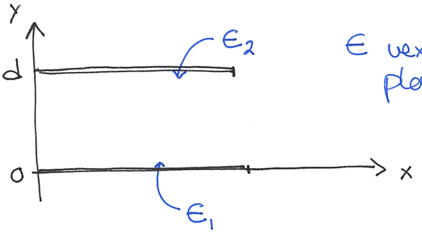
Plötuþéttir



ϵ vex línulega milli platnar

Finna rýmd þéttis

línulegur vöxtur ϵ $\longrightarrow$ $\epsilon(y) = \frac{\epsilon_2 - \epsilon_1}{d} y + \epsilon_1$

Engin jaðarhrif, óendanlegar plötur

$\longrightarrow$ $\bar{E}(y) = E(y)\,\hat{a}_y$, ef við gerum ráð fyrir jákvæðri hleðslu Q á efri plötu gefur Gauss

$$\bar{E}(y) = -\hat{a}_y \left(\frac{Q}{S}\right) \frac{1}{\epsilon(y)}, \quad \rho_s = \frac{Q}{S}$$

$$= -\hat{a}_y \frac{Q}{S\left(\frac{\epsilon_2 - \epsilon_1}{d} y + \epsilon_1\right)}$$

$$V = -\int_{y=0}^{y=d} \bar{E} \cdot d\bar{l} = \frac{Qd \ln\left(\frac{\epsilon_2}{\epsilon_1}\right)}{S(\epsilon_2 - \epsilon_1)}$$

$$C = \frac{Q}{V} = \frac{S(\epsilon_2 - \epsilon_1)}{d \ln\left(\frac{\epsilon_2}{\epsilon_1}\right)}$$

$$C \xrightarrow[\epsilon_2 \to \epsilon_1]{} \frac{S\epsilon_1}{d}$$

eins og búast mátti við

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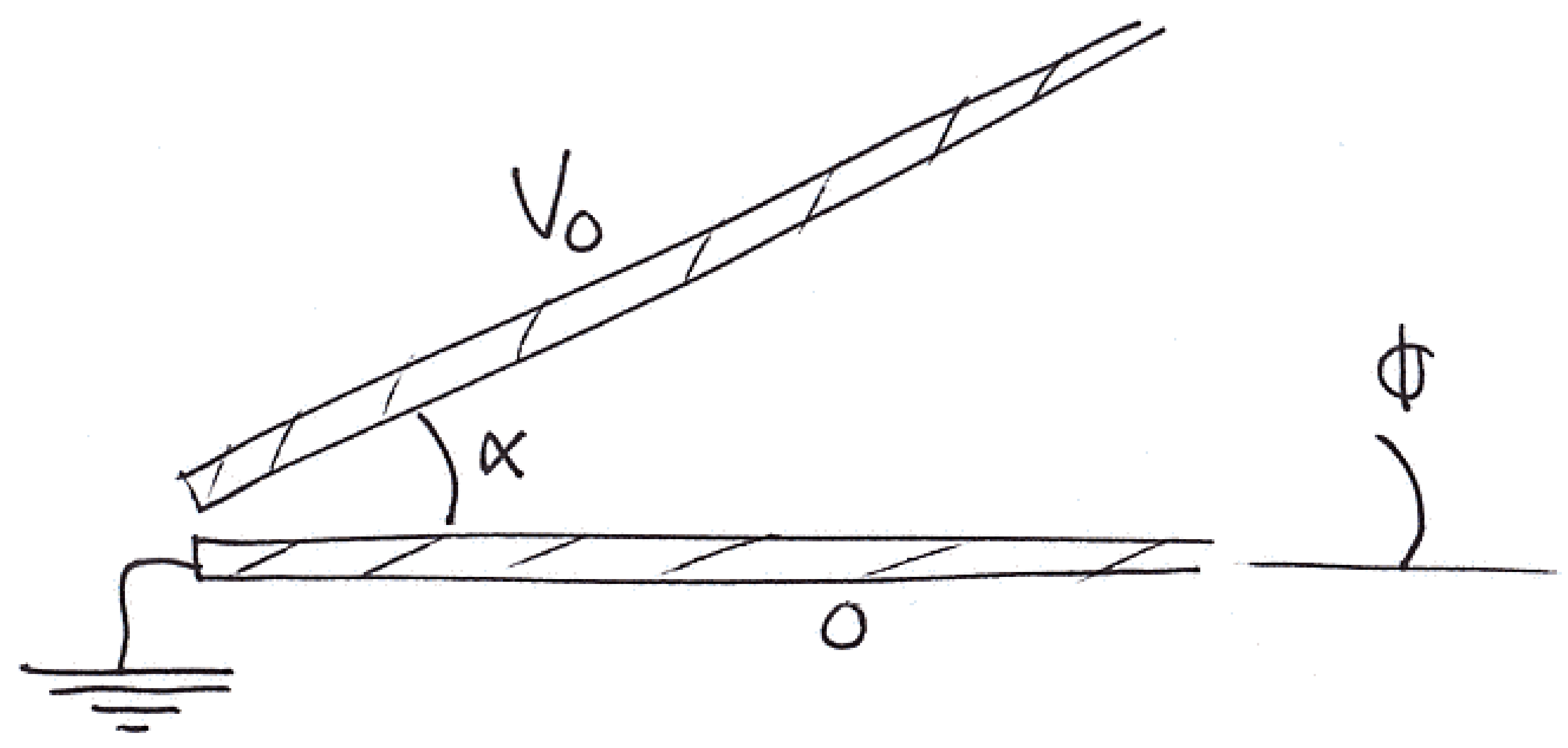
finna mottið í

a) $0 < \phi < \alpha$

b) $\alpha < \phi < 2\pi$



Ekker nýtur r-samhvørfu, $A_r r^n + B_r r^{-n}$ ekki møgulig loysn, ln(..) ekki heldur því eingin linuhleðsla kemur fyri.

a)

$$\frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} = 0$$

við loysn $V(\phi) = A_0 \phi + B_0$

jaðarskilyrði

$$\left.\begin{array}{l} V(0) = 0 \quad \rightarrow \quad B_0 = 0 \\ V(\alpha) = V_0 \quad \rightarrow \quad A_0 = \frac{V_0}{\alpha} \end{array}\right\} \quad V(\phi) = \frac{V_0}{\alpha}\phi \qquad 0 \leq \phi \leq \alpha$$

b)

$$\left.\begin{array}{l} V(\alpha) = V_0 = A_1\alpha + B_1 \\ V(2\pi) = 0 = 2\pi A_1 + B_1 \end{array}\right\} \quad A_1 = -\frac{V_0}{2\pi - \alpha}, \quad B_1 = \frac{2\pi V_0}{2\pi - \alpha}$$

$$\rightarrow V(\phi) = \frac{V_0}{2\pi - \alpha}(2\pi - \phi), \qquad \alpha \leq \phi \leq 2\pi$$

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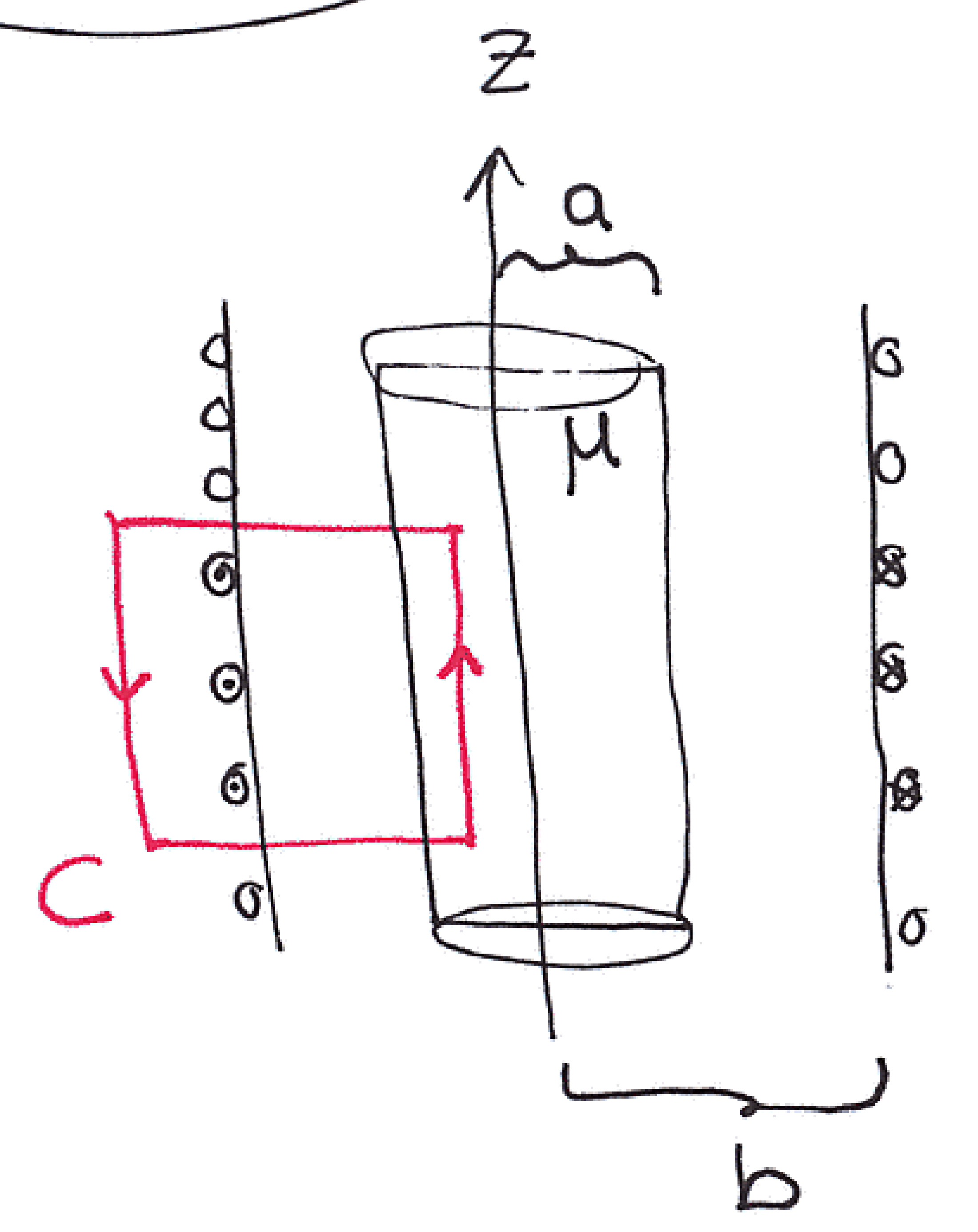

a) finna $\bar{B}$, $\bar{H}$ og $\bar{M}$ innan spólu fyrir $r<a$ og $a<r<b$

Ekki tímaháð -> Ampères lögmál

<u>$r<a$</u>

$$\oint_C \bar{H}\cdot d\bar{l} = NI$$

$$HL = NI \quad \rightarrow \quad \bar{H} = \hat{a}_z \left(\frac{N}{L}\right) I$$

$$= \hat{a}_z n I$$

<u>$r<a$</u>

$$\bar{B} = \mu \bar{H} \quad \rightarrow \quad \bar{B} = \hat{a}_z \mu n I$$

$$\bar{H} = \frac{\bar{B}}{\mu_0} - \bar{M} \quad \rightarrow \quad \bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = \hat{a}_z \left(\frac{\mu}{\mu_0} - 1\right) n I$$

$a < r < b$

Samskonar C $\quad \rightarrow \quad \bar{H} = \hat{a}_z n I$

$\bar{B} = \mu_0 \bar{H} \quad \rightarrow \quad \bar{B} = \hat{a}_z \mu_0 n I$

$\bar{M} = 0$

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b) Finna $\bar{J}_m$ og $\bar{J}_{ms}$ jafngilda $\bar{M}$

$\bar{J}_m = \bar{\nabla} \times \bar{M} = 0$ því $\bar{M}$ er fasti

$\bar{J}_{ms}$ er yfirborðsstraumur á sívalningnum með $r = a$

$$\bar{J}_{ms} = \bar{M} \times \hat{a}_n = (\hat{a}_z \times \hat{a}_r)\left(\frac{\mu}{\mu_0} - 1\right) n I$$

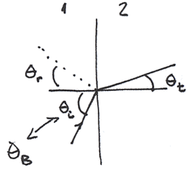
$$= \hat{a}_\phi \left(\frac{\mu}{\mu_0} - 1\right) n I$$

(4)

Ekkert endurkast, sjáum að

$$\theta_t + \theta_{B\perp} = \pi/2$$

$$\theta_t + \theta_{B\parallel} = \pi/2$$



a) þverskautun ($\mu_1 \neq \mu_2$)

$$\sin^2\theta_{B\perp} = \frac{1 - \frac{\mu_1\epsilon_2}{\mu_2\epsilon_1}}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2} \quad \text{gefur } \theta_{B\perp}$$

ekkert endurkast (8-210)

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}$$

$$\frac{n_1}{n_2} = \frac{u_2}{u_1} = \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}$$

$$= \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_{B\perp}}$$

$$= \sqrt{1 - \frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\sin^2\theta_{B\perp}} = \sqrt{\frac{1 - \frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}{1 - \frac{\mu_1^2}{\mu_2^2}}}$$

Hér sést að ef $\epsilon_1 = \epsilon_2$

$$\rightarrow \cos\theta_t = \sin\theta_{B\perp}$$

$$\longrightarrow \theta_t = \frac{\pi}{2} - \theta_{B\perp}$$

b) Samsíða stautun $\epsilon_1 \neq \epsilon_2$

$$\sin^2\theta_{B\parallel} = \frac{1 - \frac{\mu_2\epsilon_1}{\mu_1\epsilon_2}}{1 - \left(\frac{\epsilon_1}{\epsilon_2}\right)^2}$$

ef nú $\mu_1 = \mu_2$ verður þetta

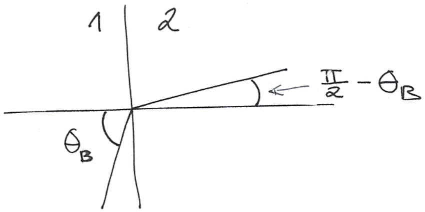
$$\sin^2\theta_{B\parallel} = \frac{1}{\sqrt{1 + \frac{\epsilon_1}{\epsilon_2}}}$$ gefur $\theta_{B\parallel}$

þá fæst aftur á svipaðan hátt

$$\cos\theta_t = \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_{B\parallel}} = \sin\theta_{B\parallel}$$

$$\rightarrow \theta_t = \frac{\pi}{2} - \theta_{B\parallel}$$

c)



(5) $a \times b$ rétthyrndur bylgjustokkur

TM_{11}

10 - 134 - 7 geta

$$E_x^0(x,y) = -\frac{i\beta_{11}}{h^2}\frac{\pi}{a}E_0 \cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$$

$$E_y^0(x,y) = -\frac{i\beta_{11}}{h^2}\frac{\pi}{b}E_0 \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)$$

$$E_z^0(x,y) = E_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$$

$$H_x^0(x,y) = \frac{i\omega\epsilon}{h^2}\frac{\pi}{b}E_0 \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)$$

$$H_y^0(x,y) = -\frac{i\omega\epsilon}{h^2}\frac{\pi}{a}E_0 \cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$$

$$h^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2$$

$$\beta_{11} = \sqrt{\omega^2\mu\epsilon - h^2}$$

a) ytri-borð streum þéttleiki

Almennt $\hat{a}_{n2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s$

Hér

$$\bar{J}_s(y=0) = \hat{a}_n \times \bar{H}\Big|_{y=0}$$

$$= \hat{a}_y \times \left[\hat{a}_x H_x^0(x,0) + \hat{a}_y H_y^0(x,0)\right]$$

$$= -\hat{a}_z H_x^0(x,0)$$

$$= -\hat{a}_z \frac{i\omega\epsilon}{h^2} \frac{\pi}{b} E_0 \sin\left(\frac{\pi x}{a}\right)$$

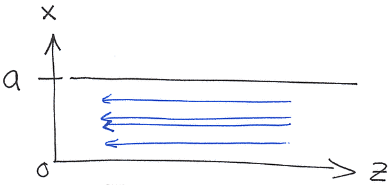
og sama fyrir $\bar{J}_s(y=b)$

$$\bar{J}_s(x=0) = \hat{a}_n \times \bar{H}\Big|_{x=0} = \hat{a}_x \times \left[\hat{a}_x H_x^0(0,y) + \hat{a}_y H_y^0(0,y)\right]$$

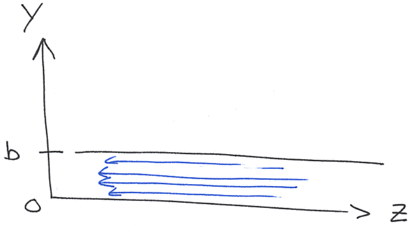
$$= \hat{a}_z H_y^0(0,y) = -\hat{a}_z \frac{i\omega\epsilon}{h^2} \frac{\pi}{a} E_0 \sin\left(\frac{\pi y}{b}\right)$$

og sama fyrir $\bar{J}_s(x=a)$

b)



$\bar{J}_s$ í $y=b$



J_s í $x=0$

Straumurinn er á móti bylgju-flutningnum í $\hat{a}_z$-átt