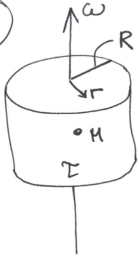


Fastur hornhraði ω, finna $n(r)$ m.v. $n(0)$ þéttleika kjörgassins í skilvindunni

Inni í skilvindunni er gervikraftur

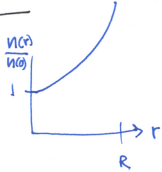
$+Mr\omega^2$ í átt að útvegg

Því er eins og bætt við mætti $\quad V(r) = -\frac{1}{2}Mr^2\omega^2$

Notum $\quad \mu = \mu_{int} + \mu_{ext} = \tau \ln\left(\frac{n}{n_Q}\right) + V(r)$

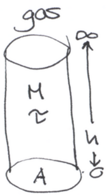
$$\rightarrow \quad \tau \ln\left(\frac{n(0)}{n_Q}\right) = \tau \ln\left(\frac{n(r)}{n_Q}\right) - \frac{1}{2}Mr^2\omega^2$$

$$\ln\left(\frac{n(0)}{n(r)}\right) = -\frac{Mr^2\omega^2}{2\tau} \quad \rightarrow \quad n(r) = n(0)\, e^{+\frac{Mr^2\omega^2}{2\tau}}$$



Hér þyrfti að útvega $n(0)$ út frá heildar þéttleika kerfisins án snúnings - - - -

5-3



fast þyngdarsvið g.

Finna meðal þyngdar og hreyfiorku á atóm

$$\mu = \tau \ln\left(\frac{n}{n_Q}\right) + Mgh$$

$$\rightarrow n(h) = n(0)\exp\left(-\frac{Mgh}{\tau}\right)$$

Fjöldi atóma á flöt A

$$\frac{N}{A} = \int_0^\infty dh\, n(h) = n(0)\frac{\tau}{Mg}$$

$$\rightarrow N = n(0)\frac{\tau A}{Mg} = n(0)\cdot A\cdot h_c$$

$$h_c = \frac{\tau}{Mg}$$

Heildar mættisorka á A

$$\frac{U_{pot}}{A} = \int_0^\infty dh\, n(h)\, Mgh$$

$$= Mg\, n(0)\int_0^\infty dh\, h\, e^{-\frac{Mgh}{\tau}}$$

notum $\frac{Mgh}{\tau} = x$

$$\frac{U_{pot}}{A} = n(0)\frac{\tau^2}{Mg}\int_0^\infty \left(\frac{Mg\,dh}{\tau}\right)\left(\frac{Mgh}{\tau}\right)e^{-\left(\frac{Mgh}{\tau}\right)}$$

$$= n(0)\frac{\tau^2}{Mg}\underbrace{\int_0^\infty dx\, x\, e^{-x}}_{1}$$

$$\rightarrow \frac{U_{pot}}{A} = n(0)\frac{\tau^2}{Mg}$$

mættis

$\rightarrow$ meðalorkan á atóm er

$$\frac{\left(\frac{U_{pot}}{A}\right)}{\frac{N}{A}} = \tau$$

Meðal hreyfiorkan fyrir kjörgas

$$U_{kin} = \frac{3}{2}\tau$$

Heildarorkan er því

$$U = \left(\frac{3}{2} + 1\right)\tau = \frac{5}{2}\tau$$

$$\rightarrow C_V = \left(\frac{\partial U}{\partial \tau}\right)_V = \frac{5}{2} \quad \text{fasti}$$

5-4

Fruma $\Delta\mu(300K) \sim 0{,}24 eV$

Lítum á jónirnar sem kjörgas

$$\frac{[K^+]_{fruma}}{[K^+]_{vatn}} \sim 10^4$$

$$\frac{n_f}{n_v} \sim 10^4$$

Kerfið er í ójafnvægi, en haldið stöðugu með $\Delta V q$

fruma: $\mu_f = \tau \ln\left(\frac{n_f}{n_Q}\right)$

vatn $\mu_v = \tau \ln\left(\frac{n_v}{n_Q}\right)$

$$\Delta\mu = \mu_f - \mu_v > 0$$

$$= \tau \ln\left(\frac{n_f}{n_v}\right) = k_B T \ln\left(\frac{n_f}{n_v}\right)$$

$$= 8.617\cdot 10^{-5}\,\frac{eV}{K}\cdot 300K \cdot \ln(10^{+4})$$

$$\simeq 0{,}24\, eV$$

5-6 tveggja stiga kerfi

ε —— 2

0 —— 1

ástönd $(\overset{1}{0},\overset{2}{0})$ $E=0$

$(1,0)$ $E=0$

$(0,1)$ $E=\varepsilon$

a)

$$z=\sum_{NS}\lambda^N\exp\left(-\frac{\varepsilon_s}{\tau}\right)$$

$$=1+\lambda+\lambda e^{-\frac{\varepsilon}{\tau}}$$

$$=1+\lambda\left(1+e^{-\frac{\varepsilon}{\tau}}\right)$$

b)

$$\langle N\rangle=\lambda\frac{\partial}{\partial\lambda}\ln z$$

$$=\lambda\frac{\partial}{\partial\lambda}\left\{\ln\left(1+\lambda\left[1+e^{-\frac{\varepsilon}{\tau}}\right]\right)\right\}$$

$$=\lambda\frac{1+e^{-\frac{\varepsilon}{\tau}}}{\ln\left(1+\lambda\left[1+e^{-\frac{\varepsilon}{\tau}}\right]\right)}$$

$$=\frac{\lambda\left(1+e^{-\frac{\varepsilon}{\tau}}\right)}{z}$$

c) meðal setni ástands með orku ε

ástand $(0,1)$

$$\langle N_\varepsilon\rangle=\frac{\lambda e^{-\frac{\varepsilon}{\tau}}}{z}$$

d) Finna U, meðal orku kerfisins

Hér er einfallt að nota

$$U=\frac{\sum_{NS}\varepsilon_s e^{\frac{N\mu-\varepsilon_s}{\tau}}}{z}$$

$$=\frac{\varepsilon e^{\frac{\mu-\varepsilon}{\tau}}}{z}$$

$$=\frac{\varepsilon\lambda e^{-\frac{\varepsilon}{\tau}}}{1+\lambda\left(1+e^{-\frac{\varepsilon}{\tau}}\right)}$$

e) Ef til viðbótar kemur ástandið (1,1) með $E = \Sigma$

$$\rightarrow \mathcal{Z} = 1 + \lambda\left(1 + e^{-\frac{\Sigma}{\tau}}\right) + \lambda^2 e^{-\frac{\Sigma}{\tau}}$$

$$= (1+\lambda)\left\{1 + \lambda e^{-\frac{\Sigma}{\tau}}\right\}$$

tvö óháð kerfi því fyrir neðra stigið fæst

$$\mathcal{Z}_0 = 1 + \lambda$$

og efra stigið

$$\mathcal{Z}_\epsilon = 1 + \lambda e^{-\frac{\Sigma}{\tau}}$$

undirkerfin voru ekki óháð þegar aðeins annað þeirra gat verið setið

8

CO eitrun

Allir viðtakar geta verið tómir eða

setnir með $O_2 \longleftrightarrow \Sigma_A$

$$CO \longleftrightarrow \Sigma_B$$

N viðtakar í jafnvægi við

O_2 : $\lambda_A(O_2) = 1 \cdot 10^{-5}$ $\qquad T = 37^\circ C$

CO : $\lambda_B(CO) = 1 \cdot 10^{-7}$

a) Án CO finna Σ_A þ.a. 90% viðtaka hafi O_2, svar í eV á O_2

7

N viðtakar, O eða Σ_A

um hvern gildir

$$z_1 = 1 + \lambda_A e^{-\frac{\Sigma_A}{\tau}}$$

og um N óháða

$$z = (z_1)^N = \left(1 + \lambda_A e^{-\frac{\Sigma_A}{\tau}}\right)^N$$

gefið að $\frac{\langle N_A \rangle}{N} = 0{,}9$

þurfum að finna Σ_A

$$\langle N_A \rangle = \lambda_A \frac{\partial}{\partial \lambda_A} \ln\left(1 + \lambda_A e^{-\frac{\Sigma_A}{\tau}}\right)^N$$

$$= \lambda_A N \frac{\partial}{\partial \lambda_A} \ln\left(1 + \lambda_A e^{-\frac{\Sigma_A}{\tau}}\right)$$

$$= \lambda_A N \frac{e^{-\frac{\Sigma_A}{\tau}}}{1 + \lambda_A e^{-\frac{\Sigma_A}{\tau}}}$$

$$\frac{\langle N_A \rangle}{N} = \frac{\lambda_A e^{-\frac{\Sigma_A}{\tau}}}{1 + \lambda_A e^{-\frac{\Sigma_A}{\tau}}} = \frac{1}{\lambda_A^{-1} e^{\frac{\Sigma_A}{\tau}} + 1}$$

$$\rightarrow \lambda_A^{-1} e^{\frac{\Sigma_A}{\tau}} + 1 = \frac{N}{\langle N_A \rangle}$$

Seda $\lambda_A^{-1} e^{\frac{\Sigma_A}{\tau}} = \frac{N}{\langle N_A \rangle} - 1$

$$e^{\frac{\Sigma_A}{\tau}} = \lambda_A \left(\frac{N}{\langle N_A \rangle} - 1 \right)$$

$$\rightarrow \Sigma_A = \tau \ln \left\{ \lambda_A \left(\frac{N}{\langle N_A \rangle} - 1 \right) \right\}$$

$$= \tau \ln \left\{ 10^{-5} \left(\frac{1}{0{,}9} - 1 \right) \right\}$$

$$\approx -13.71 \cdot \tau = \simeq -13.71 \cdot k_B T$$

$$= -13.71 \cdot 8{,}617 \cdot 10^{-5} \frac{eV}{K} \cdot (273 + 37) \approx -0.366\, eV$$

b) Finna ε_B þ.a. aðeins 10% viðtaka séu setnir með O_2

Báðar gastegundir

Einn viðtaki

ekkert atóm $\quad O_2 \quad CO$

$$\mathcal{Z}_1 = 1 + \lambda_A e^{-\frac{\varepsilon_A}{\tau}} + \lambda_B e^{-\frac{\varepsilon_B}{\tau}}$$

N óháðir viðtakar

$$\mathcal{Z} = (\mathcal{Z}_1)^N = \left\{1 + \lambda_A e^{-\frac{\varepsilon_A}{\tau}} + \lambda_B e^{-\frac{\varepsilon_B}{\tau}}\right\}^N$$

$$\langle N_A \rangle = \lambda_A \frac{\partial}{\partial \lambda_A} \ln \left\{1 + \lambda_A e^{-\frac{\varepsilon_A}{\tau}} + \lambda_B e^{-\frac{\varepsilon_B}{\tau}}\right\}^N$$

$$= \lambda_A N \frac{e^{-\frac{\varepsilon_A}{\tau}}}{1 + \lambda_A e^{-\frac{\varepsilon_A}{\tau}} + \lambda_B e^{-\frac{\varepsilon_B}{\tau}}}$$

$$\rightarrow \quad \frac{\langle N_A \rangle}{N} = \frac{\lambda_A e^{-\frac{\varepsilon_A}{\tau}}}{1 + \lambda_A e^{-\frac{\varepsilon_A}{\tau}} + \lambda_B e^{-\frac{\varepsilon_B}{\tau}}}$$

$$\frac{N}{\langle N_A \rangle} = \frac{1 + \lambda_A e^{-\frac{\varepsilon_A}{\tau}} + \lambda_B e^{-\frac{\varepsilon_B}{\tau}}}{\lambda_A e^{-\frac{\varepsilon_A}{\tau}}} = \lambda_A^{-1} e^{\frac{\varepsilon_A}{\tau}} + \lambda_A + \frac{\lambda_B}{\lambda_A} e^{-\frac{\varepsilon_B - \varepsilon_A}{\tau}}$$

$$= \lambda_A^{-1} e^{\frac{\varepsilon_A}{\tau}} + \lambda_A + \frac{\lambda_B}{\lambda_A} e^{-\frac{\varepsilon_B}{\tau}} e^{+\frac{\varepsilon_A}{\tau}}$$

$$\rightarrow \quad \frac{N}{\langle N_A \rangle} - \lambda_A^{-1} e^{\frac{\varepsilon_A}{\tau}} - \lambda_A = \frac{\lambda_B}{\lambda_A} e^{\frac{\varepsilon_A}{\tau}} e^{-\frac{\varepsilon_B}{\tau}}$$

$$\rightarrow \quad \frac{\lambda_A}{\lambda_B} e^{-\frac{\varepsilon_A}{\tau}} \left\{ \frac{N}{\langle N_A \rangle} - \lambda_A^{-1} e^{\frac{\varepsilon_A}{\tau}} - \lambda_A \right\} = e^{-\frac{\varepsilon_B}{\tau}}$$

$$\Sigma_B = -\tau \ln\left[\frac{\lambda_A}{\lambda_B} e^{-\frac{\Sigma_A}{\tau}}\left\{\frac{N}{\langle N_A\rangle} - \lambda_A^{-1} e^{\frac{\Sigma_A}{\tau}} - \lambda_A\right\}\right]$$

$$= -\tau \ln\left[100\, e^{+\frac{0.366}{8.617\cdot 10^{-5}\cdot(273.15+37)}}\left\{\frac{1}{0.1} - 10^{5} e^{-\cdots} - 10^{-5}\right\}\right]$$

$$\approx -\tau \ln\left[100\cdot 8.862\cdot 10^{5}\left\{10 - \frac{1}{8.862} - 10^{-5}\right\}\right]$$

$$\approx -\tau\cdot 20.591 \approx -(273.15+37)\cdot 8.617\cdot 10^{-5}\cdot 20.591\ \text{eV}$$

$$\approx -0.55\ \text{eV}$$

Samanborið við -0.366 eV fyrir O_2 : $\Sigma_A = -0.366$ eV

ryður út O_2 ! $\Sigma_B = -0.55$ eV