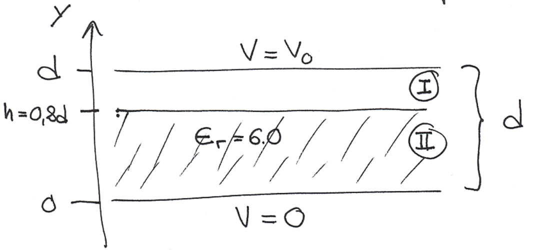
1

þéttir með samsíða plötum



a) Finna V og $\bar{E}$ í rafsvaranum (II)

Jafna Poissons $\nabla^2 V = 0$

Í okkar samhverfu : $\longrightarrow \partial_x^2 V = 0$

bæði á svæðum (I) og (II)

$$\rightarrow \quad V_{II}(y) = C_1 y + C_2$$

$$V_I(y) = C_3 y + C_4$$

þar sem C_i þurfa að ákvarðast af jaðarskilyrðum

$$V_{II}(0) = 0 \,, \qquad V_{I}(d) = V_0 \qquad (3)$$

$$V_{I}(h) = V_{II}(h) \qquad (1)$$

$$\overline{D}_{I}(h) = \overline{D}_{II}(h) \leftarrow \qquad (2)$$

$$\overline{E} = -\overline{\nabla} V \quad \text{hér} \quad \overline{E} = -\hat{a}_y \frac{\partial}{\partial y} V(y)$$

$$\rightarrow \begin{cases} \overline{E}_{I} = -\hat{a}_y C_1 \\ \overline{E}_{II} = -\hat{a}_y C_3 \end{cases}$$

$$\overline{D} = \epsilon \overline{E}$$

$$\rightarrow \begin{cases} \overline{D}_{I} = -\hat{a}_y \epsilon_0 C_3 \\ \overline{D}_{II} = -\hat{a}_y \epsilon_0 \epsilon_r C_1 \end{cases}$$

Þá eru jöfnurnar

$$(3) \rightarrow \quad C_2 = 0 \,, \qquad C_3 d + C_4 = V_0$$

$$(1) \rightarrow \quad C_3 h + C_4 = C_1 h$$

$$(2) \rightarrow \quad \epsilon_0 C_3 = \epsilon_0 \epsilon_r C_1$$

Það

$$C_3 d + C_4 = V_0$$

$$C_1 h - C_3 h - C_4 = 0$$

$$-C_1 \epsilon_r + C_3 = 0$$

Það er sama 2×2 vegna 3. jöfnunnar !

$$\begin{pmatrix} 0 & d & 1 \\ h & -h & -1 \\ -\epsilon_r & 1 & 0 \end{pmatrix} \begin{pmatrix} C_1 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} V_0 \\ 0 \\ 0 \end{pmatrix}$$

{ athugið að C_1 og C_3 hafa ekki sömu vídd og C_2 og C_4 }

Lausnin er

$$C_1 = \frac{V_0}{h + (d-h)\epsilon_r}, \qquad C_2 = 0$$

$$C_3 = \frac{\epsilon_r V_0}{h + (d-h)\epsilon_r}$$

$$C_4 = \frac{h(1-\epsilon_r)V_0}{h + \epsilon_r (d-h)}$$

$$V_{II}(y) = \frac{V_o Y}{h + (d-h)\epsilon_r}$$

$$\bar{E}_{II}(y) = -\hat{a}_y \frac{V_o}{h + (d-h)\epsilon_r}$$

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b)

$$V_I(y) = \frac{\epsilon_r V_o Y + h(1-\epsilon_r)V_o}{h + (d-h)\epsilon_r}$$

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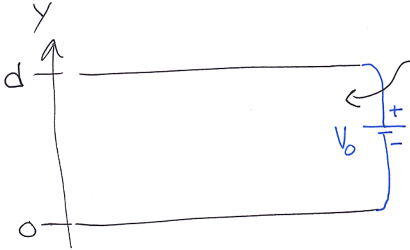
c)

$$\rho_s(d) = -D_I(d) = \frac{\epsilon_o \epsilon_r V_o}{h + (d-h)\epsilon_r}$$

$$\rho_s(0) = D_{II}(0) = -\frac{\epsilon_o \epsilon_r V_o}{h + (d-h)\epsilon_r}$$

P5-10

Þéttir með tvær samsíða plötur með flatarmál S



línuleg breyting á σ frá $\sigma(0) = \sigma_1$ til $\sigma(d) = \sigma_2$

V_0 er sett á milli platna

$$\sigma(y) = \sigma_1 + (\sigma_2 - \sigma_1)\frac{y}{d}$$

a) Finna R milli platna

$$\bar{J} = -\hat{a}_y J_0 \rightarrow \bar{E} = \frac{\bar{J}}{\sigma} = -\hat{a}_y \frac{J_0}{\sigma(y)}$$

$$V_0 = -\int_0^d \bar{E} \cdot \hat{a}_y \, dy = \int_0^d \frac{J_0 \, dy}{\sigma_1 + (\sigma_2 - \sigma_1)\frac{y}{d}} = \frac{J_0 d}{\sigma_2 - \sigma_1} \ln\left(\frac{\sigma_2}{\sigma_1}\right)$$

$$R = \frac{V_0}{I} = \frac{V_0}{J_0 S} = \frac{d}{(\sigma_2 - \sigma_1) S} \ln\left(\frac{\sigma_2}{\sigma_1}\right)$$

b) finna flatarhleðslur platna

$$\rho_s(d) = \epsilon_0 E_y(d) = \epsilon_0 \frac{J_0}{\sigma_2} = \frac{\epsilon_0 (\sigma_2 - \sigma_1) V_0}{\sigma_2 d \ln\left(\frac{\sigma_2}{\sigma_1}\right)}$$

$$\rho_s(0) = -\epsilon_0 E_y(0) = -\epsilon_0 \frac{J_0}{\sigma_1} = -\frac{\epsilon_0 (\sigma_2 - \sigma_1) V_0}{\sigma_1 d \ln\left(\frac{\sigma_2}{\sigma_1}\right)}$$

c) finna heildarhleðslu milli platna og dreifingu hennar.

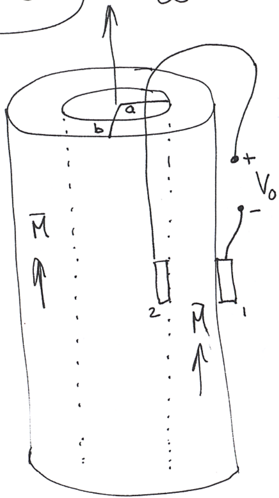
$$\rho(y) = \overline{\nabla} \cdot \overline{D} = \frac{d}{dy}(\epsilon_0 E) \quad , \quad E = -\frac{J_0}{\sigma(y)}$$

$$\rightarrow \quad \rho(y) = \frac{d}{dy}(\epsilon_0 E) = -\epsilon_0 J_0 \frac{d}{dy}\left(\frac{1}{\sigma(y)}\right)$$

$$= \epsilon_0 J_0 \frac{(\sigma_2 - \sigma_1)/d}{\left\{\sigma_1 + (\sigma_2 - \sigma_1)\frac{y}{d}\right\}^2}$$

①

7-10



$\bar{M} = \hat{a}_z M_0$

$\mu_r = 5000$

$\sigma = 10^7 \ S/m$

a) finna $\bar{H}$ og $\bar{B}$ í segulinum

$\bar{M} = \chi_m \bar{H} \rightarrow \bar{H} = \dfrac{\bar{M}}{\chi_M}$

$\mu_r = 1 + \chi_M \rightarrow \boxed{\bar{H} = \dfrac{\bar{M}}{\mu_r - 1}}$

$\bar{H} = \dfrac{\hat{a}_z M_0}{\mu_r - 1}$

$\bar{B} = \mu_0 \mu_r \bar{H}$

$\bar{B} = \mu_0 \mu_r \dfrac{\hat{a}_z M_0}{\mu_r - 1}$

$= \hat{a}_z \mu_0 M_0 \dfrac{\mu_r}{\mu_r - 1}$

b) Finna V_0 í opinni rás — bursti 1 og 2

Milli burstanna er leiðari, þvert á hann ferðast segulflæðisvið $\overline{B}$

$$V_{21} = \int_1^2 (\overline{u} \times \overline{B}) \cdot d\overline{l}$$

$\overline{B} = B\hat{a}_z$ og $\overline{u} = \omega r \, \hat{a}_\phi$

$$\rightarrow \quad V_{21} = V_0 = \int_b^a \left(\hat{a}_\phi \omega r \times \hat{a}_z B\right) \cdot \hat{a}_r \, dr$$

$$= - \frac{\omega B}{2} r^2 \Big|_b^a = - \frac{\omega B}{2}(b^2 - a^2) = - \frac{\omega \mu_0 M_0 \mu_r}{2(\mu_r - 1)}(b^2 - a^2)$$

hægri h. regla

c) Straumurinn í lokaðri rás?

Lokaða rásin er bara með viðnámi milli burstanna um segulsviðslínurnar. Snúningurinn veldur sömu íspennu og áður

$$\mathcal{E} = \int_1^2 (\bar{u} \times \bar{B}) \cdot d\bar{l} = -\frac{\omega B}{2}(b^2 - a^2) \quad \text{hér}$$

Þessi íspenna rekur straum um viðnámið R (sem við ætlum eftir að finna) i. Spennufallið um R vegna i verður að jafna út Σ.

$$\Sigma - iR = 0$$

$$\rightarrow \quad i = \frac{\Sigma}{R} = -\frac{\omega B}{2R}(b^2 - a^2)$$

Átt straumsins má snúa við með stefnu ω

(4)

Finnum R

Eðlisviðnám σ er hátt -> allur straumurinn leiðir

$$\bar{J}(\bar{r}) = \frac{i\,\hat{a}_r}{A(r)h} = \frac{i\,\hat{a}_r}{2\pi r h}, \quad \bar{E}(r)\sigma = \bar{J}(\bar{r})$$

$$\Rightarrow \bar{E}(r) = \frac{\hat{a}_r\, i}{2\pi r h \sigma}$$

Spennufallið yfir sívalninginn er þá

$$V = \int_b^a E(r)\,dr = -\frac{i}{2\pi h\sigma}\ln\left(\frac{b}{a}\right) = -iR$$

$$\Rightarrow R = \frac{1}{2\pi h\sigma}\ln\left(\frac{b}{a}\right)$$

og því

$$\boxed{i = -\frac{\omega B(b^2-a^2)}{2\ln\left(\frac{b}{a}\right)}\,2\pi h\sigma}$$

P8-44 Innbylgja með samhliða skautun

a) finna tengsl θ_c og $\theta_{B\parallel}$ ef $\mu_i = \mu_o$

fyrir ósegulvertandi efni fékkst

$$\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \quad \text{og} \quad \sin\theta_{B\parallel} = \frac{1}{\sqrt{1+\left(\frac{\epsilon_1}{\epsilon_2}\right)}}$$

(8-187) (8-226)

$$\left(\sin^2\theta_{B\parallel}\right)^{-1} = 1 + \left(\frac{\epsilon_1}{\epsilon_2}\right) \rightarrow \left(\frac{\epsilon_1}{\epsilon_2}\right) = \frac{1}{\sin^2\theta_{B\parallel}} - 1$$

$$= \frac{1-\sin^2\theta_{B\parallel}}{\sin^2\theta_{B\parallel}}$$

$$= \frac{1}{\tan^2\theta_{B\parallel}}$$

$\rightarrow \left(\frac{\epsilon_2}{\epsilon_1}\right) = \tan^2\theta_{B\parallel}$ og $\sqrt{\frac{\epsilon_2}{\epsilon_1}} = \tan\theta_{B\parallel}$

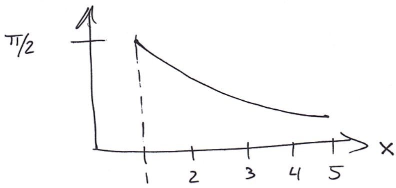
Við höfum

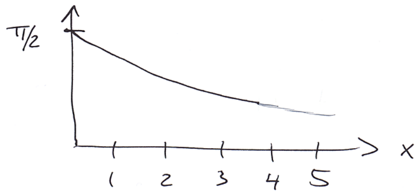
$$\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\rightarrow \quad \tan\theta_{B\parallel} = \sin\theta_c$$

b) Gröf af θ_c og $\theta_{B\parallel}$ vs $x = \frac{\epsilon_1}{\epsilon_2}$

$\theta_c = \arcsin\frac{1}{\sqrt{x}}$ $\qquad$ $\theta_{B\parallel} = \arcsin\frac{1}{\sqrt{1+x}}$





P11-4

Hertz-tvískaut með longd L á z-ás

Spóltvískaut með flöt S í x-y-sléttu

Sama I_0 og ω

fjarsvið

EP: $E_\theta(R) = i\frac{I_0 L}{4\pi}\left(\frac{e^{-i\beta R}}{R}\right)\eta_0 \beta \sin\theta$

$\rightarrow E_\theta(Rt) = -\frac{I_0 \eta_0 \beta \sin\theta}{4\pi R} L \cdot \sin(\omega t - \beta R)$

MP: $E_\phi(R) = \frac{\omega\mu_0 m}{4\pi}\left(\frac{e^{-i\beta R}}{R}\right)\beta \sin\theta$,

$\rightarrow E_\phi(Rt) = \frac{I_0 \eta_0 \beta \sin\theta}{4\pi R}\left(\frac{2\pi S}{\lambda}\right)\cos(\omega t - \beta R)$

$m = I_0 S$

$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$

$\lambda = \frac{2\pi}{\beta}$

$\beta = \frac{\omega}{c} = \omega\sqrt{\epsilon_0\mu_0}$

bui fost

$$\frac{E_\theta^2(Rt)}{\left(\frac{I_0 \eta_0 \beta \sin\theta}{4\pi R}\right)^2 L^2} + \frac{E_\phi^2(Rt)}{\left(\frac{I_0 \eta_0 \beta \sin\theta}{4\pi R}\right)^2 \left(\frac{2\pi S}{\lambda}\right)^2} = 1$$

→ Ellipse stanten

og hringstanten ef $L = \frac{2\pi S}{\lambda}$