03-04

$$P(x)\,dx = A e^{-x/\lambda}\,dx\,, \qquad x \geq 0$$

(a) Finna A, ~~staðla~~

$$A\int_0^\infty dx\, e^{-x/\lambda} = A\lambda\int_0^\infty \frac{dx}{\lambda}\, e^{-\frac{x}{\lambda}} = A\lambda\int_0^\infty du\, e^{-u}$$

$$= -A\lambda e^{-u}\Big|_0^\infty = A\cdot 1\cdot\lambda$$

$$\rightarrow \quad A = \frac{1}{\lambda} \quad \text{og} \quad P(x) = \frac{1}{\lambda}e^{-\frac{x}{\lambda}}$$

(b) Meðalgildi $\langle x\rangle$

$$\langle x\rangle = \frac{1}{\lambda}\int_0^\infty dx\, x\, e^{-\frac{x}{\lambda}} = \lambda\int_0^\infty \frac{dx}{\lambda}\,\frac{x}{\lambda}\,e^{-\frac{x}{\lambda}}$$

$$\langle x \rangle = \lambda \int_0^\infty du\, u\, e^{-u} = \lambda$$

(c) Standardavvik

$$\langle x^2 \rangle = \frac{1}{\lambda} \int_0^\infty dx\, x^2\, e^{-\frac{x}{\lambda}} = \lambda^2 \int_0^\infty \frac{dx}{\lambda} \left(\frac{x}{\lambda}\right)^2 e^{-\frac{x}{\lambda}}$$

$$= \lambda^2 \int_0^\infty du\, u^2\, e^{-u} = \lambda^2 \cdot 2$$

$$\rightarrow \sigma_x = \sqrt{(\langle x^2 \rangle - \langle x \rangle^2)} = \sqrt{2\lambda^2 - \lambda^2} = \lambda$$

03-05

fyrir $\theta \in [0, \pi]$ einsleitdreifing á bilinu

$\rightarrow \quad P(\theta) = \frac{1}{\pi} \quad$ fyrir $\theta \in [0, \pi]$

$$\int_0^\pi P(\theta)\, d\theta = 1$$

(a) $\langle \theta \rangle = \frac{1}{\pi} \int_0^\pi d\theta\, \theta = \frac{1}{\pi} \frac{\theta^2}{2} \Big|_0^\pi = \frac{1}{\pi} \frac{\pi^2}{2} = \frac{\pi}{2}$

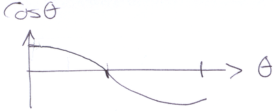
(b) $\langle \theta - \frac{\pi}{2} \rangle = \langle \theta \rangle - \frac{\pi}{2} = 0$

(c) $\langle \theta^2 \rangle = \frac{1}{\pi} \int_0^\pi d\theta\, \theta^2 = \frac{1}{\pi} \frac{\theta^3}{3} \Big|_0^\pi = \frac{1}{\pi} \frac{\pi^3}{3} = \frac{\pi^2}{3}$

(d) $\langle \theta^n \rangle = \frac{\pi^n}{(n+1)}$ fyrir $n \geq 0$

(4)

(e) $$\langle \cos\theta \rangle = \frac{1}{\pi}\int_0^{\pi} d\theta \cos\theta = 0$$



oddstætt á bilinu

(f) $$\langle \sin\theta \rangle = \frac{1}{\pi}\int_0^{\pi} d\theta \sin\theta = \frac{2}{\pi}$$ jafnstætt

(g) $$\langle |\cos\theta| \rangle = \frac{1}{\pi}\left[\int_0^{\pi/2} d\theta \cos\theta - \int_{\pi/2}^{\pi} d\theta \cos(\theta)\right] = \frac{2}{\pi}$$

(h) $$\langle \cos^2\theta \rangle = \frac{1}{\pi}\int_0^{\pi} d\theta \cos^2\theta = \frac{1}{2}$$

verður að vera

(i) $$\langle \sin^2\theta \rangle = \frac{1}{2},$$

(j) $$\langle \underbrace{\cos^2\theta + \sin^2\theta}_{=1} \rangle = 1$$

03-08 vogisframleiðandi fall

$$M(t) = \langle e^{tx} \rangle$$

$$\frac{d^n M(t)}{dt^n} = \langle x^n e^{tx} \rangle \longrightarrow \left.\frac{d^n M(t)}{dt^n}\right|_{t=0} = \langle x^n \cdot 1 \rangle = \langle x^n \rangle$$

Þá táknum sem $\langle x^n \rangle = \underline{M^{(n)}(0)}$

$$\longrightarrow \langle x \rangle = M^{(1)}(0)$$

$$\sigma_x^2 = M^{(2)}(0) - \left\{ M^{(1)}(0) \right\}^2 = \langle x^2 \rangle - \langle x \rangle^2$$

(a) Bernoulli tilraun: p Þá $1-p$

Ef $M(t) = pe^t + 1 - p \longrightarrow \langle x \rangle = p$

$M^{(1)}(t) = pe^t$, $M^2(t) = pe^t$ $\quad \langle x^2 \rangle = p$

(b) Tvíliðadreifing

$$M(t) = (pe^t + 1 - p)^n$$

$$M^{(1)}(t) = n(pe^t + 1 - p)^{n-1} \cdot pe^t \rightarrow M^{(1)}(0) = np = \langle k \rangle$$

$$M^{(2)}(t) = (n-1)n(pe^t + 1 - p)^{n-2} \cdot pe^t \cdot pe^t$$
$$+ n(pe^t + 1 - p)^{n-1} \cdot pe^t$$

$$\rightarrow M^{(2)}(0) = (n-1)np^2 + np$$

$$M^{(2)}(0) - \{M^{(1)}(0)\}^2 = (n-1)np^2 + np - n^2p^2$$
$$= np - np^2 = np(1-p)$$
$$= \sigma_k^2$$

(d) $$M(t) = \frac{\lambda}{\lambda - t}$$

$$M^{(1)}(t) = \frac{+\lambda}{(\lambda - t)^2} \longrightarrow M^{(1)}(0) = \frac{1}{\lambda}$$

$$M^{(2)}(t) = \frac{2\lambda}{(\lambda - t)^3} \longrightarrow M^{(2)}(0) = \frac{2}{\lambda^2}$$

Hér fæst greinilega ekki $\langle x \rangle = M^{(1)}(0)$

Giska á að fallið þurfi að vera

$$M(t) = \frac{1/\lambda}{1/\lambda - t} \longrightarrow M^{(1)}(t) = \frac{1/\lambda}{(1/\lambda - t)^2} \longrightarrow M^{(1)}(0) = \lambda$$

$$M^{2}(t) = \frac{2 \cdot 1/\lambda}{(1/\lambda - t)^3} \longrightarrow M^{(2)}(0) = 2\lambda^2$$

þá passar allt

7b

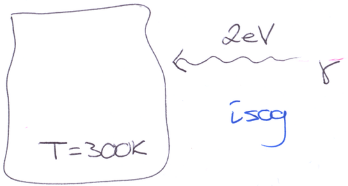
Ég sannreyndi bara, en leiddi ekki út $M(t)$
Botnum úr því fyrir (d)

$$P(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}, \qquad M(t) = \langle e^{tx} \rangle$$

$$\rightarrow M(t) = \frac{1}{\lambda}\int_0^\infty dx\, e^{tx} e^{-\frac{x}{\lambda}} = \frac{1}{\lambda}\int_0^\infty dx\, e^{x(t-\frac{1}{\lambda})}$$

$$= \boxed{\frac{\lambda^{-1}}{\lambda^{-1} - t}} \quad \text{ef} \quad t > \frac{1}{\lambda}$$

04-05



hvernig breytist Ω fyrir stórsæja hlutinn

Notum (4.12) $\Omega(E-\epsilon) = \Omega(E)\, e^{-\epsilon/k_B T}$

og snúum við

$$\frac{\Omega(E)}{\Omega(E-\epsilon)} = e^{\epsilon/k_B T} = \exp\left\{\frac{2\,eV}{8.617\cdot 10^{-5}\,\frac{eV}{K}\, 300K}\right\}$$

$$\simeq 4\cdot 10^{33}$$

fyrir γ ~~með~~ 100 MHz $\longrightarrow$ $E = h\nu = 4.135\cdot 10^{-15}\, eVs \cdot 100\cdot 10^{6}\,\frac{1}{s}$

$\sim 4.135\cdot 10^{-7}\, eV$

$$\longrightarrow \frac{\Omega(E)}{\Omega(E-\epsilon)} \simeq 1.00$$

04-08

$k_B T \sim 25{,}9$ meV við $T = 300K$

eða $0{,}0259$ eV

(a) Jönunarorka H er 13.6 eV

→ engin jónun við $T = 300K$

(b) 10^{-4} eV eða $0{,}1$ meV þarf til að auka snúning sameinda → við $T = 300\,K$ er hring-snúningar sameinda mjög örvaðar