3. project

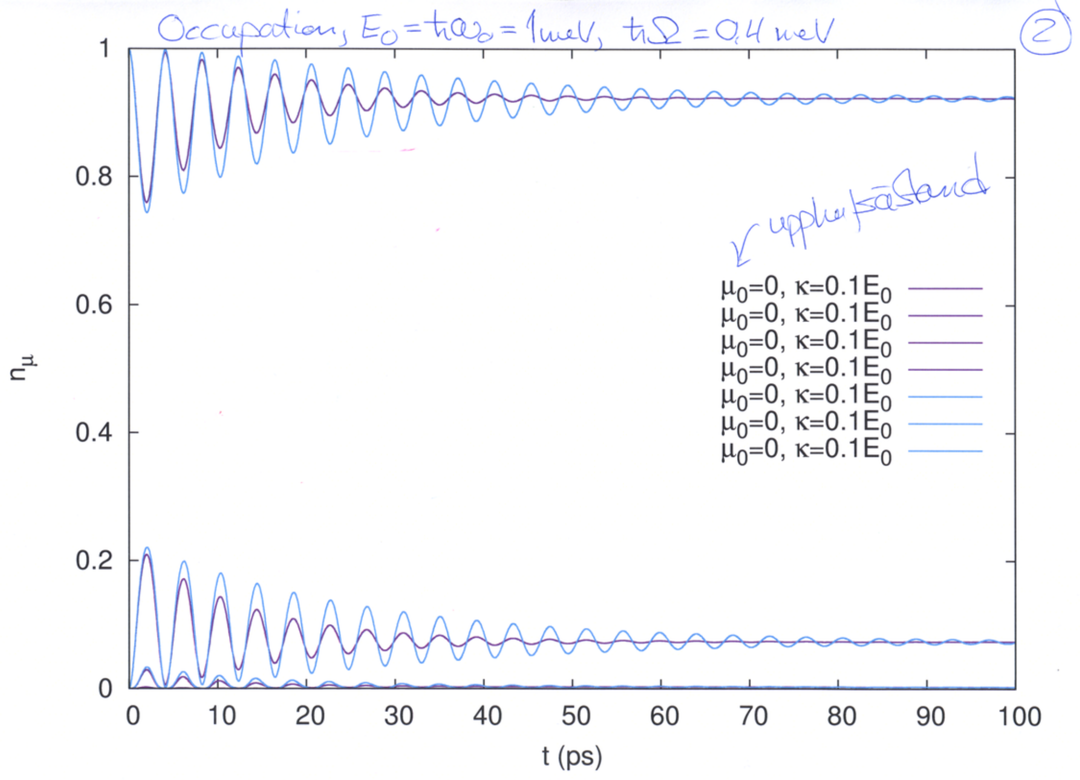
We revisit the damped harmonic oscillator in projet 2:

$$i\hbar\dot{\rho} = [H,\rho] - \frac{i\kappa}{2}\left\{[a\rho,a^+] + [a,\rho a^+]\right\} \qquad (1)$$

$$H(t) = H_0 + H'(t)\ , \quad H_0 = \hbar\omega\left\{a^+a + \frac{1}{2}\right\}$$

$$H'(t) = \hbar\Omega\left\{a^+ + a\right\}\theta(t)$$

There we solved Eq. (1) to find the time-evolution of $\rho(t)$. With the finite damping we saw that the solution tends to a steady state as $t \to \infty$.


Occupation, $E_0 = \hbar\omega_0 = 1$ meV, $\hbar\Omega = 0.4$ meV
2
uppluppsättand
$\mu_0=0, \kappa=0.1E_0$
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$\mu_0=0, \kappa=0.1E_0$
n_μ
t (ps)
0
0.2
0.4
0.6
0.8
1
0
10
20
30
40
50
60
70
80
90
100

The question now is if we can find the steady-state directly without doing the time-integration?

We can rewrite the dissipative eq. of motion (1) as

$$\dot{\rho}(t) = \Lambda[\rho(t)]$$

In the steady state $\dot{\rho} = 0$, and we need to solve

$$0 = \Lambda[\rho(t)]$$

As we have

$$i\Lambda[\rho] = \frac{1}{\hbar}[H,\rho] - \frac{i\kappa}{2\hbar}\left\{[a\rho,a^{+}] + [a,\rho a^{+}]\right\}$$

Our steady state equation can be written as

$$A\rho + \rho B = C \quad (2)$$

with $C=0$ ← homogeneous

which is Lyapunov's equation, or the sylvester equation.

Numerically, it is the ~~best~~ practice here to solve it directly, which can be accomplished by applying several Lapack-routines related to linear algebra transformations and generalized eigenvalue problems

But for fun let's try a different method here, that will introduce to us some new matrix manipulations worth to know.

In the Matrix Cookbook a solution is given for (2) in a <u>N^2-space</u> if the original matrices have dimension N.

$$\mathrm{vec}(\rho) = (I \otimes A + B^T \otimes I)^{-1} \mathrm{vec}(C)$$

where <u>vectorization</u> of matrices together with the tensorproduct of Kronecker is used.

More cleverly we have now to solve <u>the linear equation</u>

$$\{I \otimes A + B^T \otimes I\} \mathrm{vec}(\rho) = \mathrm{vec}(C) = 0$$

↑ here

The best way is thus to use the routine GEEV for nonsymmetric complex eigenvalue problem to find the complex eigenvalues. <u>There should be a zero-eigenvalue with the steady state ρ_s as an eigenvector</u>

In the next page we see how the calculation for the steady state agree.

Do this calculation, and find some interesting properties to explore further.

If in liouville-space we use the notation $\dot{\rho} = -i\mathbb{L}\rho$ then we get now left eigenvectors U and right eigenvectors V

$$\mathbb{L}V = V\lambda$$
$$U\mathbb{L} = \lambda U$$

where λ is a diagonal matrix containing the eigenvalues

U. Hohenester, PRB 81, 155303 (2010)

And $V U^{+} = U^{+} V = \mathbb{1}$

so, in case H is not t-dependent, except through $\theta(t)$ we have the direct evolution

$$\rho(t) = \left(V \exp\{-i\lambda t\} U^{+}\right) \rho_0 \quad \text{in Liouville space}$$

There is a small technical problem when using GEEV due to normalization of eigenvectors, the solution there to will be discussed in lecture

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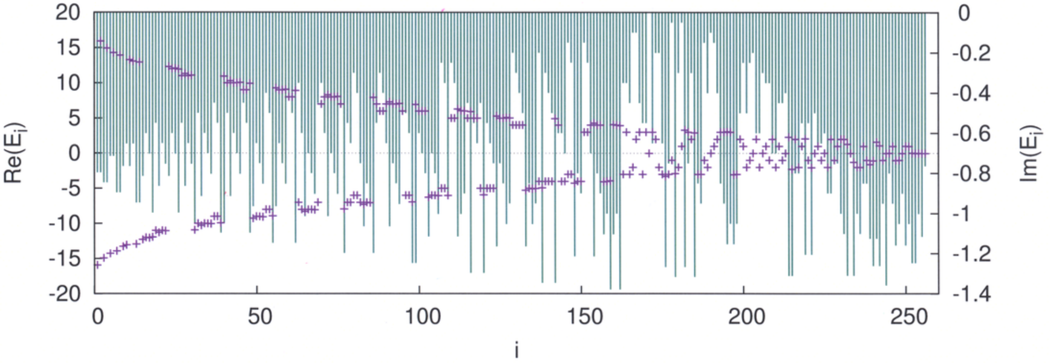
Try this time evolution and compare it to the one in project (2)

The eigen spectrum of the damped oscillator

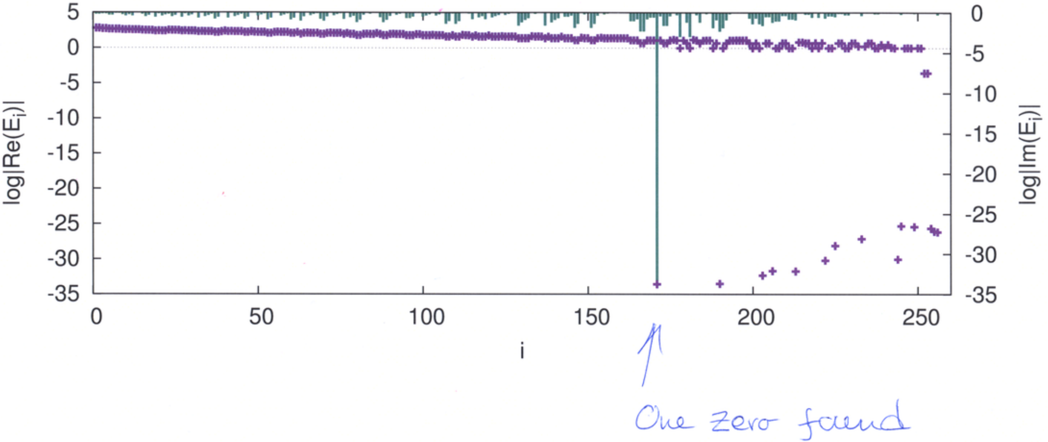
$\hbar\omega = 1.0$ meV, $\hbar\Omega = 0.4$ meV, $K = 0.2$ meV

(8)

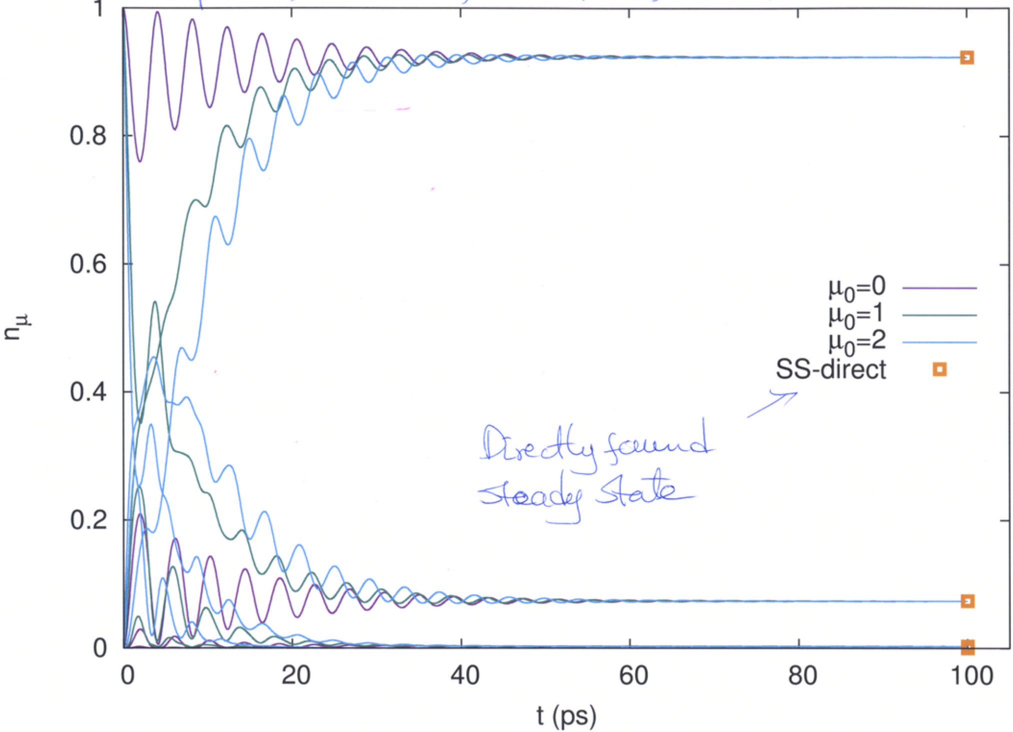


The eigenspectrum of the damped oscillator

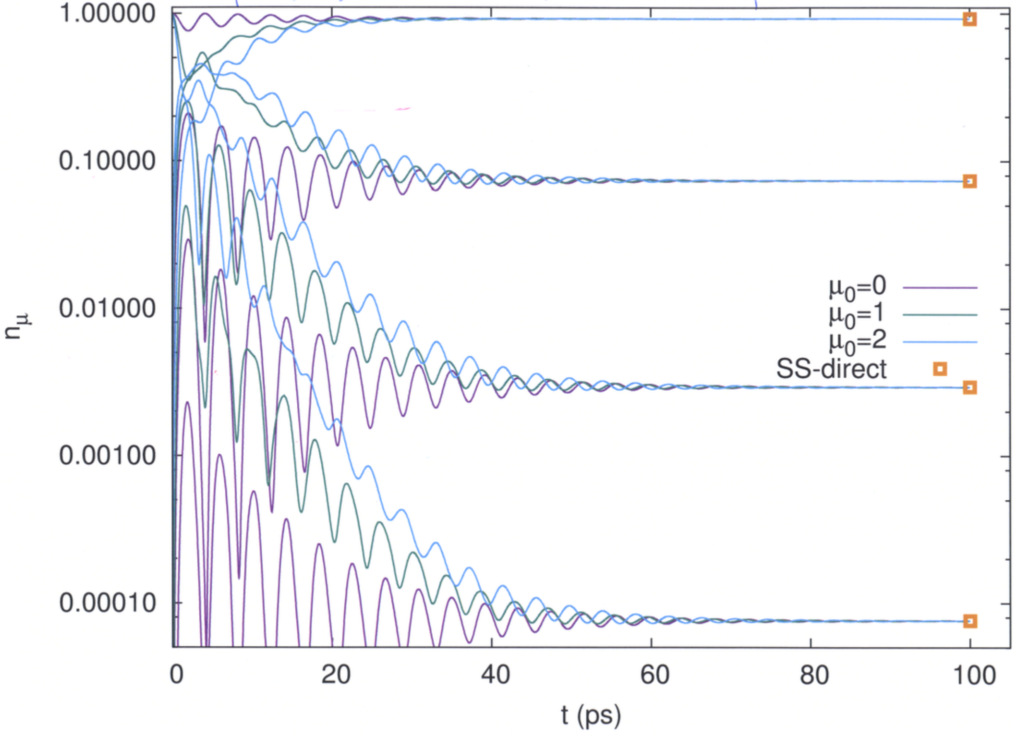
$\hbar\omega = 1.0$ meV, $\hbar\Omega = 0.4$ meV, $\kappa = 0.2$ meV



Occupation, $\hbar\omega = 1.0$ meV, $\hbar\Omega = 0.4$ meV, $\kappa = 0.2$ meV



Occupation, $\hbar\omega$ = 1.0 meV, $\hbar\Omega$ = 0.4 meV, K = 0.2 meV



Occupation, $\hbar\omega = 1{,}0$ meV, $\hbar\Omega = 0{,}4$ meV, $\kappa = 0{,}2$ meV