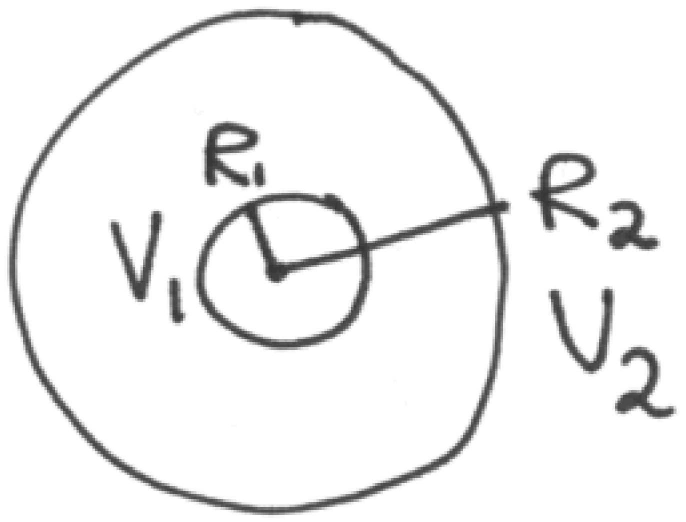
1 Tvær langar sívalningsskeljar



φ og z - samhverfa gera mættið aðeins fall af r

Hér má beita margs konar aðferðum, ég sýni beina heildun

$$\nabla^2 V(r) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0$$

leysum því

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = 0 \quad \rightarrow \quad \frac{d}{dr}\left(r\frac{dV}{dr}\right) = 0$$

$$\rightarrow \quad r\frac{dV}{dr} = C_1 \quad \text{eða} \quad \frac{dV}{dr} = \frac{C_1}{r}$$

$$\rightarrow V(r) = C_1 \ln r + C_2$$

Nú þarf að uppfylla jaðarskilyrðin

$$V(R_1) = V_1 \qquad C_1 \ln R_1 + C_2 = V_1$$

$$V(R_2) = V_2 \qquad C_1 \ln R_2 + C_2 = V_2$$

$$\begin{pmatrix} \ln R_1 & 1 \\ \ln R_2 & 1 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$C_1 = \frac{V_1 - V_2}{\ln R_1 - \ln R_2}$$

$\rightarrow$

$$C_2 = V_1 - \frac{V_1 - V_2}{\ln R_1 - \ln R_2} \ln R_1$$

Lausnin er því

$$V(r) = \frac{(V_1 - V_2)}{(\ln R_1 - \ln R_2)} \ln r + V_1 - \frac{(V_1 - V_2)}{(\ln R_1 - \ln R_2)} \ln R_1$$

$$= \frac{V_1 - V_2}{\ln R_1 - \ln R_2} \cdot \ln\left(\frac{r}{R_1}\right) + V_1$$

$$= \frac{V_1 - V_2}{\ln\left(\frac{R_1}{R_2}\right)} \ln\left(\frac{r}{R_1}\right) + V_1$$

Sem greinilega uppfyllir bæði jaðarskilyrðin.

Samræyrea lausnina

$$\nabla^2 V(r) = 0$$

Lausnin er á almenna forminu

$$V(r) = C_1 \ln r + C_2$$

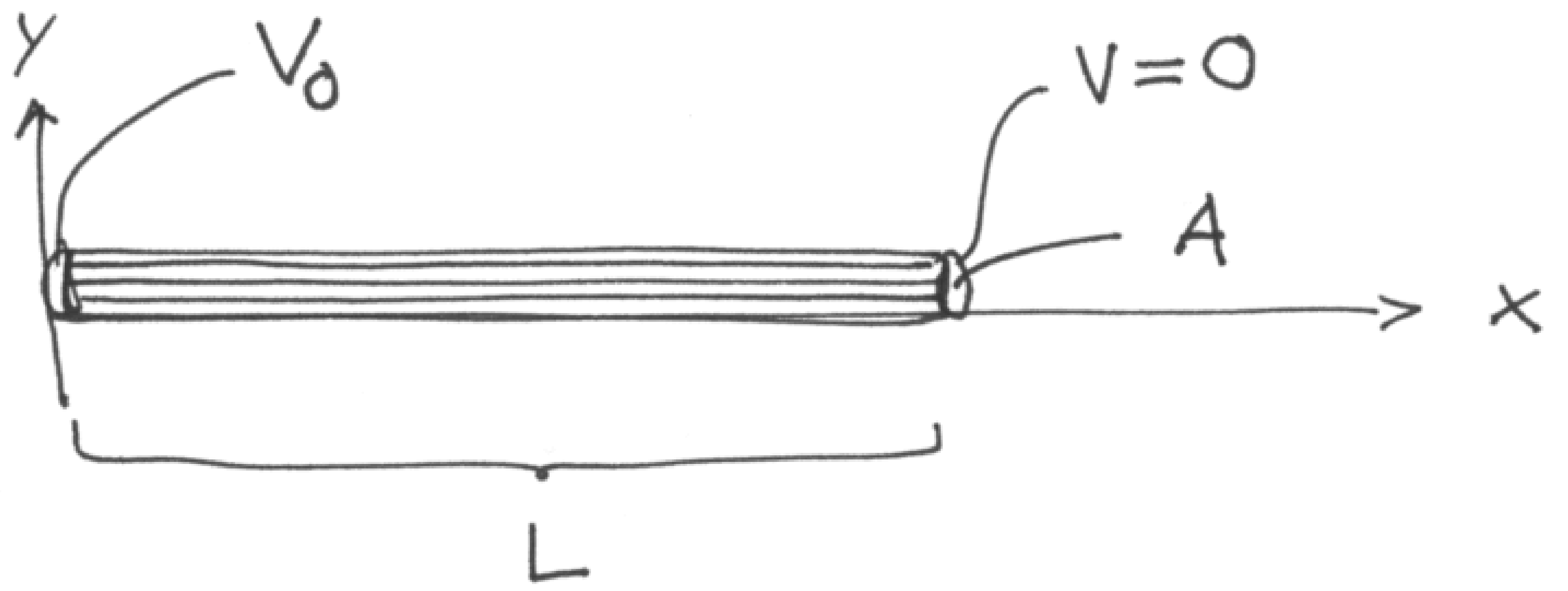
$$\longrightarrow \quad r\frac{d}{dr}V(r) = \text{fasti}$$

$$\longrightarrow \quad \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}V(r)\right) = 0$$

sem gæti litið sérstaklega

fegnir okkar lausn

2



$$\rho(x) = \rho_0 + \rho_1 e^{-x/d}$$

Ákveða I og $\frac{dP}{dx}$ (afltap á lengdar ein.)

Gerum ráð fyrir ómskri leiðni

þekkjum $\bar{J} = \sigma \bar{E}$

$\bar{E} = \rho \bar{J}$

$$\bar{E} = \rho(x)\,\hat{a}_x J = \rho(x)\,\hat{a}_x \frac{I}{A}$$

$$V_0 = -\int_L^0 \bar{E}\cdot\hat{a}_x\,dx = \frac{I}{A}\int_0^L \rho(x)\,dx$$

$$V_0 = \frac{I}{A}\int_0^L \left(\rho_0 + \rho_1 e^{-x/d}\right) dx = \frac{I}{A}\left(\rho_0 L - \rho_1 d\left(e^{-L/d} - 1\right)\right)$$

$$= \frac{I}{A}\left[\rho_0 L + \rho_1 d\left(1 - e^{-L/d}\right)\right] = \frac{IL}{A}\rho_0\left[1 + \frac{\rho_1 d}{\rho_0 L}\left(1 - e^{-L/d}\right)\right]$$

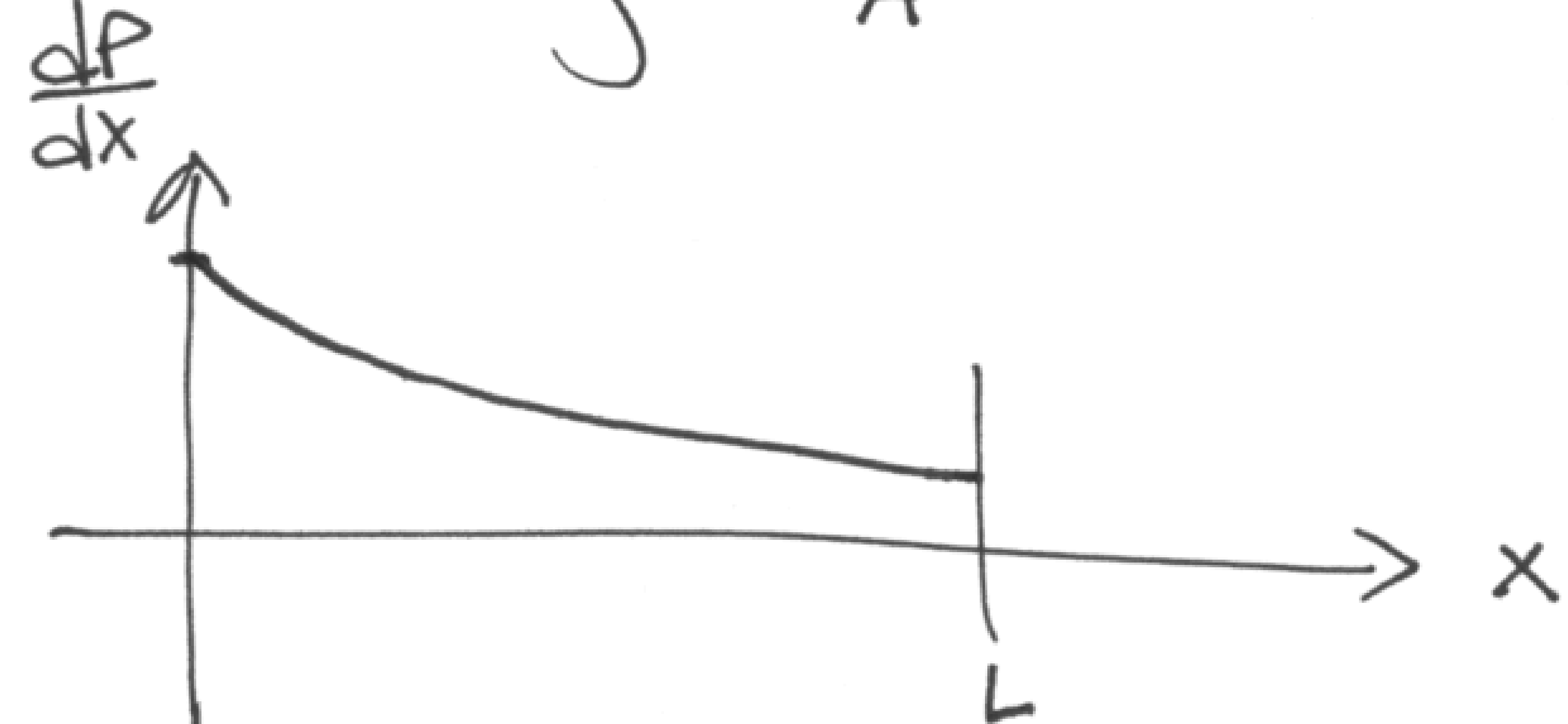
$$\longrightarrow \boxed{I = V_0\left(\frac{A}{L\rho_0}\right)\frac{1}{1 + \frac{\rho_1 d}{\rho_0 L}\left(1 - e^{-L/d}\right)}}$$

Höfum

$$P = \int_V \bar{E}\cdot\bar{J}\, dv \longrightarrow dP = \bar{E}\cdot\bar{J}\, A\, dx$$

$$= \rho(x)\frac{I}{A}\, I\, dx$$

$$\longrightarrow \frac{dP}{dx} = \rho(x)\frac{I^2}{A}$$



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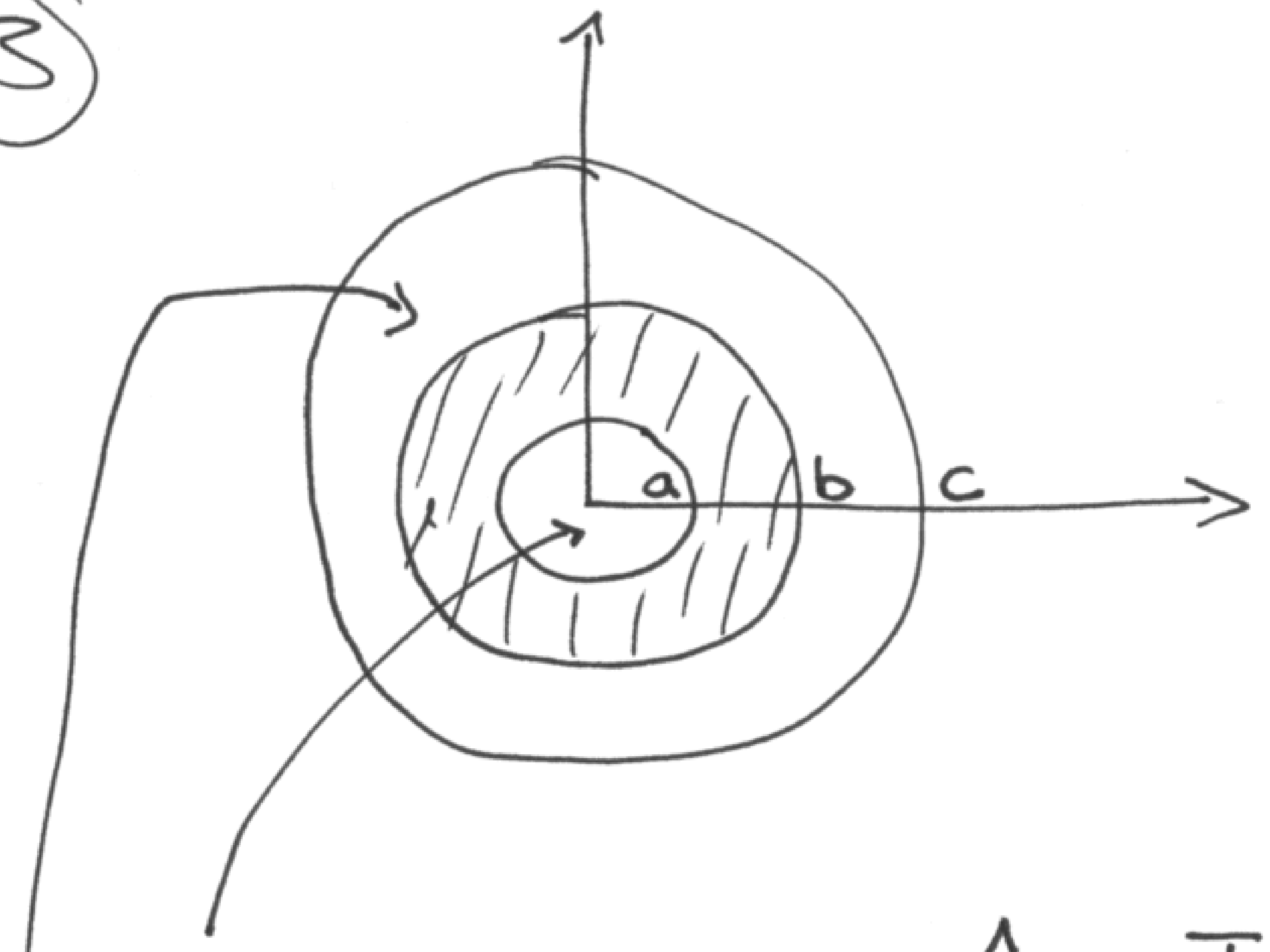

$+I \longrightarrow \quad \bar{J}_i = \hat{a}_z \dfrac{I}{\pi a^2}$

$-I \longrightarrow \quad \bar{J}_o = -\hat{a}_z \dfrac{I}{\pi(c^2-b^2)}$

straumþéttleikinn í innri og ytri leiðara

Nú gildir $\bar{\nabla} \times \bar{H} = \bar{J}$

eða $\oint \bar{H} \cdot d\bar{l} = I$

Samhverfa kerfisins leiðir til:

$$H_\phi = \frac{Ir}{2\pi a^2} \quad \text{fyrir } r \leq a$$

$$H_\phi = \frac{I}{2\pi r} \quad \text{f. } a \leq r \leq b$$

$$H_\phi = \frac{I(c^2-r^2)}{2\pi r(c^2-b^2)} \quad \text{f. } b \leq r \leq c$$

$$H_\phi = 0 \quad \text{f. } r \geq c$$

Reiknum $\overline{B}$

Munum $\overline{B} = \mu_0(1+\chi_m)\overline{H} = \mu_0\mu_r\overline{H} = \mu\overline{H}$

því fæst fyrir seglandi efnið $a \le r \le b$

$$\overline{B} = \mu_0(1+\chi_m)\overline{H}$$

fyrir hin svæðin er $\chi_m = 0$ og þar fæst því alltaf $\overline{B} = \mu_0\overline{H}$

Reiknum $\overline{M}$

$\overline{M} = \chi_m\overline{H}$ því fæst $\overline{M} = 0$ nema í seglandi efninu

þar fæst $\overline{M} = \chi_m \dfrac{I}{2\pi r}\hat{a}_\phi$

yfirborðs- eða rúmseglunarstraumar ?

$$\bar{J}_{ms} = \bar{M} \times \hat{a}_n \qquad \bar{J}_m = \bar{\nabla} \times \bar{M}$$

$\bar{J}_m = 0 \rightarrow$ engir rúmstraumar

<u>yfirborðsstraumar</u>

<u>$r = a$</u> hér er $\hat{a}_n$ út úr segulvírtaefninu $-\hat{a}_r$

$$\rightarrow \bar{J}_{ms}(a) = \bar{M} \times (-\hat{a}_r) = \chi_m \frac{I}{2\pi a} \hat{a}_z$$

með heildar yfirborðsstraum hér

$$I_{ms}(a) = \chi_m I \quad \text{í stefnu } \hat{a}_z$$

$\underline{r=b}$

hér er $\hat{a}_n$ út úr seguldvirkta efninu $\hat{a}_r$

$$\rightarrow \bar{J}_{ms}(b) = \bar{M} \times \hat{a}_r = -\chi_m \frac{I}{2\pi b} \hat{a}_z$$

~~með~~ heildar yfirborðsstraum hér

$$I_{ms}(b) = \chi_m I \quad \text{í stefnu} \quad -\hat{a}_z$$

Summa yfirborðstraumanna er 0

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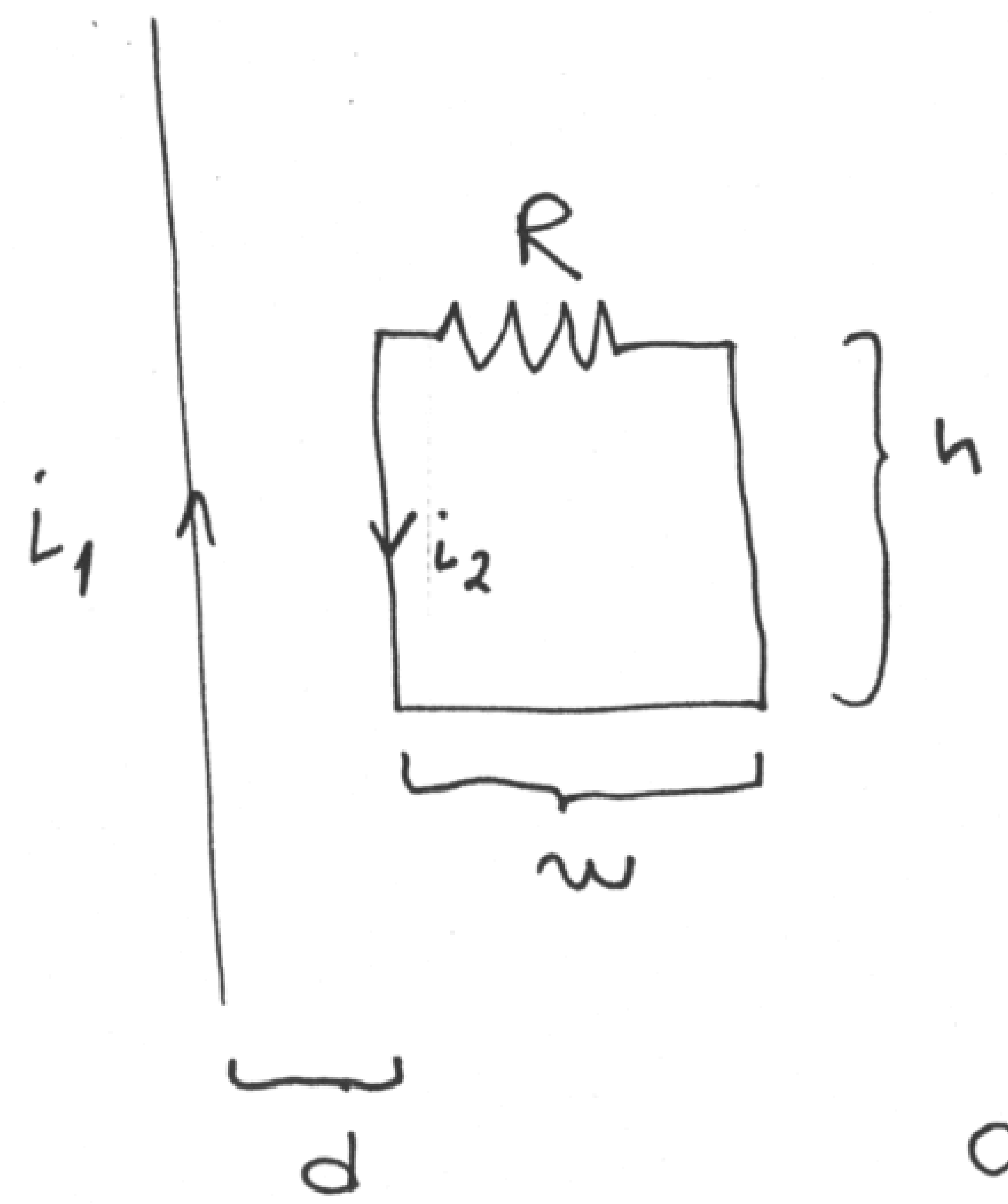
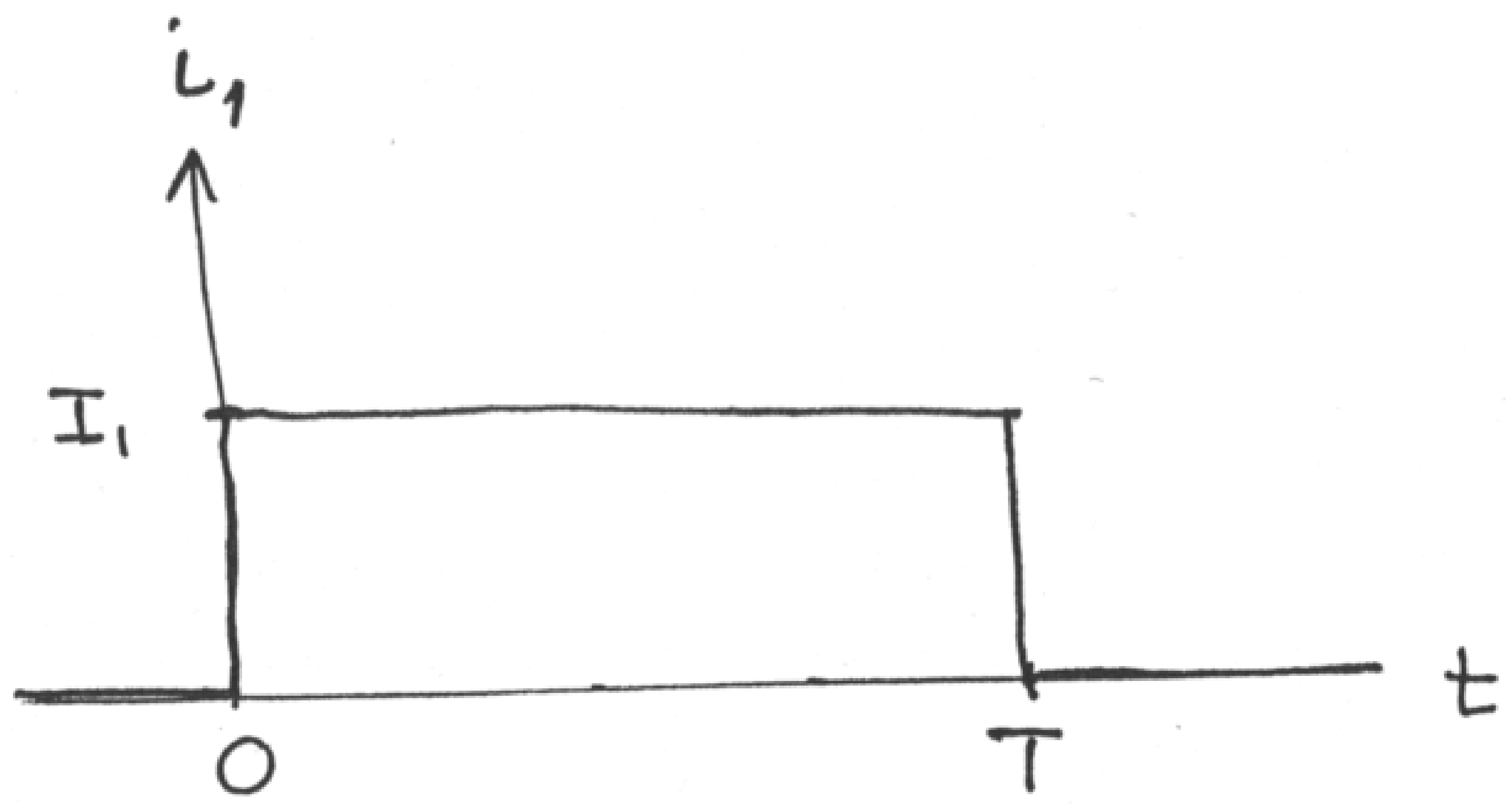


Gerum ráð fyrir að i_1 sé kassalaga púls



a) Finna spanstrauminn i_2 í lykkjunni með sjálfspanið L

b) finn orkuna sem eyðist í R ef $T \gg L/R$

Kirchhoff $\rightarrow$ $L_{12}\frac{di_1}{dt} - L\frac{di_2}{dt} - Ri_2 = 0$

spennufall í viðnámi

íspenna vegna straumpúls víxlspan

Andóf vegna sjálfspans

$$L_{12} = \frac{\Phi_{12}}{i_1} = \frac{h}{i_1}\int_d^{d+w} B_{12}\,dr = \frac{h}{i_1}\int_d^{d+w} \frac{\mu_0 i_1}{2\pi r}\,dr = \frac{\mu_0 h}{2\pi}\ln\left(1+\frac{w}{d}\right)$$

a) Afleiðan $\frac{di_1}{dt}$ af kassapúlsinum gefur

$$\frac{L_{12}}{L}\frac{di_1}{dt} = \frac{L_{12}}{L} I_1\left\{\delta(t) - \delta(t-T)\right\}$$

Dirac δ-fallið

$$\rightarrow \quad \frac{di_2}{dt} + \frac{R}{L}i_2 = \frac{L_{12}}{L} I_1\left\{\delta(t) - \delta(t-T)\right\}$$

fyrir jöfnuna $\dot{y} + p(t)y = q(t)$ er lausnin

$$y(t) = y(t_0)\, e^{-P(t)} + e^{-P(t)} \int_{t_0}^{t} e^{P(s)} q(s)\, ds\,, \qquad P(t) = \int_{t_0}^{t} p(s)\, ds$$

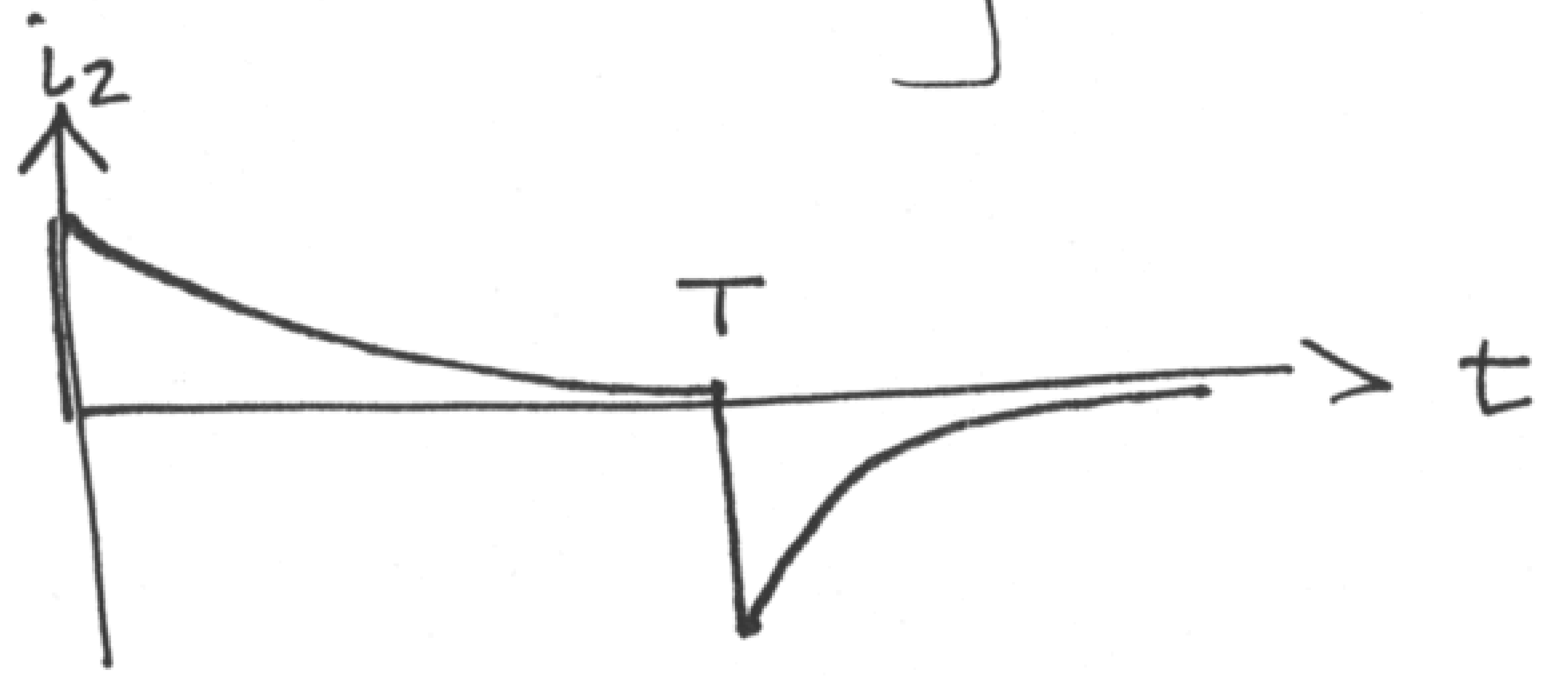
því er okkar lausn

$$i_2(t) = i_2(0)\, e^{-\frac{Rt}{L}} + \frac{L_{12} I_1}{L} \left\{ e^{-\frac{Rt}{L}} - \theta(t-T)\, e^{-\frac{R}{L}(t-T)} \right\}$$

með

$$\theta(t-T) = \begin{cases} 0 & \text{ef } t<T \\ 1 & \text{ef } t>T \end{cases} \qquad i_2(0) = 0$$

$$\rightarrow \quad i_2(t) = \frac{L_{12} I_1}{L} \left\{ e^{-\frac{Rt}{L}} - \theta(t-T)\, e^{-\frac{R}{L}(t-T)} \right\}$$



Ef $T >> \frac{L}{R} \rightarrow e^{-\frac{RT}{L}} << 1$

og løysnin etter $t > T$ er på

$$i_2(t) \simeq -\frac{L_{12} I_1}{L} e^{-\frac{R}{L}(t-T)}$$

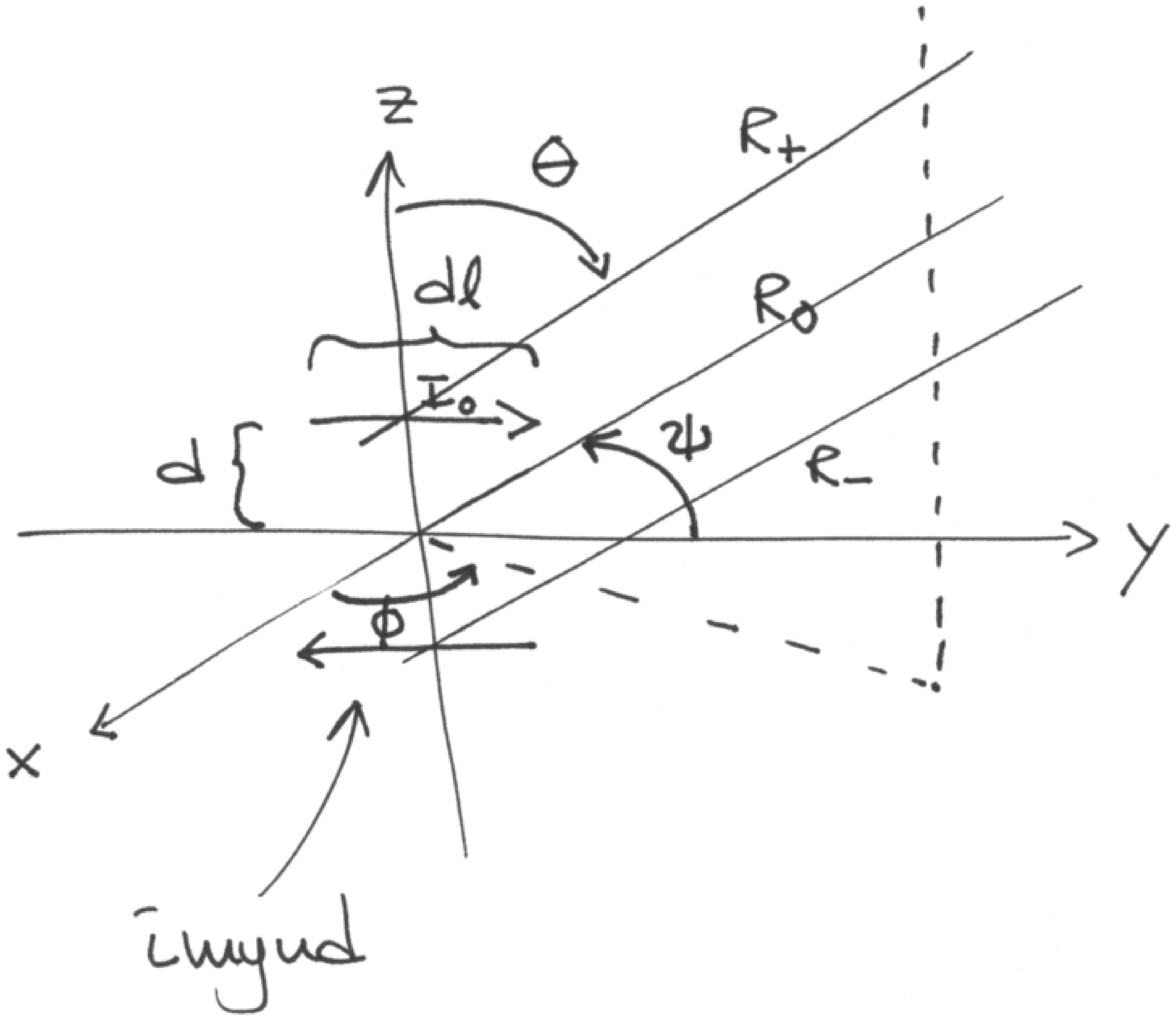
b) Orkau eydd i R ef $T >> \frac{L}{R}$

$$W = \int_0^\infty i_2^2(t) R\, dt \simeq \left(\frac{L_{12} I_1}{L}\right)^2 R \int_0^\infty e^{-\frac{2Rt}{L}} dt$$

$$+ \left(\frac{L_{12} I_1}{L}\right)^2 R \int_T^\infty e^{-\frac{2R(t-T)}{L}} dt$$

$$= 2\left(\frac{L_{12} I_1}{L}\right)^2 R \int_0^\infty e^{-\frac{2Rt}{L}} dt = \frac{1}{L}(L_{12} I_1)^2$$

(5)



(15)

Notum því (11-19b) með

ψ sem hefur tekið hlutverk

θ

Einnig er fasaþáttur með

$e^{-i\beta R_\pm}$ þar sem hliðrunin

um $d \ll R_0$ leiðir til

$\rightarrow R_\pm \simeq R_0 \mp d\cos\theta$

En í nefnara nægum við

$R_\pm \sim R_0$

$$R_+ = \sqrt{R_0^2 + d^2 - 2R_0 d\cos\theta}$$
$$= R_0\left(1 + \frac{d^2}{R_0^2} - 2\frac{d}{R_0}\cos\theta\right)^{1/2}$$
$$\simeq R_0 - d\cos\theta + o\left(\left(\frac{d}{R_0}\right)^2\right)$$

því fæst

$$E_\psi^+ = \frac{i I_0 dl\, \eta_0 \beta}{4\pi R_0} e^{-i\beta(R_0 - d\cos\theta)} \sin\psi$$

$$E_\psi^- = -\frac{i I_0 dl\, \eta_0 \beta}{4\pi R_0} e^{-i\beta(R_0 + d\cos\theta)} \sin\psi$$

spegilmynd

Frá rúmfræði fæst: $\quad \hat{a}_R \times \hat{a}_y = 1 \cdot 1 \cdot \sin\Psi$

$$\rightarrow \sin\Psi = |\hat{a}_R \times \hat{a}_y| = |(\hat{a}_x \sin\theta\cos\phi + \hat{a}_y \sin\theta\sin\phi + \hat{a}_z\cos\theta) \times \hat{a}_y|$$

$$= |\hat{a}_z \sin\theta\cos\phi - \hat{a}_x \cos\theta|$$

$$= \sqrt{\sin^2\theta\cos^2\phi + \cos^2\theta} = \sqrt{\sin^2\theta(1-\sin^2\phi) + \cos^2\theta}$$

$$= \sqrt{1 - \sin^2\theta\sin^2\phi}$$

$$E_\Psi = E_\Psi^+ + E_\Psi^- = i\frac{I_0 dl}{2\pi}\left(\frac{e^{-i\beta R_0}}{R_0}\right)\eta_0\beta\sin(\beta d\cos\theta)\sqrt{1-\sin^2\theta\sin^2\phi}$$

Mynsturfallið er því

$$F(\theta,\phi) = |\sin(\beta d\cos\theta)|\sqrt{1-\sin^2\theta\sin^2\phi}$$

a) i xy-slettu $\theta = \frac{\pi}{2}$ $F_{xy}(\frac{\pi}{2}, \phi) = 0$

b) i xz-slettu $\phi = 0$ $F_{xz}(\theta, 0) = |\sin(\beta d \cos\theta)|$

c) i yz-slettu $\phi = \frac{\pi}{2}$ $F_{yz}(\theta, \frac{\pi}{2}) = |\sin(\beta d \cos\theta) \cdot \cos\theta|$

d) $d = \frac{\lambda}{4} \rightarrow \beta d = \frac{2\pi}{\lambda} \frac{\lambda}{4} = \frac{\pi}{2}$

$F_{xz}(\theta, 0) = |\sin(\frac{\pi}{2} \cos\theta)|$ $F_{yz}(\theta, \frac{\pi}{2}) = |\sin(\frac{\pi}{2} \cos\theta) \cos\theta|$