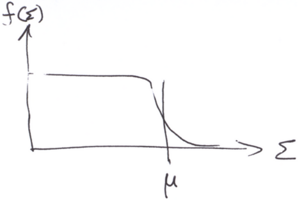
6-1 Reikna $-\left.\dfrac{\partial f}{\partial \varepsilon}\right|_{\varepsilon=\mu}$



$$f(\varepsilon) = \frac{1}{e^{\frac{\varepsilon-\mu}{\tau}}+1}$$

$$-\frac{\partial f}{\partial \varepsilon} = \frac{e^{\frac{\varepsilon-\mu}{\tau}}\cdot\frac{1}{\tau}}{\left(e^{\frac{\varepsilon-\mu}{\tau}}+1\right)^2}$$

$$-\left.\frac{\partial f}{\partial \varepsilon}\right|_{\varepsilon=\mu} = \frac{1\cdot\frac{1}{\tau}}{(1+1)^2} = \frac{1}{4\tau}$$

$$\to \quad -\left.\frac{\partial f}{\partial \varepsilon}\right|_{\varepsilon=\mu} \xrightarrow[\tau\to 0]{} \infty$$

$f(\varepsilon)$ nálgast þrepafall $\Theta(\mu-\varepsilon)$ fyrir $\tau=0$

6-2 Samhverfa $f(\varepsilon)$ um $\varepsilon = \mu$

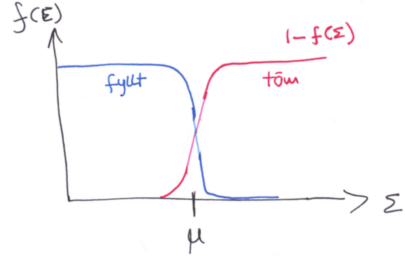
Setjum $\varepsilon = \mu + \delta$

$$f(\mu+\delta) = \frac{1}{e^{\frac{\delta}{\tau}} + 1}$$

$$f(\mu-\delta) = \frac{1}{e^{-\frac{\delta}{\tau}} + 1} = \frac{e^{\frac{\delta}{\tau}}}{1 + e^{\frac{\delta}{\tau}}} = 1 - f(\mu+\delta)$$

$$\frac{1+e^{\frac{\delta}{\tau}}}{1+e^{\frac{\delta}{\tau}}} - \frac{1}{1+e^{\frac{\delta}{\tau}}} = \frac{e^{\frac{\delta}{\tau}}}{1+e^{\frac{\delta}{\tau}}}$$

$\rightarrow f(\mu+\delta) = 1 - f(\mu-\delta)$



6-3

Sveigrúmasætin 0, 1, 2 með orku 0, ε, 2ε

a) Finna $\langle N \rangle$ fyrir kerfið í jafnvægi við einda og varmageymi með τ og μ

<u>athugum ástönd</u> (0,0) (0,1) (1,0) (1,1)

0 ε ε 2ε orka

$$\rightarrow \quad \mathcal{Z} = 1 + 2\lambda e^{-\frac{\varepsilon}{\tau}} + \lambda^2 e^{-\frac{2\varepsilon}{\tau}}$$

$$\langle N \rangle = \frac{0 \cdot 1 + 1 \cdot 2\lambda e^{-\frac{\varepsilon}{\tau}} + 2 \cdot \lambda^2 e^{-\frac{2\varepsilon}{\tau}}}{\mathcal{Z}} = \frac{2\left\{\lambda e^{-\frac{\varepsilon}{\tau}} + \lambda^2 e^{-\frac{2\varepsilon}{\tau}}\right\}}{1 + 2\lambda e^{-\frac{\varepsilon}{\tau}} + \lambda^2 e^{-\frac{2\varepsilon}{\tau}}}$$

$$\rightarrow \langle N \rangle = \frac{2\lambda e^{-\frac{\varepsilon}{\tau}}\left\{1+\lambda e^{-\frac{\varepsilon}{\tau}}\right\}}{\left\{1+\lambda e^{-\frac{\varepsilon}{\tau}}\right\}^{2}} = \frac{2\lambda e^{-\frac{\varepsilon}{\tau}}}{\left\{1+\lambda e^{-\frac{\varepsilon}{\tau}}\right\}}$$

$$= \frac{2}{\lambda^{-1} e^{\frac{\varepsilon}{\tau}} + 1} = \frac{2}{\exp\left\{\frac{\varepsilon-\mu}{\tau}\right\} + 1}$$

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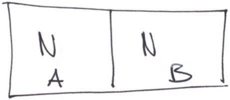
b) orkustig ε er tvöfalt

er nákvæmlega a-liðar

6-6 N-A og N-B í eindajafnvægi , τ, V sama



fullkomin blöndun

$\hookrightarrow \Delta\sigma = 2N\ln 2$

Sýna að ef $A = B \rightarrow \Delta\sigma = 0$ (Gibbs paradox)

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fjöldaföllin g_A og g_B

$$g = g_A g_B$$

$$\Downarrow$$

$$\sigma = \sigma_A + \sigma_B$$

fyrir og eftir blöndun

En við höfum

$$\sigma = N\left\{\ln\left(\frac{n_Q}{n}\right) + \frac{5}{2}\right\}$$

Fyrir

$$\sigma_i = N\left\{\ln\left(\frac{n_Q^A}{n_i}\right) + \ln\left(\frac{n_Q^B}{n_i}\right) + 5\right\}$$

því N er það sama

Eftir $n_f = \frac{N}{2V} = \frac{n_i}{2}$

$$\sigma_f = N\left\{ \ln\left(\frac{n_Q^A}{n_f}\right) + \ln\left(\frac{n_Q^B}{n_f}\right) + 5 \right\}$$

$$= N\left\{ \ln\left(2\frac{n_Q^A}{n_i}\right) + \ln\left(2\frac{n_Q^B}{n_i}\right) + 5 \right\}$$

$$= \sigma_i + \underbrace{2N\ln 2}_{\text{blöndunaróreiða}}$$

Ef eindirnar þekkjast ekki í sundur þá gildir

$n_f = \frac{2N}{2V} = n_i$

$$\sigma_f = 2N\left\{ \ln\left(\frac{n_Q}{n_i}\right) + \frac{5}{2} \right\}$$

engin munur á n_Q^A og n_Q^B

sama fyrir og eftir