2. project

We continue with the harmonic oscillator

$$H_0 = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 , \qquad [x, p] = i\hbar$$

or

$$H_0 = \hbar\omega \left\{ a^\dagger a + \frac{1}{2} \right\}$$

in the book of Griffiths

$a^\dagger = a_+$
$a = a_-$

We use the more standard notation here

At $t=0$ it finds itself all of a sudden in an external „static" electric field

$$H(t) = H_0 + H'(t)$$

with

Heaviside step function

$$H'(t) = \hbar\Omega \left\{ a^\dagger + a \right\} \theta(t)$$

In http://hartree.raunvis.hi.is/~vidar/Nam/IaSF/22F.pdf you can find an exact analytical solution for the state $|\Psi(t)\rangle$

Here we want to use numerical methods to find information about the time evolution of the system. We want to use a method that allows us later to add new terms to this model. We will solve the Liouville-von Neumann equation

$$i\hbar\dot{\rho}(t) = [H(t), \rho(t)]$$

for the density or probability operator $\rho(t)$. When we have $\rho(t)$ it's diagonal elements give us the occupation of the states of the H.O. The mean value of its location is for example

$$\langle x \rangle = \mathrm{tr}\{x\rho\} = \sum_n \langle n|x\rho|n\rangle$$

$$= \sum_{nm} \langle n|x|m\rangle\langle m|\rho|n\rangle$$

the trace of the matrices of x and ρ multiplied.

Usually the best solution method is to find the unitary time-evolution operator $T(t)$

$$\rho(t) = T(t)\,\rho(0)\,T^{+}(t)$$

substitution into L-vN-equation then gives

$$i\hbar\,\dot{T}(t) = H(t)\,T(t)$$

$$-i\hbar\,\dot{T}^{+}(t) = T^{+}(t)\,H(t)$$

but we want later to add terms that might not lead to unitary evolution, so we solve directly the L-vN-eq.

$$i\hbar\,\dot{\rho}(t) = [H(t), \rho(t)] \quad = \underbrace{i\Lambda[\rho(t)]}$$

by introducing a time grid

Λ or felli of $\rho(t)$ is a functional

and use the Cranck-Nicolson method that is good for hermitian operators. The time grid suggests 2 equations

$$① \quad \left\{ \rho(t_{n+1}) - \rho(t_n) \right] \simeq \frac{\Delta t}{\hbar} \Lambda \left[\rho(t_n) \right]$$

$$② \quad \left\{ \rho(t_n) - \rho(t_{n-1}) \right\} \simeq \frac{\Delta t}{\hbar} \Lambda \left[\rho(t_n) \right]$$

forward and backward time steps

$$① \rightarrow \rho(t_{n+1}) \simeq \left\{ \rho(t_n) + \frac{\Delta t}{\hbar} \Lambda \left[\rho(t_n) \right] \right\}$$

$$② \rightarrow \left\{ \rho(t_{n+1}) - \rho(t_n) \right\} \simeq \frac{\Delta t}{\hbar} \Lambda \left[\rho(t_{n+1}) \right]$$

$$\rightarrow \left\{ \rho(t_{n+1}) - \frac{\Delta t}{\hbar} \Lambda \left[\rho(t_{n+1}) \right] \right\} \simeq \rho(t_n)$$

Við leggjum saman jöfnurnar og deilum með 2

$$\to \underbrace{\left\{\rho(t_{n+1}) - \frac{\Delta t}{2\hbar}\Lambda\left[\rho(t_{n+1})\right]\right\}}_{\text{Framtíð}} = \underbrace{\left\{\rho(t_n) + \frac{\Delta t}{2\hbar}\Lambda\left[\rho(t_n)\right]\right\}}_{\text{nútíð}}$$

known

and interprete as

$$\rho(t_{n+1}) = \rho(t_n) + \frac{\Delta t}{2\hbar}\left\{\Lambda\left[\rho(t_n)\right] + \Lambda\left[\rho(t_{n+1})\right]\right\}$$

unknown

want to find

We scale time $\frac{\Delta t}{\hbar}$ appropriately by comparison to $\hbar\omega$ and select small steps. We need to <u>iterate</u> the equation in <u>each time step</u>

a) Find the time-evolution of the occupation of few lowest states. Check accuracy with respect to $\frac{\hbar\Omega}{\hbar\omega}$

b) Find $\langle x\rangle(t)$, $\langle x^2\rangle(t)$.

I add here a graph for a).

Because we are working with the L-vN-equation it is possible to add dissipation to our model.
We use a common method for the H.O., see for example
L.S. Bishop, E. Ginossar, and S.M. Girvin, PRL 105, 100505 (2010)

$$i\hbar\dot{\varrho} = [H,\varrho] - i\frac{\kappa}{2}\left\{[a\varrho,a^{+}] + [a,\varrho a^{+}]\right\}$$

This damping is caused by coupling to an external reservoir at a high temperature
κ is the damping strength

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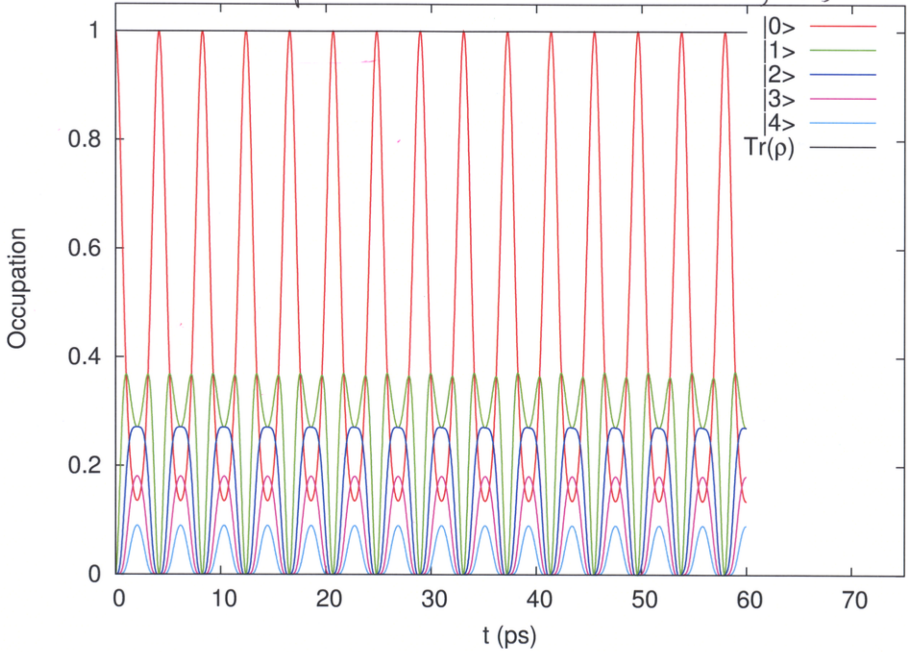
No dissipation $\hbar\omega = \hbar\Omega = 1\,meV$, $\omega_f = 16$



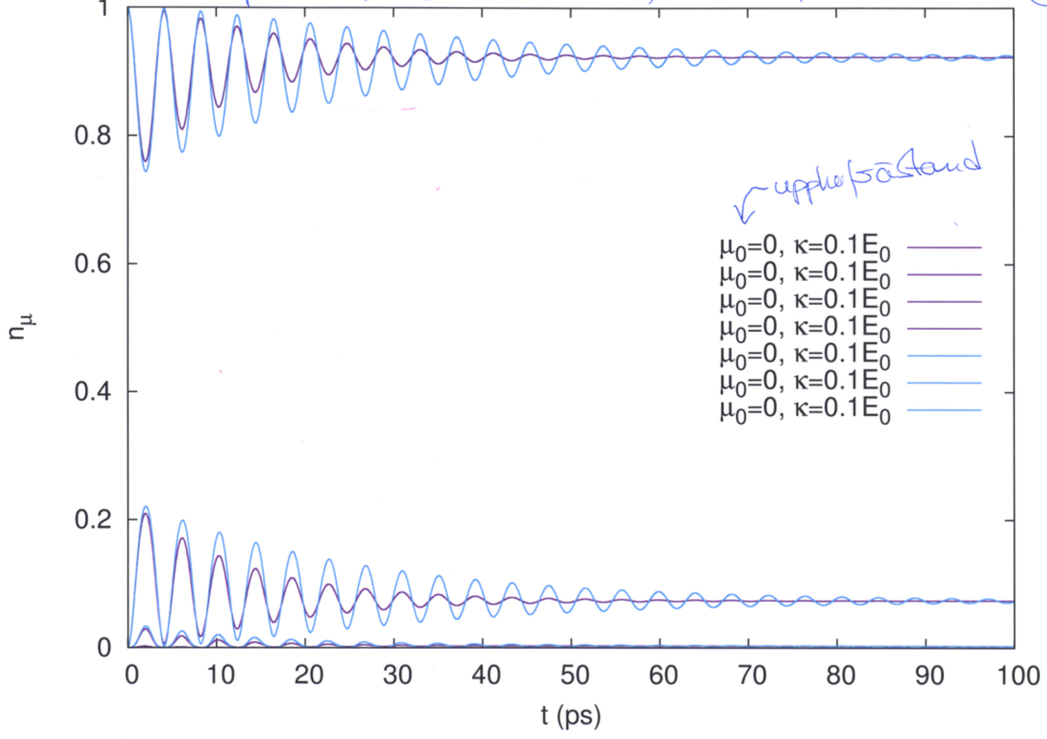
c) Find the time-evolution of the occupation like in a)

d) Find again $\langle x \rangle (t)$ and $\langle x^2 \rangle (t)$

The dissipation only changes $\mathcal{L}[\rho(t)]$ in the model.

Here are some results

Occupation, $E_0 = \hbar\omega_0 = 1\,\text{meV}$, $\hbar\Omega = 0{,}4\,\text{meV}$



Occupation, $E_0 = \hbar\omega_0 = 1$ meV, $\hbar\Omega = 0.4$ meV