2-1 Gerum ráð fyrir að $g(U) = CU^{3N/2}$, C er fasti

Kjörgas N : fjöldi einda

a) Sýna að $U = \frac{3}{2}N\tau$

Höfum $\frac{1}{\tau} = \left(\frac{\partial \sigma}{\partial U}\right)_N$ og $\sigma(N,U) = \ln\{g(N,U)\}$

$$\rightarrow \sigma = \ln\{CU^{3N/2}\} = \ln C + \frac{3N}{2}\ln U$$

$$\frac{1}{\tau} = \left(\frac{\partial \sigma}{\partial U}\right)_N = \frac{3N}{2}\cdot\frac{1}{U} \rightarrow U = \frac{3}{2}N\tau$$

Vex línulega með N og τ

b) sýna að $\left(\frac{\partial^2 \sigma}{\partial U^2}\right)_N < 0$

$$\left(\frac{\partial^2 \sigma}{\partial U^2}\right)_N = \left(\frac{\partial}{\partial U}\left(\frac{\partial \sigma}{\partial U}\right)_N\right)_N = \left(\frac{\partial}{\partial U}\left\{\frac{3N}{2U}\right]\right)_N =$$

$$= -\frac{3}{2}\frac{N}{U^2} < 0$$

Óreiðan er íhvolft fall af U

(2-2)

<u>Meðseglun</u>, N spunar með segulvægi m í segulsviði B. $N_\uparrow - N_\downarrow = 2s$

finna jafnvægisgildi hlutfallsseglunarinnar

$$\boxed{\frac{M}{Nm} = 2\langle s\rangle \frac{1}{N}}$$

nota

$$g(N,s) \simeq g(N,0)\exp\left\{-\frac{2s^2}{N}\right\}$$

$$\to \sigma(N,s) = \ln\{g(N,s)\} \simeq \ln(g(N,0)) - \frac{2s^2}{N}$$

því $|s| \ll N$ og $N \gg 1$

Hér gildir $U = -2smB \rightarrow s = -\frac{U}{2mB}$

$$\rightarrow \sigma(N,U) \approx \sigma_0 - \frac{2s^2}{N} = \sigma_0 - \frac{U^2}{2m^2B^2N}$$

með $\sigma_0 = \ln(g(N,0))$

$$\frac{1}{\tau} = \left(\frac{\partial\sigma}{\partial U}\right)_N = -\frac{U}{m^2B^2N}$$

$$\rightarrow U = -\frac{m^2B^2N}{\tau}$$

munum að τ hefur vídd orku hér

$$s = -\frac{U}{2mB} \rightarrow \langle s \rangle = -\frac{\langle U \rangle}{2mB}$$

og $$\langle U \rangle = -\frac{m^2B^2N}{\tau}$$

$$\rightarrow \langle s \rangle = \frac{m^2B^2N}{\tau\, 2mB} = \frac{mBN}{2\tau}$$

og þá

$$\boxed{\frac{M}{Nm} = 2\langle s \rangle \frac{1}{N} = \frac{mB}{\tau}}$$

→ síðar finnum við að nákvæmlega gildi

$= \tanh\left(\frac{mB}{\tau}\right)$

að þegar

$\tau \gg mB$

Curie lögmálið fyrir meðseglum

gildir fyrir ekki of lágt τ

2-3 QHO, N - HO með tíðni ω
n - orkuskammtar

finna σ

jafna (1.55) gefur $g(N,n) = \frac{(N+n-1)!}{n!\,(N-1)!}$

$U = \hbar\omega n$ ← óháðir sveiflar

notum $\ln N! \simeq N\ln N - N$

$$\rightarrow \sigma(N,n) = \ln\left\{\frac{(N+n-1)!}{n!\,(N-1)!}\right\} = \ln(N+n-1)! - \ln n! - \ln(N-1)!$$

$$\simeq (N+n)\ln(N+n) - (N+n) - n\ln n + n$$

$$- N\ln N + N = \underline{(N+n)\ln(N+n) - N\ln N - n\ln n}$$

b) $U = \hbar\omega \cdot n$ fima $\sigma(U,N)$

$$\boxed{n = \frac{U}{\hbar\omega}} \rightarrow \sigma(U,N) = \left(\frac{U}{\hbar\omega}+N\right)\ln\left(\frac{U}{\hbar\omega}+N\right) - N\ln N - \left(\frac{U}{\hbar\omega}\right)\ln\left(\frac{U}{\hbar\omega}\right)$$

$$\frac{1}{\tau} = \left(\frac{\partial\sigma}{\partial U}\right)_N = \frac{1}{\hbar\omega}\ln\left(\frac{U}{\hbar\omega}+N\right) + \left(\frac{U}{\hbar\omega}+N\right)\frac{1}{\left(\frac{U}{\hbar\omega}+N\right)}\frac{1}{\hbar\omega}$$

$$-\ln\left(\frac{U}{\hbar\omega}\right)\frac{1}{\hbar\omega} - \left(\frac{U}{\hbar\omega}\right)\frac{1}{\left(\frac{U}{\hbar\omega}\right)}\frac{1}{\hbar\omega}$$

$$= \frac{1}{\hbar\omega}\left\{\ln\left(\frac{U}{\hbar\omega}+N\right) + 1 - \ln\left(\frac{U}{\hbar\omega}\right) - 1\right\}$$

$$= \frac{1}{\hbar\omega}\left\{\ln\left(\frac{U}{\hbar\omega}+N\right) - \ln\left(\frac{U}{\hbar\omega}\right)\right\}$$

$$\rightarrow \frac{\hbar\omega}{\tau} = \ln\left\{\frac{\left(\frac{U}{\hbar\omega}+N\right)}{\frac{U}{\hbar\omega}}\right\} \rightarrow \exp\left(\frac{\hbar\omega}{\tau}\right) = \frac{\frac{U}{\hbar\omega}+N}{\frac{U}{\hbar\omega}}$$

$$\rightarrow \exp\left(\frac{\hbar\omega}{\tau}\right) = \frac{U + N\hbar\omega}{U} = 1 + \frac{N\hbar\omega}{U}$$

$$\rightarrow \exp\left(\frac{\hbar\omega}{\tau}\right) - 1 = \frac{N\hbar\omega}{U}$$

og

$$\boxed{U = \frac{N\hbar\omega}{\exp\left(\frac{\hbar\omega}{\tau}\right)-1}}$$