Háskóli Íslands
Raunvísindadeild
Eðlisfræðiskor

09.21.52 Safneðlisfræði

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Leyfileg hjálpargögn eru: Vasatölva, kennslubók og stærðfræðihandbækur.

Select any 4 of the 5 problems:

1. A monoatomic gas consists of atoms with two energy levels: a ground state of degeneracy g_1 and a low-lying excited state of degeneracy g_2 at an energy E above the ground state. Find the specific heat of this gas.

2. Consider a classical system whose Hamiltonian can be expressed as $H = H_0 + \lambda H_1$, where $\lambda << 1$. Show that the expansion of the Helmholtz free energy in powers of λ has the form

$$F = F_0 + \lambda \langle H_1 \rangle_0 + \cdots,$$

   where F_0 and $\langle \cdots \rangle_0$ denote the free energy and an expectation value calculated with $\lambda = 0$, and find the next term in this series. Within this expansion, find the internal energy $U = \langle H \rangle$ correct to the first order in λ.

3. For an ideal gas of N two-dimensional fermions find an expression for the chemical potential μ as a function of the temperature T. How are μ and ϵ_F related at a low temperature?

4. Evaluate the entropy per spin S/N, and the free energy per spin F/N for the one-dimensional Ising model in no external magnetic field in the thermodynamic limit.

5. A lattice of $N+1$ sites has spins $S_i = \pm 1$ at each site, all of which are acted on by a magnetic field. There are interactions of equal strength between one of the spins, S_0, and each of the others. Thus the Hamiltonian is

$$H = -h \sum_{i=0}^{N} S_i - J \sum_{i=1}^{N} S_i S_0.$$

   (a) Find the canonical partition function $Z(T, N)$.

   (b) Find the average energy $\langle E \rangle$.

   (c) For $i \neq 0$ find the statistical averages $\langle S_i \rangle$ and $\langle S_0 S_i \rangle$, and their limits as $h \to 0$ with $J \neq 0$, and when $J \to 0$ with $h \neq 0$.