Samlagning hverfiþunga

Heildarhverfiþungi agnakerfis m.t.t. einhvers punkt er fasti (ekkert ytra vægi)

Víxlverkun á milli einda

-> vægi á eindir

-> hverfiþungi einstakra einda ekki varðveittur

undantekning er venjulega Hartree nálgun
þ.s. allar eindir hreyfast í sama mættinu
-> vantar „fylgnihrif" í Hartree nálgun

spuna-brautar víxlverkun veldur því að hvorki $\bar{L}$ né $\bar{S}$ eru varðveitt heldur $\bar{J}$

Sem dæmi ; samlagning spuna

tvær eindir með $s = 1/2$ spuna

Hornréttur grunnur

fjórvítt tensor margfeldi 2×2

$$\{|\varepsilon_1 \varepsilon_2\rangle\} = \{|+,+\rangle, |+,-\rangle, |-+\rangle, |--\rangle\}$$

eiginvektorar S_1^2, S_2^2 og S_{1z}, S_{2z}

(útreikn...)

$$S_1^2 |\varepsilon_1, \varepsilon_2\rangle = S_2^2 |\varepsilon_1, \varepsilon_2\rangle = \frac{3}{4}\hbar^2 |\varepsilon_1, \varepsilon_2\rangle$$

margfeldi I

$$\left.\begin{aligned} S_{1z} |\varepsilon_1, \varepsilon_2\rangle &= \varepsilon_1 \frac{\hbar}{2} |\varepsilon_1, \varepsilon_2\rangle \\ S_{2z} |\varepsilon_1, \varepsilon_2\rangle &= \varepsilon_2 \frac{\hbar}{2} |\varepsilon_1, \varepsilon_2\rangle \end{aligned}\right\} \text{CSCO}$$

Heildar spuni

$$\bar{S} = \bar{S}_1 + \bar{S}_2$$

(x, y, z)

er hverfiþungi því $[S_i, S_j] = i\hbar \epsilon_{ijk} S_k$

$$[S_x, S_y] = [S_{1x} + S_{2x}, S_{1y} + S_{2y}]$$
$$= [S_{1x}, S_{1y}] + [S_{2x}, S_{2y}]$$
$$= i\hbar S_{1z} + i\hbar S_{2z}$$
$$= i\hbar S_z$$

$$[S^2, S_{1z}] = [S_1^2 + S_2^2 + 2S_1\cdot S_2, S_{1z}]$$

$$= 2[S_1\cdot S_2, S_{1z}]$$

$$= 2[S_{1x}S_{2x} + S_{1y}S_{2y}, S_{1z}]$$

$$= 2i\hbar(-S_{1y}S_{2x} + S_{1x}S_{2y}) \neq 0$$

$$S^2 = S_1^2 + S_2^2 + 2\bar{S}_1\cdot\bar{S}_2$$

$$= S_1^2 + S_2^2 + S_{1+}S_{2-} + S_{1-}S_{2+} + 2S_{1z}S_{2z}$$

$\bar{S}_1$ og $\bar{S}_2$ víxlast við S_1^2 og S_2^2

$$\rightarrow \quad [S_z, S_1^2] = [S_z, S_2^2] = 0$$

$$[S^2, S_1^2] = [S^2, S_2^2] = 0$$

$$[S_z, S_{1z}] = [S_z, S_{2z}] = 0$$

en

$$[S^2, S_{1z}] \neq 0$$

síða 2

Þrí heildar spinni er varðveittur

Finna nýjan grunn

eiginvektorar $\bar{S}$ og S_z $\quad |S,M\rangle$

$\{S_1^2, S_2^2, S^2, S_z\}$: CSCO

ekki nauðsynlegir

eiginvigrar S^2 :

$$\begin{pmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\updownarrow \qquad \updownarrow \qquad \updownarrow \qquad \updownarrow$$

$$0 \qquad 2\hbar^2 \qquad 2\hbar^2 \qquad 2\hbar^2$$ eigingildi

~~roeoe~~ í vegur

$|++\rangle$

$|+-\rangle$

$|-+\rangle$

$|--\rangle$

$\frac{3}{4}+\frac{3}{4}+ \qquad + 2\cdot\frac{1}{2}\cdot\frac{1}{2}$

framsetning S^2 og S_z fundin
í grunninum $\{|+-\rangle \ldots\}$
og síðan sett á hornalínuform

$$S_z = \hbar \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad S^2 = \hbar^2 \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

S M eiginvigrar S^2

$|0,0\rangle = \frac{1}{\sqrt{2}}\{|+-\rangle - |-+\rangle\}$ andsamhverft $1 \rightleftarrows 2$ einstig

$|1,1\rangle = |++\rangle$

$|1,0\rangle = \frac{1}{\sqrt{2}}\{|+-\rangle + |-+\rangle\}$

$|1,-1\rangle = |--\rangle$

samhverft $1 \rightleftarrows 2$ þrístig

hornréttur grunnur
4-vídd rúm
→ CSCO

engin sigild samsvörun
vegna skömmtunar hverfiþungans
og $s = 1/2$ spuna

Almennar aðferðir

hverfiþungi $\bar{J}$: staðreyndir, upprifjun

grunnur $\leftarrow$ eiginvigrar J^2 og J_z: $|k,j,m\rangle$

$$J^2 |k,j,m\rangle = j(j+1)\hbar^2 |k,j,m\rangle$$

$$J_z |k,j,m\rangle = m\hbar |k,j,m\rangle$$

$$J_\pm |k,j,m\rangle = \ldots\ldots |k,j,m\pm 1\rangle$$

$\Sigma(k,j)$ vigurrúmið spannað af $\{|k,j,m\rangle\}$

$(2j+1)$ – vítt

Í heild óbreytt eftir verkun $J^2, J_z, J_\pm$

Innan $\Sigma(k,j)$ eru fylkisstök $F(\bar{J})$ óháð k

Samlagning tveir undir

rúmin $$\Sigma(k_1,k_2\,;\,j_1,j_2) = \Sigma_1(k,j_1)\otimes\Sigma_2(k_2,j_2)$$

$$\bar{J} = \bar{J}_1 + \bar{J}_2$$

$[J_i, J_j] = i\hbar\,\epsilon_{ijk} J_k$ hagfræðsjúa $\rightarrow$ hverfiþungi, J

$[J_{1i}, J_{1j}] = i\hbar\,\epsilon_{ijk} J_{1k}$ og fyrir 2

$$\underbrace{[\bar{J}_1, \bar{J}_2] = 0}$$

$\rightarrow$ $[J_z, J_1^2] = [J_z, J_2^2] = 0$

$$[J^2, J_1^2] = [J^2, J_2^2] = 0$$

$$[J_{1z}, J_z] = [J_{2z}, J_z] = 0$$

en $[J_{1z}, J^2] \neq 0$ og f. 2

grunnaskipti

$|k_1 k_2; j_1 j_2; m_1 m_2\rangle$ er sameiginlegt

eiginástand $J_1^2, J_2^2, J_{1z}, J_{2z}$

Viljum finna sameiginleg eiginástönd

$$J_1^2,\ J_2^2,\ J^2,\ J_z$$

Eigingildi

$j_1 \geq j_2$ athugum fyrst

J_z

$$J_z |j_1 j_2; m_1 m_2\rangle = (J_{1z} + J_{2z})|\cdots\rangle$$

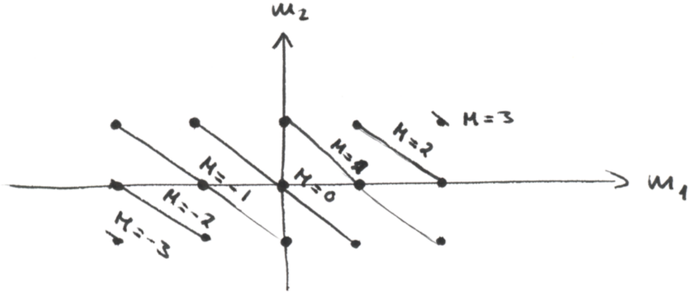
$$= (m_1 + m_2)\hbar\, |\cdots\rangle$$

$\rightarrow$ $M = m_1 + m_2$ tekur gildin

$$j_1 + j_2,\ j_1 + j_2 - 1, \ldots\ldots -(j_1 + j_2)$$

Margfeldni M : $g_{j_1 j_2}(M)$

dæmi $j_1 = 2$ $j_2 = 1$



Eigingildi J^2

$$j = j_1 + j_2,\ j_1 + j_2 - 1, \ldots, |j_1 - j_2|$$

$j_1 \geq j_2$

fjöldi ástanda $\displaystyle\sum_{j=j_1-j_2}^{j_1+j_2} (2j+1) = (2j_2+1)(2j_1+1)$

sjá bók

$\rightarrow$ J^2 og J_z eru CSCO á $\Sigma(j_1, j_2)$

eiginvigrar

$$J^2 |J,M\rangle = J(J+1)\hbar^2 |J,M\rangle$$

$$J_z |J,M\rangle = M\hbar |J,M\rangle$$

$$|J,M\rangle = \sum_{m_1=-j_1}^{j_1} \sum_{m_2=-j_2}^{j_2} |j_1,j_2;m_1,m_2\rangle \underbrace{\langle j_1,j_2;m_1,m_2|J,M\rangle}_{\text{Clebsch-Gordan stuðlar}}$$

CG-stuðlar valdir þ.a. $\in \mathbb{R}$

$$\neq 0 \text{ ef } \left\{ \begin{array}{l} M = m_1+m_2 \\ |j_1-j_2| \leq J \leq j_1+j_2 \end{array} \right\} \text{ þríhyrnings regla}$$

$|J,M\rangle$ er einingarréttur grunnur

$$\rightarrow |j_1 j_2 m_1 m_2\rangle = \sum_{J=j_1-j_2}^{j_1+j_2} \sum_{M=-J}^{J} |J,M\rangle\langle J,M|j_1 j_2 m_1 m_2\rangle$$

Rauntölur

$$\rightarrow \langle JM|j_1 j_2 m_1 m_2\rangle = \langle j_1 j_2 m_1 m_2|JM\rangle$$

Snúningur og tensorvirkjar

(Lectures on QM Gordon Baym) W.A. Benja ~~Wolog~~

(QM Merzbacher)

Framsetning snúnings

Ef $\bar{J}$ er heildar hverfiþungi kerfis þá snýr

$$R_{\bar{\alpha}} = e^{-i\bar{J}\cdot\bar{\alpha}/\hbar}$$

kerfinu í pósitíva stefnu um $\bar{\alpha}$ um hornið $|\alpha|$ m.þ.a. verka á ástandið frá vinstri

Ef $R_{\bar{\alpha}}$ verkar á eigin ástand J^2 $|jm\rangle$ þá breytist ekki j

snúningur breytir ekki hverfiþunga (lengd)

$$\rightarrow [R_{\bar{\alpha}}, J^2] = 0$$

$$J^2 R_{\bar{\alpha}} |jm\rangle = R_{\bar{\alpha}} J^2 |jm\rangle = j(j+1) R_{\bar{\alpha}} |jm\rangle$$

en m breytist, snúna ástandið
er ekki lengur eiginástand J_z
(nema $\bar{\alpha} \sim \hat{z}$) þá breytist ekki z-hnit J

fullkomin grunnur

$$\rightarrow R_{\bar{\alpha}} |jm\rangle = \sum_{m''=-j}^{j} |jm''\rangle d^{(j)}_{m''m}(\bar{\alpha})$$

með

$$d^{(j)}_{m'm}(\bar{\alpha}) = \langle jm'| e^{-i\bar{J}\cdot\bar{\alpha}/\hbar} |jm\rangle$$

$(2j+1) \times (2j+1)$ fylki $d^{(j)}(\bar{\alpha})$ ← óháð afhverfi

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Einn snúning $\bar{\gamma}$ má brjóta niður
í tvo $\bar{\alpha}, \bar{\beta}$

$$R_{\bar{\gamma}} = R_{\bar{\beta}} R_{\bar{\alpha}}$$