20-04 Hreintóna sveifill

$$Z = \frac{e^{-\frac{1}{2}\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}}$$

$$U = -\frac{d\ln Z}{d\beta} = \hbar\omega\left[\frac{1}{2} + \frac{1}{e^{\beta\hbar\omega}-1}\right]$$

$$F = -\frac{1}{\beta}\ln Z = \frac{\hbar\omega}{2} + \frac{\ln\left(1-e^{-\beta\hbar\omega}\right)}{\beta}$$

$$\beta\hbar\omega = \frac{\hbar\omega}{k_BT}$$ $\longrightarrow$ þegar $k_BT \ll \hbar\omega$ ætti aðeins grunnástandið að vera setið

$$\beta\hbar\omega \to \infty$$

grunnástandið hefur þessa orku

$$U \xrightarrow[\beta\hbar\omega\to\infty]{} \frac{\hbar\omega}{2}, \qquad F \xrightarrow[\beta\hbar\omega\to\infty]{} \frac{\hbar\omega}{2}$$

$$\frac{S}{k_B} = \frac{U-F}{k_B T} = \beta(U-F)$$

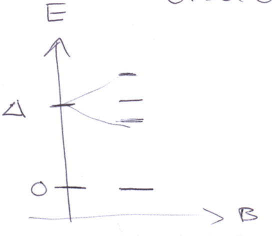
$$= \left\{ \frac{\beta\hbar\omega}{e^{\beta\hbar\omega}-1} - \ln\left(1-e^{-\beta\hbar\omega}\right) \right\}$$

og

$$\lim_{x\to\infty} \left\{ \frac{x}{e^{x}-1} - \ln\left(1-e^{-x}\right) \right\} = 0 \quad \to \frac{S}{k_B} \to 0$$

þegar $k_B T \ll \hbar\omega$

20-06 þéttleiki n óháðra sameinda með fjögur orkuástönd $0,\ \Delta - g\mu_B B,\ \Delta,\ \Delta + g\mu_B B$



finna Z, F og M, χ

$$n = \frac{N}{V}$$

fyrir eina sameind

$$Z = \sum_{\alpha} e^{-\beta E_\alpha} = \Big\{ 1 + \exp(-\beta\Delta) + \exp\big(-\beta(\Delta - g\mu_B B)\big) + \exp\big(-\beta(\Delta + g\mu_B B)\big) \Big\}$$

$$= \Big\{ 1 + \exp(-\beta\Delta)\big[1 + 2\cosh(\beta g \mu_B B)\big] \Big\}$$

$$U = -\frac{d}{d\beta} \ln Z^N$$ fyrir N óháðar sameindir

$$= -N \, \frac{-\Delta \exp(-\beta\Delta)\Big\{\big[1 + 2\cosh(\beta g\mu_B B)\big] + 2 g\mu_B B \sinh(\beta g \mu_B B)\Big\}}{1 + \exp(-\beta\Delta)\big[1 + 2\cosh(\beta g \mu_B B)\big]}$$

$$F = -\frac{1}{\beta}\ln Z^N = -\frac{N}{\beta}\ln\left\{1+\exp(-\beta\Delta)\left[1+2\cosh(\beta g\mu_B B)\right]\right\}$$ 4

$$m = -\left(\frac{\partial F}{\partial B}\right)_T = +\frac{N}{\beta}\frac{e^{-\beta\Delta}2\beta g\mu_B \sinh(\beta g\mu_B B)}{1+e^{-\beta\Delta}\left[1+2\cosh(\beta g\mu_B B)\right]}$$

$$M = \frac{m}{V} = n\frac{e^{-\beta\Delta}2g\mu_B \sinh(\beta g\mu_B B)}{1+e^{-\beta\Delta}\left\{1+2\cosh(\beta g\mu_B B)\right\}}$$

$$\lim_{B\to 0} M = \frac{n\, e^{-\beta\Delta}2g\mu_B(\beta g\mu_B B)}{1+3e^{-\beta\Delta}}$$

$\sinh(x) \simeq x + \frac{x^3}{6} + \dots$

$\cosh(x) \simeq 1 + \frac{x^2}{2} + \dots$

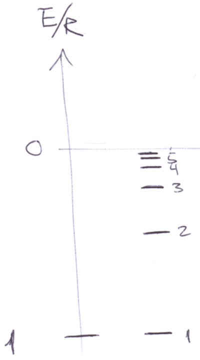
$$\chi = \lim_{B\to 0}\frac{\mu_0 M}{B} = \frac{\mu_0 n\, e^{-\beta\Delta}2g\mu_B\beta g\mu_B}{1+3e^{-\beta\Delta}} = \frac{2n\mu_0 g^2\mu_B^2 e^{-\beta\Delta}}{k_B T(1+3e^{-\beta\Delta})}$$

$$\boxed{\chi = \frac{2n\mu_0 g^2 \mu_B^2}{k_BT\left(3 + e^{\frac{\Delta}{k_BT}}\right)}}$$

eins og ~~bedið~~ ver um

20-08 Vetnis atóm $E = -\frac{R}{n^2}$, $R = 13.6$ eV

með margfeldni $2n^2$. strjála orkuróf bundinna ástanda. Utar er til samfellt róf með $E > 0$



$$Z = \sum_\alpha e^{-\beta E_\alpha} = \sum_{n=1}^{\infty} 2n^2 \exp\left\{\frac{R}{n^2 k_BT}\right\}$$

Nálgum

$$Z \simeq \sum_{n=1}^{2} 2n^2 \exp\left\{\frac{R}{n^2 k_BT}\right\}$$

$$\langle E \rangle = -\frac{2Re^{\beta R} + \frac{8R}{4}e^{\frac{\beta R}{4}}}{2e^{\beta R} + 8e^{\frac{\beta R}{4}}}$$

Við $T = 300K$
er $\beta = 25.9$ meV
$\rightarrow \beta R = \frac{13.6}{0{,}0259} \simeq 525{,}1$

$= 13.600$ eV

hverfandi örvun úr grunnástandinu

05-03 Hve mikil skekkja er að nota $\langle v \rangle$ í stað $\sqrt{\langle v^2 \rangle}$

$$\langle v \rangle = \sqrt{\frac{8k_BT}{\pi m}} \qquad \langle v^2 \rangle = \frac{3k_BT}{m}$$

$$\rightarrow \frac{\langle v \rangle}{\sqrt{\langle v^2 \rangle}} = \sqrt{\frac{8}{\pi \cdot 3}} \simeq 0{,}922$$

7,8 %

05-04 vitumoe $\langle v \rangle = \sqrt{\dfrac{8k_BT}{\pi m}}$

$$f(v)dv = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2k_BT}\right)^{3/2} v^2 dv \, e^{-mv^2/(2k_BT)}$$

Fimmmm $\langle 1/v \rangle$

$$\left\langle \frac{1}{v} \right\rangle = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2k_BT}\right)^{3/2} \int_0^\infty dv \, v \, e^{-mv^2/(2k_BT)}$$

$$= \frac{4}{\sqrt{\pi}} \left(\frac{m}{2k_BT}\right)^{3/2} \cdot \frac{k_BT}{m} = \frac{4}{\sqrt{\pi}\sqrt{8}} \sqrt{\frac{m}{k_BT}} = 4\sqrt{\frac{m}{\pi k_BT \cdot 8}}$$

$$\rightarrow \langle v \rangle \left\langle \frac{1}{v} \right\rangle = 4\sqrt{\frac{8k_BT}{\pi m}} \sqrt{\frac{m}{8\pi k_BT}} = \frac{4}{\pi}$$