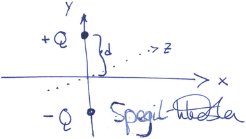
Punktladdning Q



a) finna ρ_s

Ur bok (4-37)

$$V(\bar{r}) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_+} - \frac{1}{R_-}\right)$$

$$R_+ = \sqrt{x^2 + (y-d)^2 + z^2}$$

$$R_- = \sqrt{x^2 + (y+d)^2 + z^2}$$

1

$$\frac{\partial}{\partial y}\frac{1}{R_+} = -\frac{1}{2}\frac{2(y-d)}{(x^2+(y-d)^2+z^2)^{3/2}}$$

$$\frac{\partial}{\partial y}\frac{1}{R_-} = -\frac{1}{2}\frac{2(y+d)}{(x^2+(y+d)^2+z^2)^{3/2}}$$

I shall need the electric field at the plane $y=0$

$$\bar{E}(x, y=0, z) = \left\{-(\bar{\nabla}V(\bar{r}))\cdot\hat{a}_y\right\}\hat{a}_y$$

$$= -\hat{a}_y \frac{Qd}{2\pi\epsilon_0 (x^2+d^2+z^2)^{3/2}}$$

and

$$\rho_s(x,0,z) = \hat{a}_y \cdot \epsilon_0 \bar{E}(x,0,z)$$

$$= -\frac{Qd}{2\pi(x^2+d^2+z^2)^{3/2}}$$

b) The total polarized charge on the surface

In the plane polar coordinates are convenient

$$x^2 + z^2 = r^2$$

$$\rightarrow \rho_s(r) = -\frac{Qd}{2\pi(r^2+d^2)^{3/2}}$$

$$\int_0^\infty r\,dr \int_0^{2\pi} d\varphi\, \rho_s(r) = -Qd \int_0^\infty \frac{r\,dr}{(r^2+d^2)^{3/2}} =$$

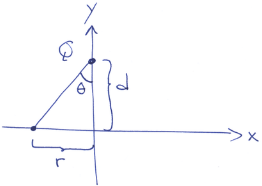
$$= +Qd \left. \frac{1}{\sqrt{r^2+d^2}} \right|_0^\infty = \underline{\underline{-Q}}$$

c) The force on $\rho_s(r)$ due to Q

$\bar{F} = q\bar{E}$ generally, or $d\bar{F} = \bar{E}\,dq$

$$d\bar{F} = \bar{E}(x,0,z)\; 2\pi r\,dr\, \rho_s(r)$$

due to φ-symmetry



x og z components of $d\bar{F}$ cancel

Field of Q only

$$dF_y = |d\bar{F}| \cdot \cos\theta = |E(x,0,z)| \cos\theta \; 2\pi r\,dr\, \rho_s(r)$$

$$= \frac{Q}{4\pi\epsilon_0 R_+^2(y=0)} \cdot 2\pi r\,dr\, \rho_s(r) \cdot \frac{d}{\sqrt{r^2+d^2}}$$

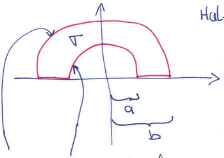
$$F_y = \frac{(Qd)^2}{4\pi\epsilon_0} \int_0^\infty r\,dr \underbrace{\frac{1}{(r^2+d^2)}}_{\sim E} \underbrace{\frac{1}{\sqrt{r^2+d^2}}}_{\sim \cos\theta} \underbrace{\frac{1}{(r^2+d^2)^{3/2}}}_{\sim \rho_s}$$

$$= \frac{(Qd)^2}{4\pi\epsilon_0} \int r\,dr \frac{1}{(r^2+d^2)^3} = \frac{(Qd)^2}{4\pi\epsilon_0 4d^4}$$

$$= \underbrace{\frac{Q^2}{4\pi\epsilon_0 (2d)^2}}$$

which should answer item d)

We could not have used the $\bar{E}$ found in a) since it includes the effects of Q and ρ_s, or Q and its image !!



Half-circular washer with thickness h

Calculate R between the arched surfaces.

$\hookrightarrow$ only radial current

Choose potential $V(a) = V_0$, $V(b) = 0$

$$\nabla^2 V(r) = 0 \iff \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0$$

$$\rightarrow \frac{d}{dr}\left(r\frac{dV}{dr}\right) = 0 \quad \rightarrow r\frac{dV}{dr} = C_1 \quad \text{or} \quad \frac{dV}{dr} = \frac{C_1}{r}$$

$$\rightarrow V(r) = C_1 \ln r + C_2$$

$\hookrightarrow$ B.C.

$$\left.\begin{aligned} V_0 &= C_1 \ln a + C_2 \\ 0 &= C_1 \ln b + C_2 \end{aligned}\right\} \rightarrow C_2 = -\frac{V_0 \ln(b)}{\ln\left(\frac{a}{b}\right)}$$

$$C_1 = \frac{V_0}{\ln\left(\frac{a}{b}\right)}$$

$$V(r) = V_0 \frac{\ln\left(\frac{r}{b}\right)}{\ln\left(\frac{a}{b}\right)} \rightarrow \bar{E}_r = -\hat{a}_r \frac{\partial V}{\partial r}$$

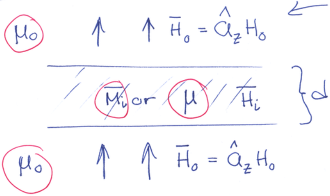
$$= -\hat{a}_r \frac{V_0}{\ln(a/b)}$$

$\bar{E}$ is in the direction of $\hat{a}_r$
since $\ln\left(\frac{a}{b}\right) < 0$ for $a < b$

$$\bar{J}(r) = \sigma \bar{E}(r) \rightarrow I = \int_S \bar{J}(r) \cdot d\bar{s}$$

$$\rightarrow I = \int_0^{\pi} J(r)\, h r\, d\varphi = -\frac{\pi \sigma h V_0}{\ln(a/b)}$$

$$R = \frac{V_0}{I} = -\frac{\ln\left(\frac{a}{b}\right)}{\pi \sigma h} = \frac{\ln\left(\frac{b}{a}\right)}{\pi \sigma h} > 0$$



given external magnetic field

Generally

$$\bar{H} = \frac{1}{\mu}\bar{B}$$

$$\bar{B} = \mu \bar{H}$$

$$\bar{H} = \frac{\bar{B}}{\mu_0} - \bar{M}$$

$$\bar{B} = (\bar{H} + \bar{M})\mu_0$$

no parallel field here and no surface current → B.C.

$$B_{1n} = B_{2n}$$

$$\mu_1 H_{1n} = \mu_2 H_{2n}$$

Use the B.C.

a) The slab has μ

$$\mu H_i = \mu_o H_o \rightarrow \boxed{\bar{H}_i = \hat{a}_z H_i = \hat{a}_z \frac{\mu_o}{\mu} H_o}$$

b) The slab has magnetization $\bar{M}_i$

$$\rightarrow \boxed{\bar{B}_i = (\bar{H}_i + \bar{M}_i)\mu_o}$$ and outside $\bar{B}_o = \mu_o \bar{H}_o$

B.C.

$$B_o = \mu_o H_o \stackrel{\downarrow}{=} B_i = (H_i + M_i)\mu_o$$

$$\rightarrow \boxed{\bar{H}_i = \hat{a}_z (H_o - M_i)}$$

4 Spherical wave

$$\overline{E} = \hat{a}_\theta \frac{E_0}{R} \sin\theta \cos(\omega t - kR)$$

In phasor notation

$$\overline{E} = \hat{a}_\theta \frac{E_0}{R} \sin\theta \, e^{-ikR}$$

only θ-component

a) Find $\overline{H}$. Use Faradays Law

$$\frac{i}{\omega\mu_0} \overline{\nabla} \times \overline{E} = \overline{H}$$

$$\overline{\nabla} \times \overline{E} = \hat{a}_\phi \frac{1}{R} \frac{\partial}{\partial R}(R E_\theta) = \hat{a}_\phi (-ik) E_\theta$$

$$\rightarrow \overline{H} = \hat{a}_\phi \frac{k}{\omega\mu_0} E_\theta$$

b) $$\mathcal{S}_{ave} = \frac{1}{2}\,\mathcal{R}e\left(\bar{E} \times \bar{H}^*\right)$$

$$= \frac{1}{2}\,\mathcal{R}e\left\{ \hat{a}_\theta \times \hat{a}_\phi \left(\frac{k}{\omega\mu_0} |E_\theta|^2 \right) \right\}$$

$$= \frac{1}{2}\,\hat{a}_r\,\frac{k E_0^2}{\omega\mu_0 R^2} \sin^2\theta$$

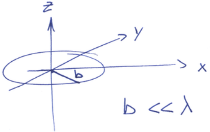
c) The total time-average power (flowing out)

$$P_{ave}^{out} = \int_0^{4\pi} R^2 d\Omega\, \mathcal{S}_{ave}\cdot\hat{a}_r = 2\pi \int_0^{\pi} R^2 \sin\theta\, d\theta\, \bar{\mathcal{S}}_{ave}\cdot\hat{a}_r$$

$$= 2\pi\,\frac{1}{2}\,\frac{R^2 k E_0^2}{\omega\mu_0 R^2}\,\frac{4}{3} = \frac{4\pi}{3}\,\frac{k E_0^2}{\omega\mu_0} = \frac{4\pi}{3}\,\frac{E_0^2}{\eta_0}$$

nonvanishing as $R \to \infty$

(5)



Dipole $I(t) = I_0 \cos(\omega t)$

$b \ll \lambda$ far field

$$E_\phi = \frac{\omega \mu_0 m}{4\pi} \left(\frac{e^{-i\beta R}}{R} \right) \beta \sin\theta$$

$$H_\theta = -\frac{\omega \mu_0 m}{4\pi \eta_0} \left(\frac{e^{-i\beta R}}{R} \right) \beta \sin\theta$$

with $m = I_0 \pi b^2$, $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \simeq 120\pi$

a)
$$P_r = \oint \overline{S}_{ave} \cdot d\overline{s} \;, \qquad S_{ave} = \frac{1}{2} \mathrm{Re} \left| \overline{E} \times \overline{H}^* \right|$$

$$P_r = -\frac{1}{2} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \; E_\phi H_\theta^* R^2$$

$$= \frac{1}{2} \left(\frac{\omega \mu_0 m}{4\pi} \right)^2 \frac{\beta^2}{\eta_0} \, 2\pi \int_0^{\pi} \sin^3\theta \, d\theta$$

$$= \frac{2}{3} \left(\frac{\omega \mu_0 m}{4\pi} \right) \frac{\beta^2}{\eta_0} \, 2\pi \;, \qquad \beta = \frac{2\pi}{\lambda}$$

$$= \frac{2}{3}\left(\frac{\omega\mu_0 I_0 \pi b^2}{4\pi}\right)^2 2\pi \left(\frac{2\pi}{\lambda}\right)^2 \frac{1}{120\pi}$$

$$= \frac{2}{3}\left(\frac{\omega\mu_0 I_0 \pi b^2}{2\lambda}\right)^2 \frac{1}{60} = \frac{1}{2} I_0^2 \left\{\frac{\omega^2\mu_0^2\pi^2 b^4}{180\lambda^2}\right\}$$

$$= \frac{1}{2} I_0^2 R_r \rightarrow R_r = \frac{\omega^2\mu_0\pi^2 b^4}{180\lambda^2} = 80\pi^2\left(\frac{\beta\pi b}{\lambda}\right)^2$$

b)

$$\eta_r = \frac{R_r}{R_r + R_\ell}$$

loss in wire

we know σ_0, length of wire $2\pi b = \ell$ and cross section $s = \pi a^2$

$$\rightarrow R_\ell = \frac{\ell}{\sigma_0 S} = \frac{2\pi b}{\sigma_0 \pi a^2} = \frac{2b}{a^2 \sigma_0}$$

only true for low frequency, for higer f we have surface conduction

$$R_s = \sqrt{\frac{\omega \mu_0}{2\sigma_0}} \qquad \text{see } (11\text{-}49)$$

$$R_\ell = R_s \frac{2\pi b}{2\pi a} = R_s \frac{b}{a}$$

$$\rightarrow \eta_r = \frac{1}{1 + \frac{R_\ell}{R_r}} = \frac{1}{1 + \frac{\frac{b}{a}\sqrt{\frac{\omega\mu_0}{2\sigma_0}}}{80\pi^2\left(\frac{\beta\pi b}{\lambda}\right)^2}}$$

c) If b and λ are fixed it would be ~~best~~ to have several windings of wire in the antenna $\Leftarrow$ see $\mathcal{R}_r$

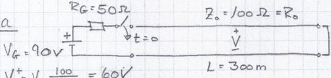
d) We have fixed $I_0 \cos(\omega t)$ in the beginning

$\rightarrow$ R_r, or higher value of R_r represent how large resistance we would need to dissipate the radiated power

RAF 402G EXAM SPRING 2009

SOLUTIONS FOR TRANSMISSION LINE QUESTONS

6a

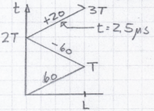


$T = \frac{L}{c} = \frac{300}{3\cdot 10^8} = 1\cdot 1($

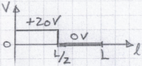
$= 1\mu sek$

$V_1^+ = V_G \frac{100}{100+50} = 60V$

$\Gamma_G = \frac{50-100}{50+100} = \frac{-1}{3}$. $\Gamma_L = \frac{0-100}{0+100} = -1$

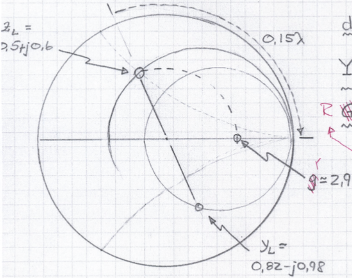


Voltage at $t = 2{,}5\mu s$:



6b

$Z_L = 50 + j60\,\Omega \Rightarrow z_L = \frac{1}{R_0} \cdot Z_L = 0{,}5 + j0{,}6$



$d = 0{,}15\lambda$

$Y_L = \frac{1}{R_0} y_L = 8{,}2 - 9{,}8\,mS$

$R = R_0 \cdot r \approx 290\,\Omega$

Var leiðrétt í prófinu