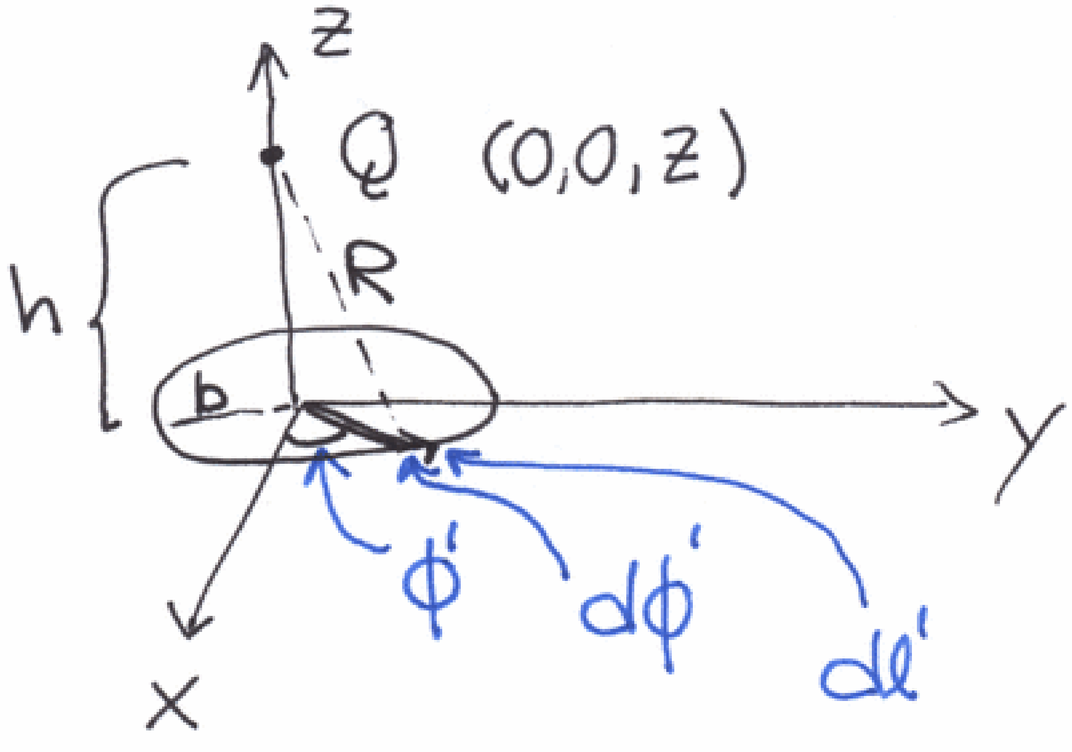
1



línuþrýmið $d\ell'$ veldur mætti í punktinum Q

$$dV_Q = \frac{\rho_\ell \, b \, d\phi'}{4\pi\epsilon_0 \sqrt{(z^2+b^2)}}$$

Heilda þarf upp hrif ~~hleðslu~~ hleðslunnar í öllum hringnum, eina breytan er ϕ'

$$V_Q = \frac{\rho_\ell b}{4\pi\epsilon_0 \sqrt{z^2+b^2}} \int_0^{2\pi} d\phi'$$

$$= \frac{\rho_\ell b}{2\epsilon_0 \sqrt{z^2+b^2}}$$

Kraftsviðið á Q er $\bar{F}_Q = Q \bar{E}_Q$

því þarf að finna $\bar{E}_Q = -\bar{\nabla} V_Q$

Eina breytan er nú z í Q

$$\bar{E}_Q = -\bar{\nabla} V_Q = -\hat{a}_z \frac{dV_Q}{dz}$$

$$= \hat{a}_z \frac{\rho_\ell b z}{2\epsilon_0 \left(\sqrt{z^2+b^2}\right)^3} = \hat{a}_z \frac{\rho_\ell b z}{2\epsilon_0 (z^2+b^2)^{3/2}}$$

Þegar $\rho_\ell, Q > 0$ þá er krafturinn frá hringnum ofan og neðan hans

Í punktinum $z = h$

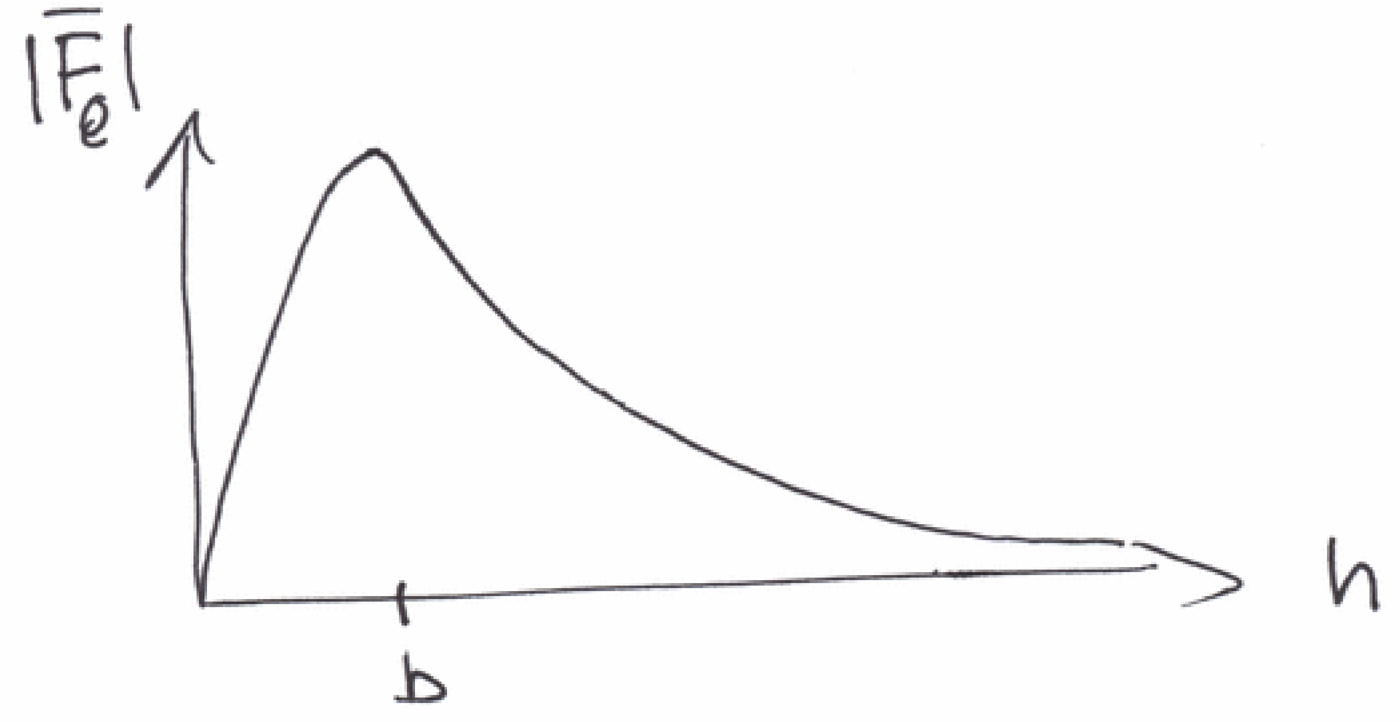
$$\bar{F}_Q = Q\bar{E}_Q = \hat{a}_z \frac{Q \rho_\ell b h}{2\epsilon_0 (h^2+b^2)^{3/2}}$$

$$\bar{F}_Q \xrightarrow[h \gg b]{} \hat{a}_z \frac{Q \rho_\ell b}{2\epsilon_0 h^2}$$

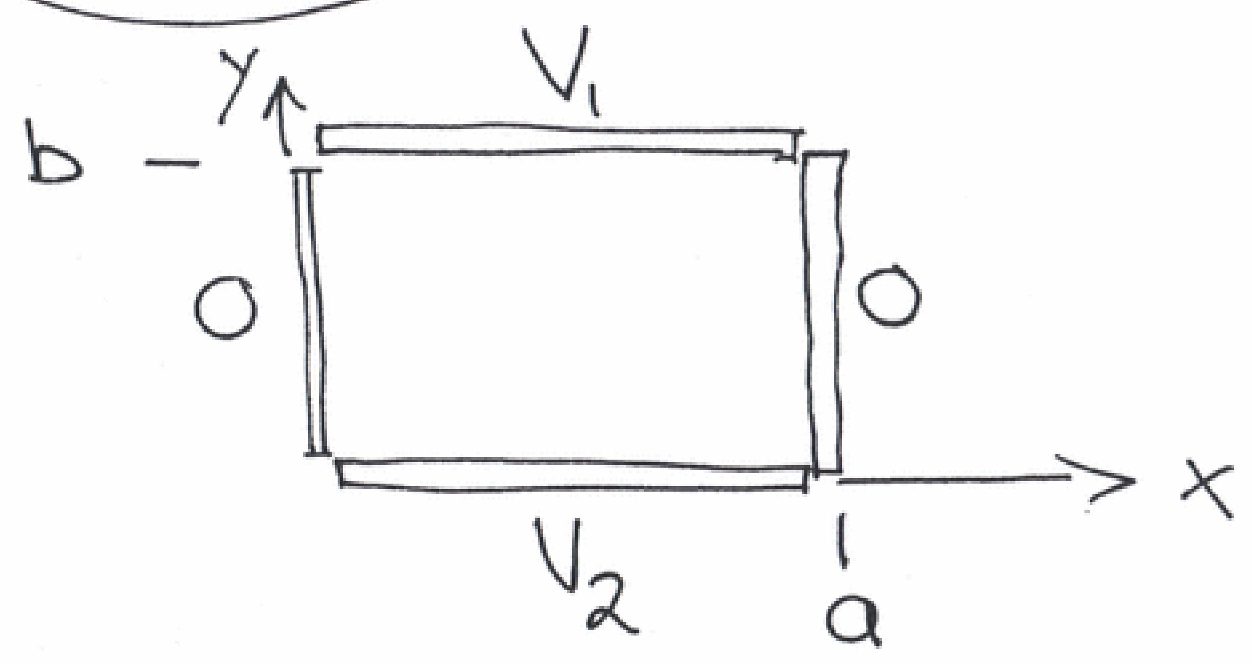
fellur með $\sim h^{-2}$

$$\bar{F}_Q \xrightarrow[h \to 0]{} 0$$

óstöðugur jafnvægispunktur fyrir Q



2



Kartísk hnit

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)V = 0$$

$$k_x^2 + k_y^2 = 0$$

Veljum $k_x \in \mathbb{R}$ og $k_x > 0$, þá verður lausnin að vera

$$V(x,y) = \sum_n \sin(k_n x)\left[A_n \sinh(k_n y) + B_n \cosh(k_n y)\right]$$

með $k_n = \frac{n\pi}{a}$

til þess að uppfylla að $V(0,y) = 0$ og $V(a,y) = 0$

Hin jaðarskilyrðin

$$\underline{V(x,0) = V_2} = \sum_n B_n \sin(k_n x)$$

Bera saman við fourier röðina á bls 179-180

þá fæst

$$B_n = \begin{cases} \frac{4V_2}{n\pi} & \text{ef } n \text{ er oddatala} \\ 0 & \text{ef } n \text{ er slétt} \end{cases}$$

$$\underline{V(x,b) = V_1} = \sum_n \sin(k_n x)\left[A_n \sinh(k_n b) + B_n \cosh(k_n b)\right]$$

samskonar fourier röð aftur. Því fæst

$$A_n \sinh(k_n b) + \underbrace{B_n}_{\frac{4V_2}{n\pi}} \cosh(k_n b) = \begin{cases} \frac{4V_1}{n\pi} , & n\ \cancel{er}\ \text{oddatala} \\ 0 , & n \text{ er slétt} \end{cases}$$

Þui fæst

$$A_n = \begin{cases} \dfrac{4}{\pi n \sinh(k_n b)} \left(V_1 - V_2 \cosh(k_n b) \right) & , n \text{ er oddetalar} \\ 0 & , n \text{ er slett talar} \end{cases}$$

3

Lögmál Ampères

$$\oint \overline{B} \cdot d\overline{l} = \mu_0 I$$

getur fyrir gegnheila sívalninginn

$$\overline{B}_1 = \hat{a}_\phi \frac{\mu_0 I}{2\pi b^2} r \qquad 0 \leq r \leq b$$

$$\overline{B}_2 = \hat{a}_\phi \frac{\mu_0 I}{2\pi r} \qquad r \geq b$$

$$\overline{B} = \overline{\nabla} \times \overline{A} \qquad \overline{B} = \hat{a}_\phi \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right)$$

sívalningurinn er einsleitur óendanlega langur

-> r er eina rúmbreytan

$$\rightarrow \overline{B} = -\hat{a}_\phi \frac{\partial A_z}{\partial r}$$

Heildum

$$\overline{A}_1 = \hat{a}_z \left\{ -\frac{\mu_0 I}{4\pi} \left(\frac{r}{b}\right)^2 + C_1 \right\} \qquad 0 \leq r \leq b$$

$$\overline{A}_2 = \hat{a}_z \left\{ -\frac{\mu_0 I}{2\pi} \ln r + C_2 \right\} \qquad r \geq b$$

Samfelldni í $r = b$, $\bar{A}_1 = \bar{A}_2$

$$\longrightarrow \quad C_2 = \frac{\mu_0 I}{4\pi}\left(2\ln b - 1\right) + C_1$$

$$\rightarrow \quad \bar{A}_2 = \hat{a}_z \left\{ -\frac{\mu_0 I}{4\pi}\left[\ln\left(\frac{r}{b}\right)^2 + 1\right] + C_1 \right\}$$

C_1 er frjáls stuðull samkvæmt ~~þessari~~ að því, enda hefur hann engin áhrif á B.

4

Hægri hring skautuð bylgja táknuð með fasor

$$\bar{E}_i(z) = E_{i0}(\hat{a}_x - i\hat{a}_y)e^{-i\beta z}$$

fellur lóðrétt á kjörleiðandi flöt í $z = 0$

a) Ákvarðið skautun speglaðu bylgjunnar

Gerum ráð fyrir (reynum)

$$\bar{E}_r(z) = (\hat{a}_x E_{rx} + \hat{a}_y E_{ry})e^{i\beta z}$$

Jaðarskilyrði

$$0 = \bar{E}_i(0) + \bar{E}_r(0) \rightarrow E_{i0}(\hat{a}_x - i\hat{a}_y) + \hat{a}_x E_{rx} + \hat{a}_y E_{ry} = 0$$

$$\rightarrow E_{rx} = -E_{i0}\,, \quad +iE_{i0} = E_{ry}$$

$$\rightarrow \quad \overline{E}_r(z) = E_{io}\left(-\hat{a}_x + i\,\hat{a}_y\right)e^{i\beta z}$$

Vinstri hring skautuð bylgja

$*$

b) finna yfirborðsstrauminn á leiðandi fletinum í $z=0$

Almennt gildir

$$\hat{a}_{n2} \times (\overline{H}_1 - \overline{H}_2) = \overline{J}_s$$

í kjörleiðara er $\overline{H}_2 = 0$

$$\overline{H}_i(0) = \frac{1}{\eta_0}\hat{a}_z \times \overline{E}_i(0) = \frac{E_{io}}{\eta_0}(\hat{a}_x i + \hat{a}_y)$$

$$\overline{H}_r(0) = \frac{1}{\eta_0}(-\hat{a}_z) \times \overline{E}_r(0) = \frac{E_{io}}{\eta_0}(\hat{a}_x i + \hat{a}_y)$$

$$\overline{H}_1(0) = \overline{H}_i(0) + \overline{H}_r(0) = \frac{2E_{io}}{\eta_0}(\hat{a}_x i + \hat{a}_y)$$

$$\boxed{\begin{aligned}\overline{J}_s &= -\hat{a}_z \times \overline{H}_1(0) \\ &= \frac{2E_{io}}{\eta_0}(\hat{a}_x - i\hat{a}_y)\end{aligned}}$$

c) finna t-háða heildar rafsviðið í 1

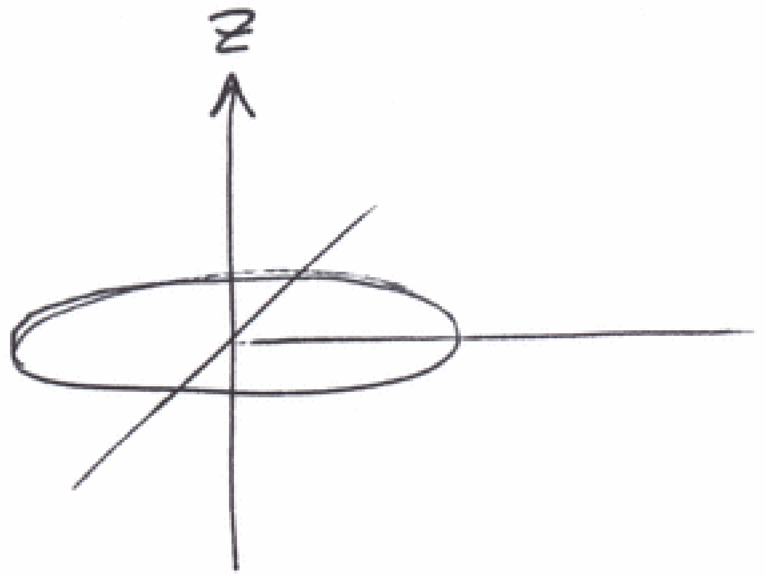
$$\overline{E}_1(z,t) = \mathrm{Re}\left\{\left[\overline{E}_i(z) + \overline{E}_r(z)\right]e^{i\omega t}\right\}$$

$$\bar{E}_1(z,t) = \mathcal{Re}\left\{E_{i0}\left[(\hat{a}_x - i\hat{a}_y)e^{-i\beta z} + (-\hat{a}_x + i\hat{a}_y)e^{i\beta z}\right]e^{i\omega t}\right\}$$

$$= \mathcal{Re}\left\{E_{i0}e^{i\omega t}\left[-2i(\hat{a}_x - i\hat{a}_y)\sin(\beta z)\right]\right\}$$

$$= 2E_0 \sin(\beta z)\left\{\hat{a}_x \sin(\omega t) - \hat{a}_y \cos(\omega t)\right\}$$

⑤



$I_0 \cos(\omega t)$

Segultvískaut, fjarsvið

$$E_\phi = \frac{\omega \mu_0 m}{4\pi}\left(\frac{e^{-i\beta R}}{R}\right)\beta \sin\theta$$

$$H_\theta = -\frac{\omega \mu_0 m}{4\pi \eta_0}\left(\frac{e^{-i\beta R}}{R}\right)\beta \sin\theta$$

$$m = I_0 \pi b^2, \qquad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \simeq 120\pi$$

a)

$$P_r = \oint \overline{\mathcal{P}}_{ave} \cdot d\bar{s}, \qquad \overline{\mathcal{P}}_{ave} = \frac{1}{2}\mathrm{Re}\left|\bar{E} \times \bar{H}^*\right|$$

$$P_r = -\frac{1}{2}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \; E_\phi H_\theta^* R^2$$

$$= \frac{1}{2}\left(\frac{\omega \mu_0 m}{4\pi}\right)^2 \frac{\beta^2}{\eta_0} 2\pi \int_0^{\pi} \sin^3\theta \, d\theta$$

$$P_r = \frac{2}{3}\left(\frac{\omega \mu_0 m}{4\pi}\right)^2 \frac{\beta^2}{\eta_0} 2\pi, \quad \beta = \frac{2\pi}{\lambda}$$

$$= \frac{2}{3}\left(\frac{\omega \mu_0 I_0 \pi b^2}{4\pi}\right)^2 2\pi \left(\frac{2\pi}{\lambda}\right)^2 \frac{1}{120\pi}$$

$$= \frac{2}{3}\left(\frac{\omega \mu_0 I_0 \pi b^2}{2\lambda}\right)^2 \frac{1}{60}$$

$$= \frac{1}{2} I_0^2 \left\{ \frac{\omega^2 \mu_0^2 \pi^2 b^4}{180 \lambda^2} \right\} = \frac{1}{2} I_0^2 R_r$$

$$\rightarrow R_r = \frac{\omega^2 \mu_0^2 \pi^2 b^4}{180 \lambda^2} = 80\pi^2 \left(\frac{\beta \pi b}{\lambda}\right)^2$$

---

b)

$$\eta_r = \frac{R_r}{R_r + R_\ell}$$

tap í vír

veit σ_0, Lengd hrings $2\pi b = \ell$

þvermál $S = \pi a^2$

$$R_\ell = \frac{\ell}{\sigma_0 S} = \frac{2\pi b}{\sigma_0 \pi a^2} = \frac{2b}{a^2 \sigma_0}$$

við lága tíðni, við hærri tíðni er leiðnin á yfirborðinu

$$R_s = \sqrt{\frac{\omega \mu_0}{2\sigma_0}} \qquad (11\text{-}49)$$

og

$$R_\ell = R_s \left(\frac{2\pi b}{2\pi a} \right) \qquad (11\text{-}48)$$

$$= R_s \frac{b}{a}$$

$$\eta_r = \frac{1}{1 + \frac{R_\ell}{R_r}} = \frac{1}{1 + \frac{\frac{b}{a}\sqrt{\frac{\omega\mu_0}{2\sigma_0}}}{80\pi^2 \left(\frac{\beta\pi b}{\lambda}\right)^2}}$$

Þetta má umskrifa á margan hátt.