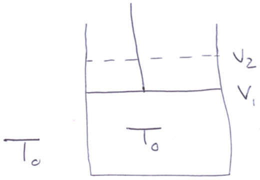
Kjørgas í strokki þenst hógt frá $V_1 \to V_2$ (eitt mól, $pV = RT$) við fast $T = T_0$




Hvers vegna breytist U ekki?

$$U = \frac{3}{2}RT \;\to\; \left(\frac{\partial U}{\partial V}\right)_T = 0$$

U er ekki fall af V

Reikna $\Delta W$ og $\Delta Q$

Vinna gassins $\quad dW = p\,dV \quad$ (á umhverfið)

$$\Delta W = \int_{V_1}^{V_2} dV\, p = RT_0 \int_{V_1}^{V_2} \frac{dV}{V} = RT_0 \ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta U = \Delta Q - \Delta W$$

vinna kerfisins á umhverfið

varma flæði inn í kerfið

$$\Delta U = 0 \longrightarrow \Delta Q = \Delta W = RT_0 \ln\left(\frac{V_2}{V_1}\right)$$

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11-02 Sýna að fyrir kjörgas gildi

$C_v, C_p$ eru varmarýmdir á mól

$$\frac{R}{C_v} = \gamma - 1 \quad \text{og} \quad \frac{R}{C_p} = \frac{\gamma - 1}{\gamma}$$

Höfum $C_p - C_v = R$ og $\gamma = \dfrac{C_p}{C_v}$

$$1 - \frac{C_v}{C_p} = \frac{R}{C_p} \longrightarrow 1 - \frac{1}{\gamma} = \frac{R}{C_p} \quad \text{eða} \quad \boxed{\frac{R}{C_p} = \frac{\gamma - 1}{\gamma}}$$

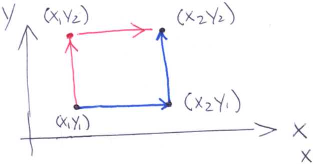
$$\frac{C_p}{C_v} - 1 = \frac{R}{C_v} \longrightarrow \boxed{\gamma - 1 = \frac{R}{C_v}}$$

11-03



$$df = 2xy\,dx + (x^2 + 2y)\,dy$$

(i) keildemervegar $(x_1 y_1) \to (x_2 y_1)$ og $(x_2 y_1) \to (x_2 y_2)$




(ii) $(x_1 y_1) \to (x_1 y_2)$ og $(x_1 y_2) \to (x_2 y_2)$

$$\text{(i)} \quad \int_i df = 2y_1 \int_{x_1}^{x_2} x\,dx + \int_{y_1}^{y_2} (x_2^2 + 2y)\,dy$$

$$= 2y_1 \frac{x_2^2 - x_1^2}{2} + x_2^2 \cdot (y_2 - y_1) + 2\frac{y_2^2 - y_1^2}{2}$$

$$= y_1 (x_2^2 - x_1^2) + x_2^2 (y_2 - y_1) + y_2^2 - y_1^2$$

$$= -y_1 x_1^2 + x_2^2 y_2 + y_2^2 - y_1^2$$

$$\int_{ii} df = \int_{y_1}^{y_2} (x_1^2 + 2y)\,dy + 2y_2 \int_{x_1}^{x_2} x\,dx$$

$$= x_1^2(\cancel{y_2} - y_1) + y_2^2 - y_1^2 + y_2(x_2^2 - \cancel{x_1^2})$$

$$= -x_1^2 y_1 + x_2^2 y_2 + y_2^2 - y_1^2$$

$$\longrightarrow \int_i dz = \int_{ii} dz$$

afleiðan er nákvæm

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Reynum

$$f(x,y) = x^2 y + y^2$$

$$\rightarrow \left(\frac{\partial f}{\partial x}\right)_y = 2xy$$

$$\left(\frac{\partial f}{\partial y}\right)_x = x^2 + 2y$$

$$\longrightarrow df = 2xy\,dx + (x^2 + 2y)\,dy$$

11-04



$$x = r\cos\theta \equiv x(r,\theta)$$
$$y = r\sin\theta \equiv y(r,\theta)$$

$$\longrightarrow \left(\frac{\partial x}{\partial r}\right)_\theta = \cos\theta = \frac{x}{r}$$

$$x^2 + y^2 = r^2 \longrightarrow x^2 = r^2 - y^2 \equiv x^2(r,y)$$

$$\longrightarrow 2x\left(\frac{\partial x}{\partial r}\right)_y = 2r \longrightarrow \left(\frac{\partial x}{\partial r}\right)_y = \frac{r}{x}$$

$$\longrightarrow \left(\frac{\partial x}{\partial r}\right)_\theta = \frac{1}{\left(\frac{\partial x}{\partial r}\right)_y} = \left(\frac{\partial r}{\partial x}\right)_y$$

ekkert rangt við það, þar sem afleiðurnar eru teknar með mismunandi stærðir.

11-05

sungið

varmi er vinna
vinna er varmi

ekki rétt því ekki er hægt að
breyta varma algerlega í vinnu

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