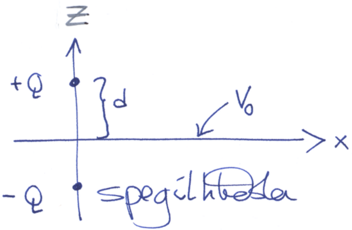
1



a) finna $V(\bar{r})$

Notum línulega samlagningar eiginleika rafsegulfræði í tómarúmi ->

leysum í tveimur prepum

i) ~~Hleðin~~ slétta

ii) Punkthleðsla

1

i)

$$\frac{\partial^2 V}{\partial z^2} = 0 \rightarrow V(z) = az + b$$

$$V(0) = V_z \rightarrow b = V_0$$

$$E_z = -\frac{\partial}{\partial z} V \rightarrow E_z = -a$$

$$\boxed{V(z) = -E_z z + V_0}$$

~~Með~~ Gauß-lögmáli má tengja ~~við~~ yfirborðshleðslu ρ_s

$$E_z A = \frac{\rho_s A}{\epsilon_0}$$

$$\rightarrow E_z = \frac{\rho_s}{\epsilon_0}$$

$$\rightarrow \boxed{V(z) = -\frac{\rho_s}{\epsilon_0} z + V_0}$$

ii) Úr bók (4-37)

$$V(\bar{r}) = \frac{Q}{4\pi\epsilon_0}\left\{\frac{1}{R_+} - \frac{1}{R_-}\right\}$$

$$R_+ = \sqrt{x^2 + y^2 + (z-d)^2}$$

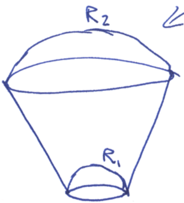
$$R_- = \sqrt{x^2 + y^2 + (z+d)^2}$$

Heildar lausnin er því

$$\boxed{V(\bar{r}) = \frac{Q}{4\pi\epsilon_0}\left\{\frac{1}{R_+} - \frac{1}{R_-}\right\} - E_z z + V_0}$$

b) Kraftur punkthleðslunnar á sléttuna

$$\bar{F} = \hat{z}\left\{\frac{Q^2}{4\pi\epsilon_0 (2d)^2} - \frac{Q\rho_s}{\epsilon_0}\right\}$$



$R_1 \le R \le R_2$

$0 \le \theta \le \theta_0$

$\sigma(R) = \frac{\sigma_0 R_1}{R}$

Ekki háð tíma $\rightarrow \bar{\nabla}\cdot\bar{J} = 0$

frá samfelldnijöfnu

Í kúluhnitum verður R eina breytan hér

$$\bar{\nabla}\cdot\bar{J} = \bar{\nabla}\cdot(\sigma\bar{E})$$

$$= \sigma\,\bar{\nabla}\cdot\bar{E} + (\bar{E}\cdot\bar{\nabla})\sigma = 0$$

Notum Ohms lögmál

$$\bar{J} = \sigma\bar{E}$$

vitum að $\bar{E} = \hat{a}_R E$

$$\rightarrow \bar{\nabla}\cdot\bar{J} = \sigma\frac{1}{R^2}\frac{d}{dR}(R^2E) + E\frac{d}{dR}\sigma(R)$$

$$= \sigma(R)\frac{1}{R^2}\left(\frac{d}{dR}R^2E\right) - E\sigma_0R_1/R^2 = 0$$

$$\rightarrow \quad \frac{\sigma_0 R_1}{R^3}\left(\frac{d}{dR}R^2E\right) = \frac{E\sigma_0 R_1}{R^2}$$

$$\rightarrow \quad \frac{d}{dR}(R^2E) = ER \quad \rightarrow \quad 2RE + R^2\frac{d}{dR}E = ER$$

$$\rightarrow \quad R^2\frac{d}{dR}E = -ER \quad \rightarrow \quad R\frac{d}{dR}E = -E$$

$$\rightarrow \quad E = \frac{c_1}{R} \quad \text{όπου} \quad \bar{E} = \hat{a}_R\frac{c_1}{R}$$

$$V = -\int_{R_2}^{R_1} \bar{E}\cdot d\bar{R} = +c_1 \ln\left(\frac{R_2}{R_1}\right)$$

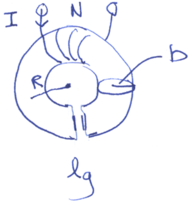
$$\rightarrow \quad C_1 = \frac{V}{\ln\left(\frac{R_2}{R_1}\right)} \qquad \rightarrow \quad \bar{E} = \hat{a}_r \frac{V}{R \ln\left(\frac{R_2}{R_1}\right)}$$

$$I = \int_S \bar{J} \cdot d\bar{s} = \int_S \sigma(R) \bar{E} \cdot d\bar{s} = \sigma_0 R_1 \int_0^{2\pi} d\phi \int_0^{\theta_0} \frac{R^2}{R^2} \sin\theta \, d\theta \, \frac{V}{\ln\left(\frac{R_2}{R_1}\right)}$$

$$= \frac{2\pi \sigma_0 R_1 V (1 - \cos\theta_0)}{\ln(R_2/R_1)}$$

$$R = \frac{V}{I} = \frac{\ln\left(\frac{R_2}{R_1}\right)}{2\pi \sigma_0 R_1 (1 - \cos\theta_0)}$$

3



a) Segulviðnám geilar

$$\boxed{R_g = \frac{l_g}{\mu_0 S} = \frac{l_g}{\mu_0 \pi b^2}}$$

Segulviðnám kjarna

$$\boxed{R_c = \frac{2\pi R - l_g}{\mu \pi b^2}}$$

b) Reikna $\bar{B}_g$, $\bar{H}_g$

og $\bar{B}_c$ og $\bar{H}_c$

Segulflæðisviðið $\bar{B}$ er samfellt $\bar{B}_c = \bar{B}_g$

og $\bar{B}_c = \bar{B}_g = \hat{a}_\phi B$ t.d., en við þekkjum ekki enn B

(7)

$$\bar{H}_c = \hat{a}_\phi \frac{B}{\mu} \quad \text{og} \quad \bar{H}_g = \hat{a}_\phi \frac{B}{\mu_0}$$

Ef segul~~flæði~~ Φ er þekkt $\rightarrow$ $\Phi = SB = \pi b^2 B$

$$\rightarrow B = \frac{\Phi}{\pi b^2}$$

$$\rightarrow \boxed{\bar{B}_c = \bar{B}_g = \hat{a}_\phi \frac{\Phi}{\pi b^2}}$$

$$\boxed{\bar{H}_c = \hat{a}_\phi \frac{\Phi}{\pi b^2 \mu}} \quad \text{og} \quad \boxed{\bar{H}_g = \hat{a}_\phi \frac{\Phi}{\pi b^2 \mu_0}}$$

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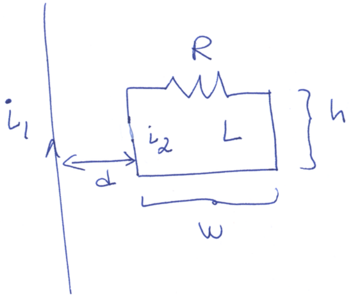
c) Straumurinn I til ~~að~~ ~~við~~halda Φ

$$\oint \bar{H} \cdot d\bar{l} = NI \rightarrow H_c(2\pi R - l_g) + H_g l_g = NI$$

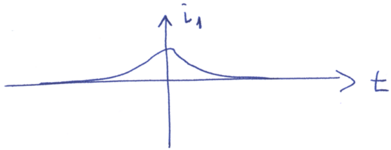
$$\frac{\Phi}{\pi b^2 \mu}(2\pi R - l_g) + \frac{\Phi}{\pi b^2 \mu_0} l_g = NI$$

$$\rightarrow \boxed{I = \frac{\Phi}{\pi b^2 N}\left\{\frac{2\pi R - l_g}{\mu} + \frac{l_g}{\mu_0}\right\}}$$

4



$$i_1 = i_0 \exp(-(\gamma t)^2)$$



a) Finna i_2. Til þess þurfum við lögmál Faradays

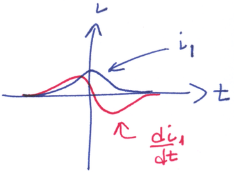
$$\oint \bar{E} \cdot d\bar{l} = -\frac{d\Phi}{dt} = -L\frac{di_2}{dt} - L_{12}\frac{di_1}{dt}$$

$$\rightarrow \quad R i_2 = -L\frac{di_2}{dt} - L_{12}\frac{di_1}{dt}$$

Þar fáum $L_{12} = \frac{\Phi_{12}}{i_1} = \frac{h}{i_1}\int_d^{d+w} B_{12}\,dr$

Segulsvið frá vír $= \frac{h}{i_1}\int_d^{d+w} \frac{\mu_0 i_1}{2\pi r}\,dr = \frac{\mu_0 h}{2\pi}\ln\left(1+\frac{w}{d}\right)$

$$\frac{di_1}{dt} = -i_0\gamma^2 2t\,\exp\left(-(\gamma t)^2\right) = -2\gamma^2 t\cdot i_1$$



$$\rightarrow \quad L\frac{di_2}{dt} + Ri_2 = -L_{12}\frac{di_1}{dt} = L_{12}i_0\gamma^2 2t\,e^{-(\gamma t)^2}$$

$$\frac{di_2}{dt} + \frac{R}{L}i_2 = \frac{L_{12}}{L}i_0\gamma^2 2t\,e^{-(\gamma t)^2}$$

fyrsta stigs afleiðujafnan

$$y' + p(t)y = q(t)$$

Lefur lausnina

$$y(t) = y(t_0)e^{-P(t)} + e^{-P(t)}\int_{t_0}^{t} e^{P(s)} q(s)ds, \quad P(t) = \int_{t_0}^{t} p(s)ds$$

$$\rightarrow$$

$$i_2(t) = i_2(t_0)e^{-\frac{R}{L}(t-t_0)} + e^{-\frac{R}{L}(t-t_0)}\int_{t_0}^{t} e^{+\frac{R}{L}(t'-t_0)}\, t' e^{-\gamma^2 t'^2} dt' \cdot \frac{L_{12}}{L} 2 i_0 \gamma^2$$

Veljum $t_0 = -\infty$ vegna forms $i_1(t)$ og setjum $i_2(-\infty)=0$

$$i_2(t) = e^{-\frac{R}{L}t}\int_{-\infty}^{t} ds\, e^{-\gamma^2 s^2 + \frac{R}{L}s}\, s \cdot \frac{L_{12}}{L} 2 i_0 \gamma$$

(12)

$$i_2(t) = e^{-\frac{R}{L}t}\left\{\frac{\sqrt{\pi}\,\frac{R}{L}\,e^{\frac{R^2}{4L^2\gamma^2}}\operatorname{erf}\left(\gamma t - \frac{R}{L\gamma}\right) - 2\gamma e^{t\left(\frac{R}{L} - \gamma^2 t^2\right)}}{4\gamma^2}\right\}\frac{L_{12}}{L}2i_0$$

$$= \frac{L_{12}}{L}i_0\left\{\frac{\sqrt{\pi}\,\frac{R}{L}\,e^{\frac{R^2}{4L^2\gamma^2}-\frac{R}{L}t}\operatorname{erf}\left(\gamma t - \frac{R}{L\gamma}\right) - 2\gamma e^{-\gamma^2 t^2}}{2\gamma^2}\right\}$$

b) Hér hefði heildis formið nægt, það sýnir okkur t.d. að

$i_2(t) \xrightarrow[t \to +\infty]{} 0$ og $i_2(t) < 0$ fyrir $t < \infty$

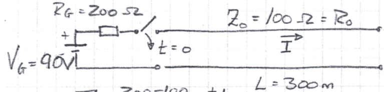
$$W = \int_{-\infty}^{\infty} i_2^2(t)\,R\,dt$$

þetta form nægir líka

RAF 402G ALT. EXAM SPRING 2009

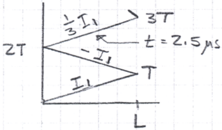
SOLUTIONS FOR TRANSMISSION LINE QUESTIONS

6a



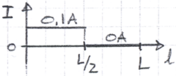
$$T = \frac{L}{c} = \frac{300}{3 \cdot 10^{8}} = 1 \cdot 10^{-6} s$$

$$\Gamma_G = \frac{200-100}{200+100} = +\frac{1}{3} \qquad \Gamma_L = +1$$

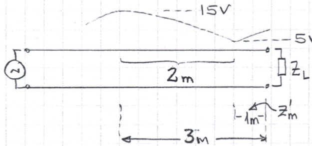


$$I_1 = \frac{V_1}{R_0} = \frac{1}{R_0} \cdot V_G \frac{100}{100+200} = 0{,}3A$$

Current at $t = 2.5\mu s$:



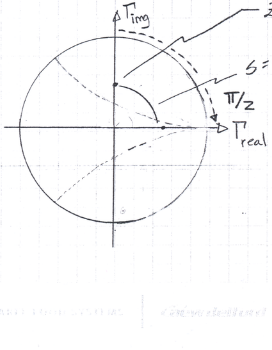
6b



$$\lambda/4 = 2m \Rightarrow \lambda = 8m$$

$$\beta = \frac{2\pi}{\lambda} = \frac{\pi}{4}$$

$$S = \frac{15}{5} = 3 \qquad |\Gamma| = \frac{S-1}{S+1} = 0{,}5 \qquad \measuredangle\Gamma = 2\beta z'_m = 2\frac{\pi}{4} \cdot 1 = \pi/2$$



$$z_L = 0{,}6 + j\,0{,}8$$

$$\Rightarrow Z_L = R_0 \cdot z_L = 60 + j80\Omega$$