6-4

$\varepsilon = pc$ Kjörgas

fyrir klassíska kjörgasið með $\varepsilon = \frac{p^2}{2m}$

notuðum við

$$\varepsilon = \frac{\hbar^2}{2M}\left\{k_x^2 + k_y^2 + k_z^2\right\}, \quad k_i = \frac{\pi n_i}{L}$$

Nú höfum við

$$\varepsilon = pc = \hbar c\left\{k_x^2 + k_y^2 + k_z^2\right\}^{1/2} \quad \text{með } k_i = \frac{\pi n_i}{L}$$

$$= \frac{\hbar c \pi}{L}\left\{n_x^2 + n_y^2 + n_z^2\right\}^{1/2} = \frac{\pi \hbar c}{L} n$$

Kórsumman fyrir eina eind í varmatengslum er

$$Z_1 = \frac{1}{8} 4\pi \int_0^\infty dn \cdot n^2 \cdot \exp\left\{-\frac{\pi\hbar c}{\tau L} n\right\} \qquad \left\{\sum_n e^{-\frac{\pi\hbar c}{\tau L} n}\right\}$$

setjum $\frac{\pi\hbar c}{\tau L} n = x$

$$Z_1 = \frac{1}{8} 4\pi \left(\frac{L\tau}{\pi\hbar c}\right)^3 \int_0^\infty dx\, x^2 e^{-x} = \frac{\pi}{2}\left(\frac{L}{\pi\hbar c}\right)^3 \cdot 2 \cdot \tau^3$$

notum síðan

$$U = \tau^2 \left(\frac{\partial \ln Z_1}{\partial \tau}\right) = \tau^2 \left(\frac{\partial (3 \ln \tau)}{\partial \tau}\right) = 3\tau$$

$\rightarrow$ $\boxed{U = 3\tau}$ í stað $\frac{3}{2}\tau$ fyrir klassíska kjörgasið

6-10 Jafnóreiðu ferli, kjörgas

$$\sigma = N\left\{\ln\left(\frac{n_Q}{n}\right) + \frac{5}{2}\right\} + \sigma_{int}, \quad n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{3/2}$$

$$\rightarrow \sigma = N\left\{\frac{3}{2}\ln\tau + \ln V + \text{fasti}\right\} + \sigma_{int}(\tau) \qquad (*)$$

Eins getum við sett $pV = N\tau \rightarrow V = \frac{N\tau}{p}$ til að fá

$$\sigma = N\left\{\frac{5}{2}\ln\tau - \ln p + \text{fasti}\right\} + \sigma_{int}(\tau) \qquad (**)$$

Notum $$C_V = \tau\left(\frac{\partial\sigma}{\partial\tau}\right)_V$$

$$C_V = \frac{3}{2}N + \tau\left(\frac{\partial\sigma_{int}}{\partial\tau}\right)_V = \frac{3}{2}N + C_{int}$$

og $$C_p = \tau\left(\frac{\partial\sigma}{\partial\tau}\right)_p$$

$$C_p = \frac{5}{2}N + \tau\left(\frac{\partial\sigma}{\partial\tau}\right)_p = \frac{5}{2}N + C_{int}$$

$\rightarrow$ $$C_p - C_V = \left(\frac{C_p}{C_V} - 1\right)C_V = (\gamma - 1)C_V = \left(\frac{\gamma-1}{\gamma}\right)C_p = N$$

med $$\gamma = \left(\frac{C_p}{C_V}\right)$$

Allmegum (*)

$$dU = \frac{3}{2}N\frac{d\tau}{\tau} + N\frac{dV}{V} + \left(\frac{\partial U_{int}}{\partial \tau}\right)_V d\tau$$

$$= \frac{3}{2}N\frac{d\tau}{\tau} + N\frac{dV}{V} + C_{int}\frac{d\tau}{\tau}$$

$$= C_V\frac{d\tau}{\tau} + N\frac{dV}{V} = 0$$

$$\rightarrow \quad \frac{d\tau}{\tau} = -\frac{N}{C_V}\frac{dV}{V} = -(\gamma-1)\frac{dV}{V}$$

$$\rightarrow \quad \boxed{\frac{d\tau}{\tau} + (\gamma-1)\frac{dV}{V} = 0}$$

Athugum (**)

$$d\sigma = \frac{5}{2} N \frac{d\tau}{\tau} - N \frac{dp}{p} + \left(\frac{\partial \sigma}{\partial \tau}\right)_p d\tau$$

$$= \frac{5}{2} N \frac{d\tau}{\tau} - N \frac{dp}{p} + C_{int} \frac{d\tau}{\tau}$$

$$= C_p \frac{d\tau}{\tau} - N \frac{dp}{p} = 0$$

nota $\frac{d\tau}{\tau} + (\gamma - 1)\frac{dV}{V} = 0$

$$\rightarrow \quad \frac{dp}{p} = \frac{C_p}{N} \frac{d\tau}{\tau} = \frac{\gamma}{\gamma - 1} \frac{d\tau}{\tau} = -\gamma \frac{dV}{V}$$

$$\rightarrow \quad \boxed{\frac{dp}{p} + \frac{\gamma}{\gamma - 1} \frac{d\tau}{\tau} = 0} \quad \text{og} \quad \boxed{\frac{dp}{p} + \gamma \frac{dV}{V} = 0}$$

b) $$B_\sigma = -V\left(\frac{\partial p}{\partial V}\right)_\sigma \quad , \quad B_\tau = -V\left(\frac{\partial p}{\partial V}\right)_\tau$$

$$\frac{dp}{p} + \gamma\frac{dV}{V} = 0 \quad \rightarrow \quad \frac{dp}{dV} = \left(\frac{\partial p}{\partial V}\right)_\sigma = -\gamma\frac{p}{V}$$

$$\rightarrow \boxed{B_\sigma = +\gamma p}$$

fyrir jafnhita

$$p = \frac{N\tau}{V}$$

$$\left(\frac{\partial p}{\partial V}\right)_\tau = -\frac{N\tau}{V^2} = -\frac{p}{V}$$

$$\rightarrow \boxed{B_\tau = p}$$

6-12 Kjörgas í 2D

Sveigrúmin eru $\quad \varepsilon_n = \frac{\hbar^2}{2M}\left(\frac{\pi}{L}\right)^2\left[n_x^2 + n_y^2\right] = \frac{(\hbar\pi)^2}{2MA} n^2$

þ.s. $n^2 = n_x^2 + n_y^2$ og $A = L^2$: flötur kertisins

$$N = \sum_n f(\varepsilon_n) = \lambda \sum_n e^{-\frac{\varepsilon_n}{\tau}}$$

$$= \lambda \frac{1}{4} 2\pi \int_0^\infty n\,dn\, e^{-\frac{\varepsilon_n}{\tau}} = \frac{\lambda\pi}{2}\int_0^\infty n\,dn\, e^{-\frac{\hbar^2\pi^2}{2MA\tau}n^2}$$

$n_x, n_y > 0$

Setjum $\frac{\hbar\pi n}{\sqrt{2MA\tau}} = x$

$$= \lambda\pi \frac{2MA\tau}{\hbar^2\pi^2 2}\int_0^\infty x\,dx\, e^{-x^2}$$

$$N = \lambda\pi \frac{MA\tau}{\hbar^2\pi^2 2} = \lambda \frac{MA}{\hbar^2\pi 2} = \lambda A n_Q^{2D}, \quad n_Q^{2D} = \frac{M\tau}{2\pi\hbar^2}$$

$$\lambda = e^{\frac{\mu}{\tau}} \rightarrow \mu = \tau \ln\lambda = \tau \ln\left(\frac{N}{A n_Q^{2D}}\right)$$

$$= \tau \ln\left(\frac{n^{2D}}{n_Q^{2D}}\right) \quad \text{p.s. } n^{2D} = \frac{N}{A}$$

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b)

$$U = \sum_n \varepsilon_n f(\varepsilon_n) = \lambda \sum_n \varepsilon_n e^{-\frac{\varepsilon_n}{\tau}}$$

$$= \lambda \tau^2 \frac{\partial}{\partial\tau} \sum_n e^{-\frac{\varepsilon_n}{\tau}} = \lambda\tau^2 \frac{\partial}{\partial\tau} A n_Q^{2D}$$

$$= \lambda A \tau^2 \left(\frac{\partial n_Q^{2D}}{\partial\tau}\right) = \lambda A n_Q^{2D}\tau = N\tau$$

$$\rightarrow \boxed{U = N\tau}$$

c) finna σ

Jafnan fyrir $\mu = \tau \ln\left(\frac{n^{2D}}{n_Q^{2D}}\right)$

er formlega eins og fyrir 3D, nema n^{2D} og n_Q^{2D} þýða annað

$$\rightarrow F = N\tau\left\{\ln\left(\frac{n^{2D}}{n_Q^{2D}}\right) - 1\right\}$$

með leiðdum eins og í bók á bls. 163

Notum síðan

$$\sigma = -\left(\frac{\partial F}{\partial \tau}\right)_{A,N} = -N\left\{\ln\left(\frac{n^{2D}}{n_Q^{2D}}\right) - 1\right\} + N$$

$$= N\left\{\ln\left(\frac{n_Q^{2D}}{n^{2D}}\right) + 2\right\}$$