jónunarorka H : $-R$

$$Z_1^H = \frac{V}{\lambda_{th}^3} \cdot e^{\beta R}$$

fyrir eina orkustig H sem við tökum með

fyrir hreyfiorku H

$$Z = \frac{\left(Z_1^H\right)^{N_H}}{N_H!} \cdot \frac{\left(Z_1^p\right)^{N_p}}{N_p!} \cdot \frac{\left(Z_1^e\right)^{N_e}}{N_e!}$$

$$\left.\begin{aligned} F &= -k_B T \ln Z \\ \mu &= \left(\frac{\partial F}{\partial N}\right)_{V,T} \end{aligned}\right\}$$

$$F = -k_B T \left\{ N_H \ln\left(Z_1^H\right) - N_H \ln N_H + N_p \ln\left(Z_1^p\right) - N_p \ln N_p + N_e \ln\left(Z_1^e\right) - N_e \ln N_e \right\}$$

$$\rightarrow \mu_H = -k_B T \left\{ \ln\left(Z_1^H\right) - \ln N_H \right\} = -k_B T \ln\left(\frac{Z_1^H}{N_H}\right)$$

á sama hátt

$$\mu_e = -k_B T \ln\left(\frac{Z_1^e}{N_e}\right) \quad \text{og} \quad \mu_p = -k_B T \ln\left(\frac{Z_1^p}{N_p}\right)$$

höfum $\mu_H = \mu_p + \mu_e$

$$\to \quad -k_B T \ln\left(\frac{V}{\lambda_{th,H}^3} \frac{e^{\beta R}}{N_H}\right) = -k_B T \left\{ \ln\left(\frac{V}{\lambda_{th,e}^3 N_e}\right) + \ln\left(\frac{V}{\lambda_{th,p}^3 N_p}\right) \right\}$$

$$-k_B T \ln\left(\frac{e^{\beta R}}{\lambda_{th,H}^3 \, n_H}\right) = -k_B T \left\{ \ln\left(\frac{1}{n_e \lambda_{th,e}^3}\right) + \ln\left(\frac{1}{n_p \lambda_{th,p}^3}\right) \right\}$$

munum að

$$\lambda_{th,x}^3 = \left\{ \frac{h}{\sqrt{2\pi m_x k_B T}} \right\}^3 = \frac{1}{n_Q^x}$$

$$-k_B T \ln\left(\frac{n_Q^H}{n_H} e^{\beta R}\right) = -k_B T \left\{ \ln\left(\frac{n_Q^e}{n_e}\right) + \ln\left(\frac{n_Q^p}{n_p}\right) \right\}$$

$$\ln\left(\frac{n_Q^H}{n_H} e^{\beta R}\right) = \ln\left(\frac{n_Q^e n_Q^p}{n_e n_p}\right)$$

$$\rightarrow \quad \frac{n_e n_p}{n_H} = \frac{n_Q^e n_Q^p}{n_Q^H} e^{-\beta R}$$

Saha jafnan

$$m_x \sim m_p \rightarrow n_Q^H \sim n_Q^p \rightarrow \boxed{\frac{n_e n_p}{n_H} \simeq n_Q^e e^{-\beta R}}$$

---

b) $H \longleftrightarrow \underbrace{p + e^-}$

$\rightarrow n_e = n_p$ því $N_e = N_p$ og V er fast.

Vinstra megin engin hleðsla

$n = n_H + n_p$ heldur ~~þéttleiki~~ ójónaðs og jónaðs vetnis

Ef $y = \frac{n_p}{n}$: hlutfall jónunar

Sýna að $\frac{y^2}{1-y} = \frac{e^{-\beta R}}{n\lambda_{th}^3} = \frac{n_Q^H e^{-\beta R}}{n}$

$n_p = ny \quad \rightarrow \quad n_H = n - n_p = n(1-y)$

$n_e = n_p = ny$

höfðum $\frac{n_e n_p}{n_H} = n_Q^e e^{-\beta R} \quad \rightarrow \quad \frac{n^2 y^2}{n(1-y)} = n_Q^e e^{-\beta R}$

$$\frac{y^2}{1-y} = \frac{n_Q^e}{n} e^{-\beta R}$$

Finna jónun við $T = 1000K$, $n = 10^{20}\,m^{-3}$

$n_Q^e \simeq 7.2 \cdot 10^{25}\,m^{-3} \rightarrow \frac{n_Q^e}{n} e^{-\beta R} \sim \frac{7.2 \cdot 10^{25}}{10^{20}} \exp\left\{-\frac{13.6}{8.617 \cdot 10^{-5} \cdot 1000}\right\}$

$\sim 2.1 \cdot 10^{-63}$

mög smá jónum

$$\rightarrow \frac{y^2}{1-y} \sim y^2 \approx 2.1 \cdot 10^{-63} \text{ og } y = 4.5 \cdot 10^{-32}$$

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c) Hvers vegna eykst y þegar n lækkar

$$\frac{y^2}{1-y} = \frac{n_e^e e^{-\beta R}}{n}$$

Þá dregur úr sameiningu $p + e^- \rightarrow H$

22-07

NaCl í H_2O 0.9% af massa NaCl
er jafnþrýst blóði (isotonic)
sýna að osmósuþrýstingur blóðs
sé nálægt 8 atm

$$\Pi = \frac{n_B}{V} RT$$

Lausninger med $n_B = 0{,}009\ \mathrm{Kg/L} = 9\ \mathrm{Kg/m^3}$ of NaCL

Loft ved 22.5 C $\sim 1{,}19\ \mathrm{Kg/m^3}$

$$\rightarrow \quad \frac{n_B^{NaCL}}{n_B^{air}} \sim 7{,}56$$