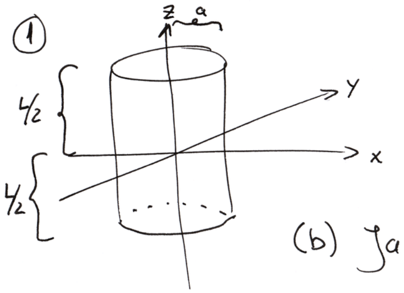
(1)




$\overline{M} = M_0 \hat{a}_x$

(a) $\overline{M} = M_0\{\hat{a}_r \cos\phi - \hat{a}_\phi \sin\phi\}$

(b) Jafngildar segulhleðslur

<u>Fyrir bol</u>

$$\rho_m = -\overline{\nabla}\cdot\overline{M} = \frac{1}{r}\frac{\partial}{\partial r}(rM_r) + \frac{1}{r}\frac{\partial}{\partial \phi}M_\phi$$

$$= \frac{M_0\cos\phi}{r} - \frac{M_0\cos\phi}{r} = 0$$

<u>Á bogayfirborðinu</u>

$$\rho_{ms} = \overline{M}\cdot\hat{a}_n$$

$$= M_0\{\hat{a}_r\cos\phi - \hat{a}_\phi\sin\phi\}\cdot\hat{a}_r = M_0\cos\phi$$

(2)

Á endunum gildir $\pm\overline{M}\cdot\hat{a}_z = 0$

→ aðeins bogayfirborðið ber segulhleðslu.

$$\rho_{ms}(\phi) = M_0\cos\phi$$

(c)
<u>Heildarhleðslan</u> $\int_0^{2\pi} d\phi\, M_0\cos\phi\cdot a\cdot L = 0$

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(d) Jafngildir straumar

$$\overline{J}_m = \overline{\nabla}\times\overline{M} \quad \text{og} \quad \overline{J}_{ms} = \overline{M}\times\hat{a}_n$$

$$\overline{J}_m = \overline{\nabla}\times\overline{M} = \hat{a}_z\frac{1}{r}\left\{\frac{\partial}{\partial r}(rM_\phi) - \frac{\partial M_r}{\partial\phi}\right\}$$

(3)

$$J_m = \hat{a}_z\frac{1}{r}\{-M_0\sin\phi + M_0\sin\phi\} = 0$$

<u>Á bogaflötinum</u>

$$\overline{J}_{ms} = \overline{M}\times\hat{a}_n = M_0\{\hat{a}_r\cos\phi - \hat{a}_\phi\sin\phi\}\times\hat{a}_n$$

$$= -M_0\sin\phi\,\hat{a}_\phi\times\hat{a}_r = M_0\sin\phi\,\hat{a}_z$$

<u>Efri endinn</u> $\hat{a}_n = \hat{a}_z$

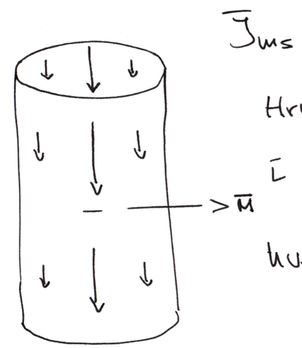
$$\overline{J}_{ms} = M_0\{\hat{a}_r\cos\phi - \hat{a}_\phi\sin\phi\}\times\hat{a}_z$$

$$= -M_0\cos\phi\,\hat{a}_\phi - M_0\sin\phi\,\hat{a}_r$$

og neðri endinn með öfugu formerki

(4)

(d)




Hringstraumur upp bogaflötinn í $\phi = \frac{\pi}{2}$ og niður í $\phi = \frac{3\pi}{2}$

hverfur í $\phi = 0$ og $\pi$

(e)

passar við hægri handar reglu þegar $J_{ms}$ og $\overline{M}$ eru bornir saman.

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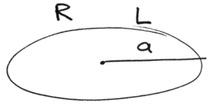
$J_{ms}$ má nota fyrir jaðarskilyrði t.þ.a. leysa $\nabla^2\overline{A} = 0$ innan og utan sívalnings.

$\rho_{ms}$ má nota á sams konar hátt til að leysa $\nabla^2 V_m = 0$ innan og utan sívalnings. . . .

(2) lykkja $\qquad \Phi(t) = \Phi_0 (\Gamma t)^2 e^{-\Gamma t}$, $\quad i(0) = i_0$ $\qquad$ (5)




(a) Finnum $i(t)$

bætum við flæðis-breytingu vegna sjálfspans lykkju

Lögmál Faradays $\qquad \oint_C \bar{E}\cdot d\bar{l} = -\frac{d\Phi}{dt}$

$$\to R\, i(t) = -\frac{d\Phi(t)}{dt} - L\frac{di(t)}{dt}$$

$$\to L\frac{di(t)}{dt} + R\, i(t) = -\Phi_0\left\{-\Gamma(\Gamma t)^2 + 2\Gamma^2 t\right\}e^{-\Gamma t}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{\Phi_0}{L}\left\{-\Gamma(\Gamma t)^2 + 2\Gamma^2 t\right\}e^{-\Gamma t}$$

Notum almenna lausn 1. stigs afleiðujöfnu $\qquad$ (6)

$$y' + p(t)y = q(t)$$

$$y(t) = y(0)e^{-P(t)} + e^{-P(t)}\int_0^t ds\, e^{P(s)} q(s)$$

ef upphaf er í $t=0$ og $P(t) = \int_0^t ds\, p(s)$

Hér $\quad P(t) = \int_0^t ds\, \frac{R}{L} = \frac{R}{L}t \equiv \alpha t$

$$\to i(t) = i_0 e^{-\frac{R}{L}t} + e^{-\frac{R}{L}t}\frac{\Phi_0}{L}\int_0^t ds\, e^{(\frac{R}{L}-\Gamma)s}\left\{-\Gamma(\Gamma s)^2 + 2\Gamma^2 s\right\}$$

notum $\int dx\, e^{-\gamma x}\cdot x^2 = -\frac{\gamma^2x^2+2\gamma x+2}{\gamma^3}e^{-\gamma x}$, $\quad \int dx\, e^{-\gamma x} x = -\frac{(\gamma x+1)e^{-\gamma x}}{\gamma^2}$

$$i(t) = i_0 e^{-\alpha t} + e^{-\alpha t}\frac{\Phi_0}{L}\left[\frac{\left\{(\Gamma^5 - 2\alpha\Gamma^4 + \alpha^2\Gamma^3)t^2 + (+2\alpha\Gamma^3 - 2\alpha^2\Gamma^2)t + 2\alpha\Gamma^2\right\}e^{(\alpha-\Gamma)t}}{\Gamma^3 - 3\alpha\Gamma^2 + 3\alpha^2\Gamma - \alpha^3} + \frac{-2\alpha\Gamma^2}{\Gamma^3 - 3\alpha\Gamma^2+3\alpha^2\Gamma-\alpha^3}\right] \quad (7)$$

$$= i_0 e^{-\alpha t} + e^{-\alpha t}\frac{\Phi_0}{L}\left[\frac{\left\{\Gamma^3(\Gamma-\alpha)^2t^2 + 2\alpha\Gamma^2(\Gamma-\alpha)t + 2\alpha\Gamma^2\right\}e^{(\alpha-\Gamma)t} - 2\alpha\Gamma^2}{(\Gamma-\alpha)^3}\right]$$

Gott er ~~að~~ athuga ~~að~~ $i(0) = i_0$, $\quad i(t) \xrightarrow[t\to+\infty]{} 0$

(b) Hve mikil hleðsla hefur flust þegar $t\to\infty$

$$Q = \int_0^\infty dt\, i(t)$$

$$i(t) = i_0 e^{-\alpha t} + \frac{\Phi_0}{L}\left[\frac{\left\{\Gamma^3(\Gamma-\alpha)^2t^2 + 2\alpha\Gamma^2(\Gamma-\alpha)t + 2\alpha\Gamma^2\right\}e^{-\Gamma t} - 2\alpha\Gamma^2 e^{-\alpha t}}{(\Gamma-\alpha)^3}\right] \quad (8)$$

notum $\quad \int_0^\infty dx\, e^{-\gamma x} = \frac{1}{\gamma}$ $\qquad \int_0^\infty dx\, e^{-\gamma x}x^2 = \frac{2}{\gamma^3}$

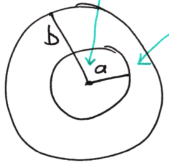
$$\int_0^\infty dx\, e^{-\gamma x}x = \frac{1}{\gamma^2}$$

$$\to Q = \frac{i_0}{\alpha} + \frac{\Phi_0}{L}\underbrace{\left[\frac{\Gamma^3}{\Gamma-\alpha}\frac{2}{\Gamma^3} + \frac{2\alpha\Gamma^2}{(\Gamma-\alpha)^2}\frac{1}{\Gamma^2} + \frac{2\alpha\Gamma^2}{(\Gamma-\alpha)^3}\frac{1}{\Gamma} - \frac{2\alpha\Gamma^2}{(\Gamma-\alpha)^3}\frac{1}{\alpha}\right]}_{=0}$$

$$= \frac{i_0}{\alpha}$$

(c) $Q = \frac{i_0}{\alpha}$ línulega háð $i_0$ þ.a. $Q = 0$ ef $i_0 = 0$

(3) Gegnheil málmkúla með geisla $a$ og hleðslu $Q$ (9)




rafsvari með $\epsilon$

Finnum spennu kúlunnar

þekkjum frjálsu hleðsluna $Q$

$$\oint \bar{D}\cdot d\bar{s} = Q \quad \longrightarrow \text{kúlusamhverfa} \longrightarrow \quad \bar{D} = \frac{Q}{4\pi R^2}\hat{a}_r \qquad r>a$$

Einfaldur línulegur rafsvari

$$\longrightarrow \bar{E} = \begin{cases} \dfrac{Q}{4\pi\epsilon R^2}\hat{a}_r & a<r<b \\[2ex] \dfrac{Q}{4\pi\epsilon_0 R^2}\hat{a}_r & r>b \end{cases}$$

(10)

$$V = -\int_\infty^a \bar{E}\cdot d\bar{l} = -\int_\infty^b dR\,\frac{Q}{4\pi\epsilon_0 R^2} - \int_b^a dR\,\frac{Q}{4\pi\epsilon R^2}$$

$$= \frac{Q}{4\pi}\left[\frac{1}{\epsilon_0 b} + \frac{1}{\epsilon a} - \frac{1}{\epsilon b}\right]$$ sem er yfirborðspenna

kúlunnar eða spenna allrar kúlunnar, þar sem við gerum ráð fyrir að hún sé úr kjörleiðni.

og $V(\infty) = 0$

(4) $$\bar{A} = \frac{B}{2a^2}\left(-y^3, x^3, 0\right)$$ í kartísum hnitum (11)

Við sjáum strax að $\bar{\nabla}\cdot\bar{A} = 0$

Reiknum til gamans strax $\bar{B} = \bar{\nabla}\times\bar{A}$

$$\bar{B} = \hat{a}_z\,\frac{3B}{2a^2}\left(x^2+y^2\right) \quad \text{eða} \quad \bar{B} = \hat{a}_z\,\frac{3B}{2a^2}r^2$$

í sívalningshnitum

Hér sést líka eins og á að vera að

$\bar{\nabla}\cdot\bar{B} = 0$

Ekki er sniðugt að reikna $\Phi$ frá $\bar{B}$ hér, en vissulega hægt. Notum heldur

C: hringur í x-y-sléttu með geisla b og miðju í 0

$$\Phi = \oint_C \bar{A}\cdot d\bar{l}$$

þá er $d\bar{l} = b(-\sin\phi, \cos\phi, 0)$ í kartísum hnitum (12)

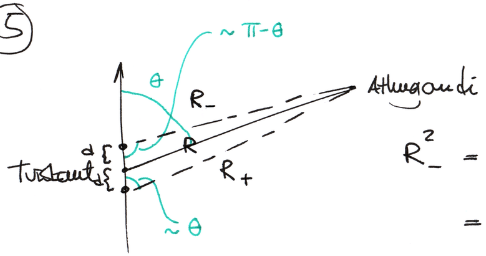
$$\bar{A} = \frac{Bb^3}{2a^2}\left(-\sin^3\phi, \cos^3\phi, 0\right)$$

$$\longrightarrow \Phi = \int_0^{2\pi} d\phi\left\{\sin^4\phi + \cos^4\phi\right\}\frac{Bb^4}{2a^2} = \frac{Bb^4}{2a^2}\,\frac{3\pi}{2}$$

þar sem við notum

$$\int_0^{2\pi} d\phi\,\cos^4\phi = \frac{3\pi}{4} \qquad \int_0^{2\pi} d\phi\,\sin^4\phi = \frac{3\pi}{4}$$

(5) (13)




$$R_-^2 = (R^2 - 2Rd\cos(\pi-\theta) + d^2)$$
$$= (R^2 + 2Rd\cos\theta + d^2)$$
$$R_+^2 = (R^2 - 2Rd\cos\theta + d^2)$$

þegar $R >> d \longrightarrow R_\pm \simeq R \mp d\cos\theta$

$$V(R) = \frac{e}{4\pi\epsilon_0}\left\{\frac{e^{ikR_+}}{R_+} - \frac{e^{-ikR_-}}{R_-}\right\}$$

tvískaut, sem gefur blikkað með tíðni $\omega$

$$\boxed{k = \frac{\omega}{c}}$$

(14)

Fjarlægur athugandi

$$\longrightarrow V(R) \simeq \frac{e}{4\pi\epsilon_0}\left\{\frac{e^{ikR - ikd\cos\theta}}{R - d\cos\theta} - \frac{e^{ikR + ikd\cos\theta}}{R + d\cos\theta}\right\}$$

$$= \frac{e\,e^{ikR}}{4\pi\epsilon_0 R}\left\{\frac{e^{-ikd\cos\theta}}{1 - \frac{d}{R}\cos\theta} - \frac{e^{ikd\cos\theta}}{1 + \frac{d}{R}\cos\theta}\right\}$$

$$\simeq \frac{e}{4\pi\epsilon_0}\frac{e^{ikR}}{R}\left[e^{-ikd\cos\theta}\left(1 + \frac{d}{R}\cos\theta + \cdots\right) - e^{ikd\cos\theta}\left(1 - \frac{d}{R}\cos\theta\right)\right]$$

$$= \frac{e}{4\pi\epsilon_0}\frac{e^{ikR}}{R}\left\{-2i\sin(kd\cos\theta) + 2\cos(kd\cos\theta)\frac{d}{R}\cos\theta\right\}$$

(15)

$$V(R) \xrightarrow[k\to 0]{} \frac{e}{4\pi\epsilon_0}\frac{1}{R}\,2\frac{d}{R}\cos\theta = \frac{e(2d)}{4\pi\epsilon_0 R^2}\cos\theta$$

sem er tvískauts rafstöðumættið fyrir tvískaut með vægi $p = e2d$, enda $k = 0$ þýðir $\omega = 0$

— — — — — — —

Ef $k$ er endanlegt þá er ráðandi liður þ. $R \to \infty$

$$V(R) \xrightarrow[R\to\infty]{} \frac{e}{4\pi\epsilon_0}\frac{e^{ikR}}{R}(-2i)\sin(kd\cos\theta)$$

Ef nú til viðbótar $kd = \frac{2\pi}{\lambda}d << 1$

(16)

þá fæst

$$V(R) \longrightarrow -\frac{ei}{2\pi\epsilon_0}\frac{e^{ikR}}{R}kd\cos\theta$$

$$= -i\frac{(2ed)}{4\pi\epsilon_0}k\frac{e^{ikR}}{R}\cos\theta$$

sem við þekkjum sem geislunarmætti tvískauts