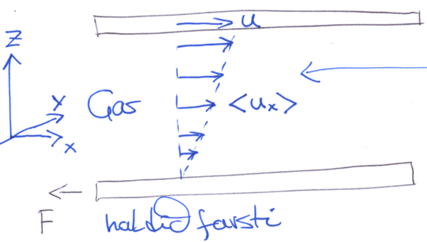
Flutningseiginleikar gasa

Flutningur er alltaf úr jafnvægi
skoðum sístæð ástönd (Steady State)

* Seigju (skriðþungi)
* Varmaleiðni (varmi)
* Sveimi (eindir)



Setur upp hraðastigul

$$\frac{d\langle u_x\rangle}{dz}$$

þrýsturinn τ_{xz} lýsir
sker- eða skúfspennunni

$$\tau_{xz} = \eta \frac{d\langle u_x\rangle}{dz}$$

Seigjan

Eining η er Pa·s $\left(\frac{N}{m^2}s\right)$, vídd $[\eta] \sim \frac{ML}{TL^2}T = \frac{M}{L}$ (2)

skriðþungaflæði
á móti stiglinum

$$\Pi_z = -\eta \frac{\partial \langle u_x \rangle}{\partial z}$$

fjöldi sameinda sem stefna á einingarflöt/s

$$v\cos\theta \cdot n f(v) dv \, \frac{1}{2}\sin\theta \cdot d\theta$$

með stefnu θ mælt frá z-ás. Þær hafa ferðast $\lambda\cos\theta$ samsíða z-ás frá síðasta árekstri

Á þeirri leið hefur $\langle u_x \rangle$ aukist um

$$\frac{\partial \langle u_x \rangle}{\partial z} \lambda \cos\theta$$

og sameind á leið upp. dregur úr skriðþunganum sem nemur

$$-m\left(\frac{\partial \langle u_x \rangle}{\partial z}\right)\lambda\cos\theta$$

því er heildar skriðþunga flutningurinn (p_x) um flöt þvert á z-ás

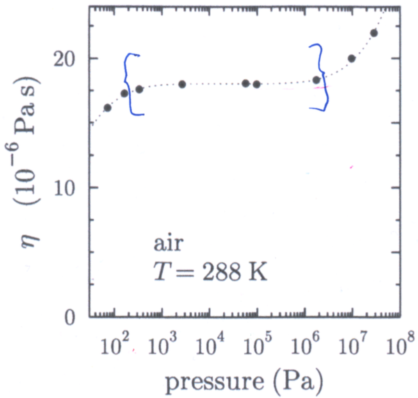
$$\Pi_z = \int_0^\infty dv \int_0^\pi d\theta \cdot v\cos\theta \cdot n f(v)\, \frac{1}{2}\sin\theta \cdot m\left(-\frac{\partial\langle u_x\rangle}{\partial z}\right)\lambda\cos\theta$$

$$= \frac{1}{2} n m \lambda \int_0^\infty dv\, v f(v)\left(-\frac{\partial\langle u_x\rangle}{\partial z}\right)\int_0^\pi d\theta \cos^2\theta \sin\theta$$

$$= -\frac{1}{3} n m \lambda \langle v\rangle \left(\frac{\partial\langle u_x\rangle}{\partial z}\right)$$

$$\Pi_z = -\eta \frac{\partial\langle u_x\rangle}{\partial z}$$

$$\rightarrow \boxed{\eta = \frac{1}{3} n m \lambda \langle v\rangle}$$



Blundell og Blundell

$$\lambda \simeq \frac{1}{\sqrt{2}\, n \sigma} \sim \frac{1}{n}$$

→ η er óháð n.

og við fast T er η því óháð p

$\{ p = n k_B T \}$

Ef η er óháð n þá er η aðeins háð T í gegnum $\langle v \rangle$

$\rightarrow \eta \sim \sqrt{T}$

$\boxed{\eta \text{ vex með } T}$ öfugt við flesta vökva

$\lambda = \frac{1}{\sqrt{2}\, n \sigma}$

$\sigma = \pi d^2$

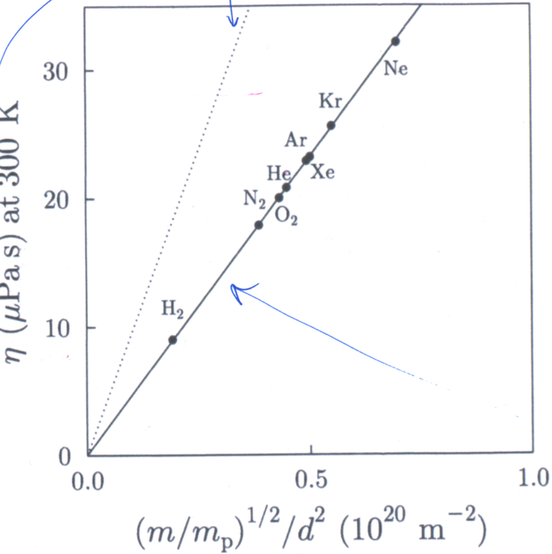
$\langle v \rangle = \sqrt{\frac{8 k_B T}{\pi m}}$

$\downarrow$

$\boxed{\eta = \frac{2}{3\pi d^2}\sqrt{\frac{m k_B T}{\pi}}}$

$\downarrow$

$\eta \sim \frac{\sqrt{m}}{d^2}$



þurfum að uppfylla

$L >> \lambda >> d$

$\eta = \frac{1}{3} n m \lambda \langle v \rangle$

↑

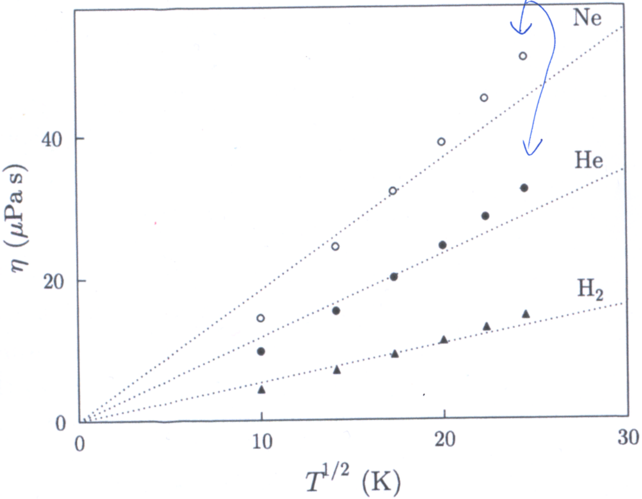
ekki alveg rétt

Ferðaheitingin er mismunandi milli brotaloga . . . .

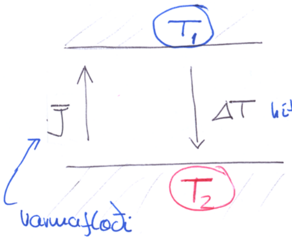
$\eta \sim \sqrt{T}$

$\sigma = \pi d^2$ er ikki alveg rætt

σ er T-háð, virðist minka við hækkandi T $\rightarrow$ η vex ødeins meira en $\sqrt{T}$



Varmaleiðni



$T_1 < T_2$

$$\boxed{\bar{J} = -\kappa \bar{\nabla} T}$$

Lögmál Fouriers

varmaleiðni

Getum við leitt út stuðulinn κ?

sameindir ferðast $\lambda \cos\theta$ samsíða z-ás frá síðasta árekstri

Hver þeirra breytir varmaorkunni um

$$C_{sameind} \cdot \Delta T = C_{sameind} \frac{\partial T}{\partial z} \lambda \cos\theta$$

$$\rightarrow J_z = \int_0^\infty dv \int_0^\pi d\theta \left(- C_{sameind} \frac{\partial T}{\partial z} \lambda \cos\theta \right) v \cos\theta \, n f(v) \frac{1}{2} \sin\theta$$

munar um

$$= -\frac{1}{2} n C_{\text{sameind}} \lambda \int_0^\infty dv\, v f(v) \frac{\partial T}{\partial z} \int_0^\pi d\theta \cos^2\theta \sin\theta$$

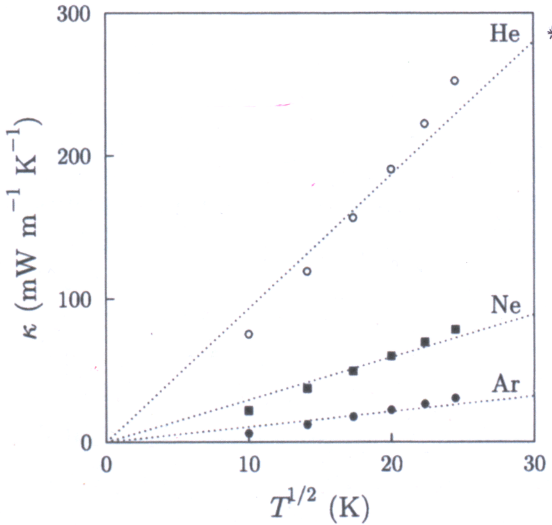
$$= -\frac{1}{3} n C_{\text{sameind}} \lambda \langle v \rangle \frac{\partial T}{\partial z}$$

$$\rightarrow \quad \boxed{\kappa = \frac{1}{3} C_V \lambda \langle v \rangle} \qquad C_V = n C_{\text{sameind}}$$

* K er óháð p: $\lambda \simeq \dfrac{1}{\sqrt{2}\, n \sigma} \sim \dfrac{1}{n}$

því er K óháð n

$\rightarrow$ óháð p fyrir $T = \text{fasti}$



* $\kappa \sim \sqrt{T}$

κ er óháð n

$\rightarrow$ T kemur aðeins frá $\langle v \rangle \sim \sqrt{T}$

sama frávik fyrir hátt T og hjá η vegna þess að σ virðist ekki óháð T

Notum $\lambda = \frac{1}{\sqrt{2}\, n \sigma}$, $\sigma = \pi d^2$

$$\langle v \rangle = \sqrt{\frac{8 k_B T}{\pi m}}$$

$L \gg \lambda \gg d$

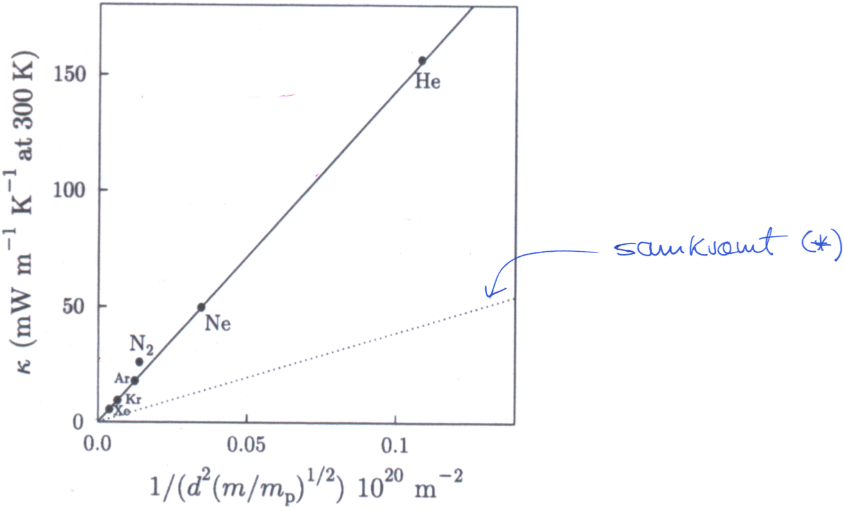
$$\longrightarrow \boxed{K = \frac{2}{3\pi d^2} C_{sameind} \sqrt{\frac{k_B T}{\pi m}}} \quad (*)$$

after sæst at $K \sim \frac{1}{\sqrt{m}\, d^2}$

Likinda K og η leiða til

$$\boxed{K = C_v \eta}$$

en okkar einföldu úttrúksur fyrir K og η halda ekki mjög vel


150
100
50
0
0.0
0.05
0.1
He
Ne
N2
Ar
Kr
Xe
κ (mW m−1 K−1 at 300 K)
1/(d2(m/mp)1/2) 1020 m−2
samkvant (*)

Sveim einda

Lögmál Ficks

$$\boxed{\overline{\Phi} = -D\overline{\nabla} n^*}$$

n^*: fjöldi merktra sameinda á rúmmál

(þar sveima um þar ómerktu)

fjöldi einda er varðveittur

$$\longrightarrow \underbrace{\oint_S \overline{\Phi}\cdot d\overline{S}}_{\text{flæði um lokað yfirborð út úr } V} = \underbrace{-\frac{\partial}{\partial t}\int_V n^* dV}_{\text{tímabreyting á heildar einda fjölda innan } V}$$

Div-setningin gefur

$$\oint_S \bar{\Phi} \cdot d\bar{s} = \int_V dv\, \bar{\nabla} \cdot \bar{\Phi}$$

hugsum okkur fast rúmmál V

$$\rightarrow \int_V dv \left\{ \bar{\nabla} \cdot \bar{\Phi} + \frac{\partial}{\partial t} n^* \right\} = 0$$

notum Fick

$$\bar{\Phi} = -D \bar{\nabla} n^*$$

$$\int_V dv \left\{ -D \bar{\nabla} \cdot \bar{\nabla} n^* + \frac{\partial}{\partial t} n^* \right\} = 0$$

$$\boxed{\frac{\partial n^*}{\partial t} - D \nabla^2 n^* = 0}$$

Getum leitt út

$$D = \frac{1}{3}\lambda\langle v\rangle$$

* $D \sim \frac{1}{p}$: $\lambda \sim \frac{1}{n} \rightarrow D \sim \frac{1}{n}$

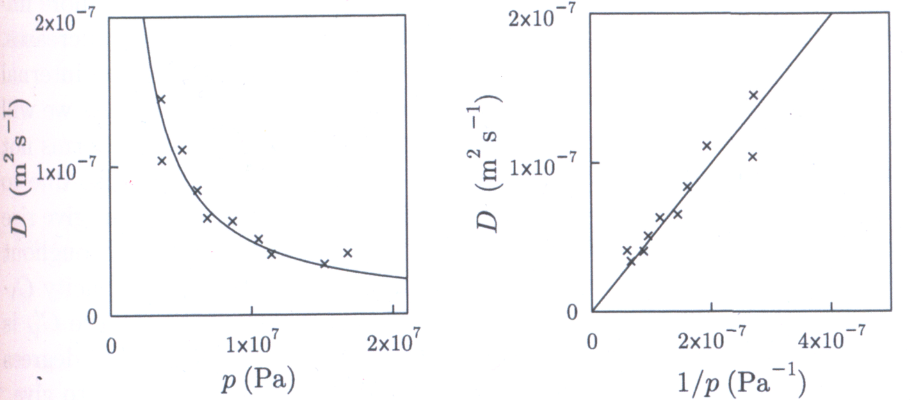
fyrir fast $T \rightarrow D \sim \frac{1}{p}$

* $D \sim T^{3/2}$: $p = nk_BT$, $\langle v\rangle \sim \sqrt{T}$

$\rightarrow D \sim T^{3/2}$ fyrir fastan p

* $D\rho = \eta$: líkindi jafna fyrir D og η

$\rho = nm$ . . . . .


2×10^{-7}
1×10^{-7}
0
D ($m^2 s^{-1}$)
0
1×10^{7}
2×10^{7}
p (Pa)
2×10^{-7}
1×10^{-7}
0
D ($m^2 s^{-1}$)
0
2×10^{-7}
4×10^{-7}
$1/p$ (Pa^{-1})