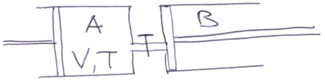
12-05

finna T (ekkert varmaflæði frá umhverfi)



A Bulla A föst, ventill opnaður
Bulla B dregin hægt út

'Overmið $đQ = 0$, kjörgas

$$dU = đQ + đW \rightarrow dU = dW$$

$$dU = C_V dT + \underbrace{\left(\frac{\partial U}{\partial V}\right)_T}_{=0} dT$$

$$đW = -p\,dV$$

$$\rightarrow C_V dT = -p\,dV = -\frac{RT}{V} dV$$

$$\rightarrow C_V \frac{dT}{T} = -R\frac{dV}{V}$$

$$C_V \int_T^{T_f} \frac{dT'}{T'} = -R \int_{V_A}^{V_A+V_B} \frac{dV'}{V'}$$

V_A+V_B → $2V$

$$C_V \ln\left(\frac{T_f}{T}\right) = -R\ln\left(\frac{2V}{V}\right)$$

$$C_V \ln\left(\frac{T}{T_f}\right) = R\ln(2)$$

$$\ln\left(\frac{T}{T_f}\right) = \frac{R}{C_V}\ln 2$$

Kjörgas $\rightarrow \gamma = \frac{5}{3}$ og $\frac{R}{C_V} = \gamma - 1 = \frac{2}{3}$

$$\ln\left(\frac{T}{T_f}\right) = (\gamma-1)\ln 2 \rightarrow \frac{T}{T_f} = 2^{\gamma-1}$$

$$\rightarrow T_f = \frac{T}{2^{\gamma-1}} = \frac{T}{2^{2/3}}$$

hitastigið lækkar
Innri orkan breytist

B

Bulla B dregin alveg út
Ventill opnað er aðeins

Bullu A ýtt
þ.a. p haldist fastur eins langt og gengur

Bullustrokkarnir leyfa
varmaflæði milli sín.

Þrýstingur í loka og upphafsástandi er sá sami

Rúmmál B verður V, en þenst inn í A við $V_A \neq V$ í A

<u>Upphaf</u>: $pV = RT$, T er hitinn í upphafi

<u>Lok</u>: $p(V+V_A) = RT_f$

<u>Varmaeinangrun frá umhverfi</u> $\longrightarrow dU = dW$

Engin vinna vegna hreyfingar bullu í B (engin hreyfing)

A þjappað

$$C_V \int_T^{T_f} dT = -p \int_V^{V_A} dV \longrightarrow C_V(T_f - T) = -p(V_A - V)$$

$U = U(T)$, ekki háð V

$$C_v(T_f - T) = +p(V - V_A)$$

Ástandsjöfnurnar gefa saman

$$\frac{V+V_A}{V} = \frac{T_f}{T} \rightarrow \frac{V_A}{V} = \frac{T_f}{T} - 1$$

$$C_v(T_f - T) = \frac{RT}{V}(V - V_A) = RT\left(1 - \frac{V_A}{V}\right)$$

$$\frac{C_v}{R}\left(\frac{T_f - T}{T}\right) = \left(1 - \frac{T_f}{T} + 1\right) = \left(2 - \frac{T_f}{T}\right)$$

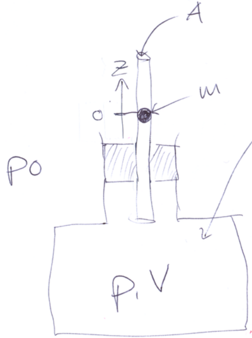
$$\frac{C_v}{R}(T_f - T) = (2T - T_f)$$

$$\frac{C_v}{R} = \frac{3}{2}$$

$$T_f\left(1 + \frac{C_v}{R}\right) = T\left(2 + \frac{C_v}{R}\right) \rightarrow T_f = \frac{7}{5}T$$

12-06

Hraðar sveiflur – óvermið ferli



$$p = p_0 + \frac{mg}{A}$$

"Örlítil hnikun kúlu um dz leiðir til rúmmálsbreytingar $dV = A\,dz$ og breytingar á kraftinum á kúluna $A\,dp = m\ddot{z}$

fyrir óvermið ferli gildir pV^{γ} = fasti

$$\to \frac{dp}{dV} = -\gamma\frac{p}{V} \quad \text{eða} \quad \frac{dp}{p} = -\gamma\frac{dV}{V}$$

$$m\ddot{z} = A\,dp = -A\gamma p\frac{dV}{V} = -A\gamma p\frac{A\,dz}{V}$$

$$\to \ddot{z} = -\frac{\gamma p A^2}{mV}dz$$

mælum z frá jafnvægisstöðunni

$$\rightarrow \quad \ddot{z} + \frac{\gamma p A^2}{mV} z = 0$$

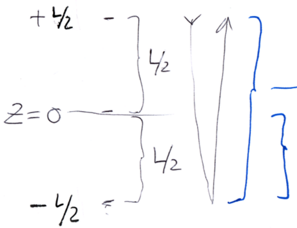
$$\ddot{z} + \omega^2 z = 0$$

með $\omega^2 = \dfrac{\gamma p A^2}{mV}$

$$\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mV}{\gamma p A^2}}$$

b) útslag kúlunnar frá punktinum þ.s. $p = p_0$

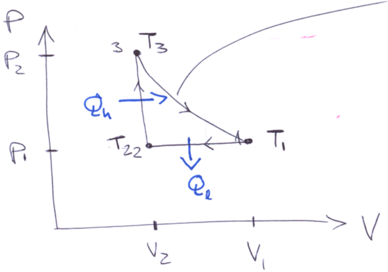
stöðuorka : $mgL = \frac{1}{2}\omega^2\left(\frac{L}{2}\right)^2$: fjaðurorkan



$$\rightarrow \quad mgL = \frac{1}{8}\frac{\gamma p A^2}{mV} L^2$$

$$\rightarrow \quad L = \left(\frac{8m^2Vg}{\gamma p A^2}\right)$$

13-04 Kjörgashringur



övermið $pV^{\gamma} = \text{fasti}$

$$Q_l = C_p (T_1 - T_2)$$

$$Q_h = C_v (T_3 - T_2)$$

$$\eta = \frac{W}{Q_h} = \frac{Q_h - Q_l}{Q_h} = 1 - \frac{Q_l}{Q_h}$$

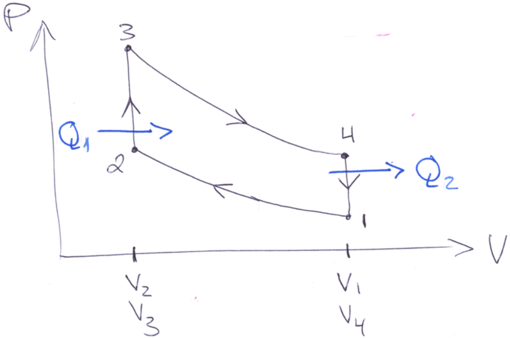
$$\rightarrow \eta = 1 - \frac{C_p}{C_v} \frac{T_1 - T_2}{T_3 - T_2}$$

fyrir 1->2 og 2->3 vatum við $pV \sim T$, það er ekki hægt fyrir 3->1, en þar er fertið övermið

$$= 1 - \underbrace{\frac{C_p}{C_v}}_{\gamma} \frac{p_1 V_1 - p_1 V_2}{p_2 V_2 - p_1 V_2} = 1 - \gamma \frac{V_1 - V_2}{\frac{p_2}{p_1} V_2 - V_2}$$

$$= 1 - \gamma \frac{\frac{V_1}{V_2} - 1}{\frac{p_2}{p_1} - 1}$$

Otto-hringurinn



$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1}$$

$$Q_1 = C_V (T_3 - T_2)$$

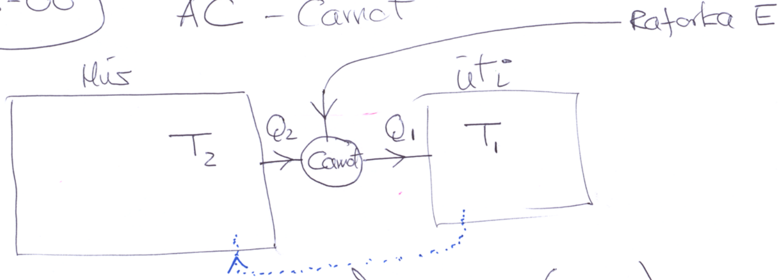
$$Q_2 = C_V (T_4 - T_1)$$

notum tengsl um óvermna breytinga $TV^{\gamma-1} = \text{fasti}$

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{\frac{C_2}{V_1^{\gamma-1}} - \frac{C_1}{V_1^{\gamma-1}}}{\frac{C_2}{V_2^{\gamma-1}} - \frac{C_1}{V_2^{\gamma-1}}}$$

$$= 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1} = 1 - \left(\frac{V_1}{V_2}\right)^{1-\gamma} = 1 - r^{1-\gamma}$$

13-06 AC - Carnot



Varmaleki inn í húsið $Q = A(T_1 - T_2)$

Staða stöðuga ástandið, þegar $Q = Q_2$

Orkuvarðveisla samkvæmt 1. lögmálinu $\to$ $Q_1 = E + Q_2$

Carnot vél

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

Finna jöfnu fyrir $T_2(T_1, E, A)$

Losa okkur við Q, Q_1 og Q_2

$$Q = Q_2$$
$$Q = A(T_1 - T_2)$$
$$Q_2 = A(T_1 - T_2)$$

$$Q_1 = E + Q_2 \longrightarrow Q_1 = E + A(T_1 - T_2)$$

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \longrightarrow \frac{A(T_1 - T_2)}{T_2} = \frac{E + A(T_1 - T_2)}{T_1}$$

$$AT_1(T_1 - T_2) - ET_2 - AT_2(T_1 - T_2) = 0$$

$$T_2^2 A + T_2\{-AT_1 - E - AT_1\} + \{AT_1^2\} = 0$$

$$T_2^2 A - T_2\{2AT_1 + E\} + AT_1^2 = 0$$

$$T_2^2 - T_2\left\{2T_1 + \frac{E}{A}\right\} + T_1^2 = 0$$

Sem hefur lausnir

$$T_2 = T_1 + \frac{E}{2A} \pm \sqrt{\left(\frac{E}{2A}\right)^2 + \frac{ET_1}{A}}$$

fyrir $T_1 = 30°C$ og $T_2 = 20°C$ er $E = 0,3\, E_{max}$

Notum $$T_2^2 - T_2\left\{2T_1 + \frac{E}{A}\right\} + T_1^2 = 0$$

sem $$T_2 - 2T_1 - \frac{E}{A} + \frac{T_1^2}{T_2} = 0$$

og $$\frac{E}{A} = -2T_1 + T_2 + \frac{T_1^2}{T_2} = T_1\left\{-2 + \frac{T_2}{T_1} + \frac{T_1}{T_2}\right\}$$

$$= 303\left\{-2 + \frac{293}{303} + \frac{303}{293}\right\} = 0,341$$

Notum síðan aftur $T_2^2 - T_2\left\{2T_1 + \frac{E}{A}\right\} + T_1^2 = 0$

til að finna

$$T_1 = T_2 \pm \frac{1}{2}\sqrt{(2T_2)^2 - 4T_2\left(-\frac{E}{A} + T_2\right)}$$

og reynum

$* \; 3,3333$

og fáum þá $T_1 \simeq 311.3\,K = 38.3\,°C$