1 Kúla með geisla a ber segul $\overline{M} = M_0 \hat{a}_z \left(\frac{\hat{a}_z \cdot \overline{R}}{a}\right)^2$

a) Finna jafngildar segulhleðslur

Sol: $\rho_m = -\overline{\nabla}\cdot\overline{M}$, yfirborðs $\rho_{ms} = \overline{M}\cdot\hat{a}_n$

yfirborðs: $\overline{R} = R\cdot\hat{a}_n$

$$\rho_{ms} = \overline{M}\cdot\hat{a}_n = M_0\, \hat{a}_z\cdot\hat{a}_n \cdot \left(\frac{\hat{a}_z\cdot\overline{R}}{a}\right)^2$$

$$= M_0 \cos\theta \cdot \left(\frac{R}{a}\right)^2 \cdot \cos^2\theta$$, því $\hat{a}_z\cdot\hat{a}_R = \cos\theta$

$$\rightarrow \boxed{\rho_{ms} = M_0 \left(\frac{a}{a}\right)^2 \cos^3\theta} \quad \boxed{= M_0 \cos^3\theta}$$

$$\hat{a}_z = \hat{a}_R \cos\theta - \hat{a}_\theta \sin\theta$$

②

<u>Bol</u>:
$$\bar{M} = M_o \hat{a}_z \left(\frac{\hat{a}_z \cdot \bar{R}}{a}\right)^2 = M_o \left(\frac{R}{a}\right)^2 \cos^2\theta \left[\hat{a}_R \cos\theta - \hat{a}_\theta \sin\theta\right]$$

$$\rho_m = -\bar{\nabla}\cdot\bar{M} = -\frac{1}{R^2}\partial_R\left(R^2 M_R\right) - \frac{1}{R\sin\theta}\partial_\theta\left(M_\theta \sin\theta\right) - \frac{1}{R\sin\theta}\partial_\phi M_\phi$$

$$= -4M_o\left(\frac{R}{a^2}\right)\cos^3\theta + 2M_o\left(\frac{R}{a^2}\right)\left(\cos^3\theta - \cos\theta\cdot\sin^2\theta\right)$$

$$= -4M_o\left(\frac{R}{a^2}\right)\left\{\cos^3\theta - \frac{1}{2}\cos\theta\left(2\cos^2\theta - 1\right)\right\}$$

$$= -4M_o\left(\frac{R}{a^2}\right)\left\{\cos^3\theta + \cos\theta\left(\frac{1}{2} - \cos^2\theta\right)\right\}$$

$$\boxed{\rho_m = -2M_o\left(\frac{R}{a^2}\right)\cos\theta}$$

b) Jamgildir straumar

BoL: $\bar{J}_m = \bar{\nabla} \times \bar{M}$, ytribords $\bar{J}_{ms} = -\bar{M} \times \hat{a}_n$

$$\bar{J}_m = \bar{\nabla} \times \bar{M} = \hat{a}_\phi \frac{1}{R} \left\{ \partial_R (R M_\theta) - \partial_\theta M_R \right\}$$

Báðir liðir hverfa

$$= \hat{a}_\phi \frac{1}{R} \left\{ -3\left(\frac{R}{a}\right)^2 M_0 \cos^2\theta \cdot \sin\theta + 3\left(\frac{R}{a}\right)^2 M_0 \cos^2\theta \cdot \sin\theta \right\} = 0$$

$$\bar{J}_{ms} = -\bar{M} \times \hat{a}_n = -\bar{M} \times \hat{a}_R = -M_0 \left(\frac{R}{a}\right)^2 \cos^2\theta \left\{ \hat{a}_R \cos\theta - \hat{a}_\theta \sin\theta \right\} \times \hat{a}_R$$

$$\rightarrow J_{ms}\Big|_{R=a} = -\hat{a}_\phi \left(\frac{a}{a}\right)^2 M_0 \cos^2\theta \cdot \sin\theta = \boxed{-\hat{a}_\phi M_0 \cos^2\theta \cdot \sin\theta}$$

c) Hver er heildar segulmer~~hleð~~sla kúlunnar?

Á yfirborði $\quad \rho_{MS} = M_0 \cos^3\theta$

$$Q_{MS} = \int d\Omega\, a^2 \rho_{MS} = 2\pi a^2 \int_0^{\pi} d\theta \cdot \sin\theta \cdot \cos^3\theta \cdot M_0$$

$$= 2\pi a^2 M_0 \int_{-1}^{+1} d(\cos\theta)\, \cos^3\theta = 0$$

Í bol

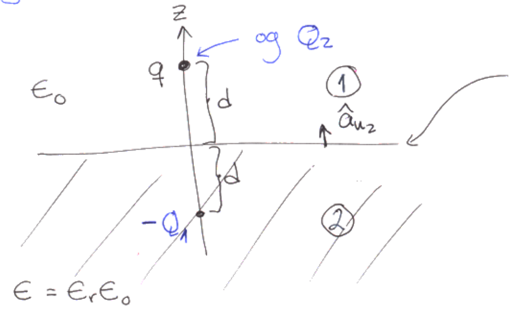
$$Q_M = \int R^2 dR\, d\Omega\, \rho_M = -4\pi M_0 \int_0^a \frac{R^3}{a^2} dR \int_0^{\pi} d\theta \sin\theta \cos\theta$$

$$= -\frac{4\pi M_0}{a^2} \int_0^a R^2 dR \int_{-1}^{1} d(\cos\theta)\, \cos\theta = 0$$

Heildarsegulflæðið er 0, kúlan er venjulegt segultvískaut

segulmögnun er öll í eina átt, z-átt, svo kerfið er svipað og stangar-segull. $\Leftrightarrow$ segultvískaut

2



á jaðrinum (x-y-sléttu) verða að gilda $V_1 = V_2$ (1)

$$(\bar{D}_1 - \bar{D}_2) \cdot \hat{a}_{n2} = 0$$

$$\epsilon \partial_z V_1 = \epsilon_0 \partial_z V_2 \qquad (2)$$

$$V_1(x,y,z) = \frac{q}{4\pi\epsilon\sqrt{x^2 + y^2 + (z-d)^2}} - \frac{Q_1}{4\pi\epsilon\sqrt{x^2+y^2+(z+d)^2}}$$

$$V_2(x,y,z) = \frac{q + Q_2}{4\pi\epsilon_0\sqrt{x^2+y^2+(z-d)}}$$

Jaðarskilyrði

$$① \rightarrow \quad \frac{q - Q_1}{\epsilon} = \frac{q + Q_2}{\epsilon_0}$$

$$② \rightarrow \quad q + Q_1 = q + Q_2$$

$$\rightarrow Q_1 = Q_2 = \frac{\epsilon_0 - \epsilon}{\epsilon_0 + \epsilon} q = -\underbrace{\left(\frac{\epsilon - \epsilon_0}{\epsilon + \epsilon_0}\right)}_{\text{jákvæð stærð}} q$$

Einangrari → engin frjáls hleðsla á skilfletinum, en bundin hleðsla, skautuð er

$$\rho_{ps} = \overline{P} \cdot \hat{a}_n$$

skautunin finnst frá $\overline{D} = \epsilon_0 \overline{E} + \overline{P} \rightarrow \overline{P} = \overline{D} - \epsilon_0 \overline{E}$

I 2 er $\bar{D}_2 = \epsilon_0 \bar{E}_2 \rightarrow \bar{P}_2 = 0$

1 $\bar{D}_1 = \epsilon \bar{E}_1 \rightarrow \bar{P}_1 = \bar{D}_1 - \epsilon_0 \bar{E}_1 = (\epsilon - \epsilon_0)\bar{E}_1$

Við þurfum að finna $\bar{P}_1$ í flötinum $z = 0$

$$\bar{P}_1 (z=0) = (\epsilon - \epsilon_0)\bar{E}_1 (z=0) = -(\epsilon - \epsilon_0)\bar{\nabla} V_1\Big|_{z=0}$$

$$-\bar{\nabla} V_1\Big|_{z=0} = -\hat{a}_z \partial_z V_1\Big|_{z=0} = -\left\{\frac{(q+Q_2)(z-d)(-\hat{a}_z)}{4\pi\epsilon\left(x^2+y^2+(z-d)^2\right)^{3/2}}\right\}\Bigg|_{z=0^-}$$

$$= -\frac{(q+Q_2)\,d\,\hat{a}_z}{4\pi\epsilon\left(x^2+y^2+d^2\right)^{3/2}}$$

$$= -\frac{\hat{a}_z\, q d}{4\pi\epsilon\left(x^2+y^2+d^2\right)^{3/2}}\left(1 + \frac{\epsilon_0 - \epsilon}{\epsilon_0 + \epsilon}\right)$$

$$\rightarrow \overline{P}_1(z=0) = -\frac{\hat{a}_z q d}{4\pi\epsilon(x^2+y^2+d^2)^{3/2}} \cdot \frac{2\epsilon_0(\epsilon-\epsilon_0)}{\epsilon_0+\epsilon}$$

og

$$\rho_{ps} = -\frac{qd}{4\pi\epsilon(x^2+y^2+d^2)^{3/2}} \frac{2\epsilon_0(\epsilon-\epsilon_0)}{\epsilon_0+\epsilon}$$

Þannig er t.d. ljóst að $\rho_{ps} \rightarrow 0$ ef $\epsilon \rightarrow \epsilon_0$ þá er enginn munur á ① og ② og engin skautuð hleðsla

$\rho_{ps} < 0$ ef $q > 0$
eins og búast má við

3

2 Jónahvolf ϵ_p

1 ϵ_0

Jörð

Jónað gas

$$\epsilon_p = \epsilon_0\left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

$$\omega_p^2 = \frac{Ne^2}{m\epsilon_0}$$, þ.s. N er þéttleiki einda

$$\gamma = i\omega\sqrt{\mu_0\epsilon_0}\sqrt{1 - \frac{\omega_p^2}{\omega^2}} \quad , \quad \eta_p = \frac{\eta_0}{\sqrt{1 - \frac{\omega_p^2}{\omega}}}$$

speglunarstuðull

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_p - \eta_0}{\eta_p + \eta_0} = \frac{\frac{1}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} - 1}{\frac{1}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} + 1}$$

$$= \frac{1 - \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2}}{1 + \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2}}$$

þegar $\omega \gtrsim \omega_p$ þá er

$\Gamma \lesssim 1$ mikið endurkast

$\omega \gg \omega_p \rightarrow \Gamma \rightarrow 0$ lítið endurkast og ~~framførar~~ ~~Stuðull~~ framstuðull τ

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2\eta_p}{\eta_p + \eta_0} = \frac{2}{1 + \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2}}$$

þegar $\omega \geq \omega_p$

$\tau \leq 2$

$\omega >> \omega_p$

$\rightarrow \tau \rightarrow 1$

full framferð

fyrir $\omega < \omega_p$ er $\sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2}$ hrein þvertala

$$\rightarrow \gamma = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\left(\frac{\omega_p}{\omega}\right)^2 - 1}$$ er hrein rauntala

engin bylgja berst mjög langt

fyrir $\omega > \omega_p$ er $\gamma = i\omega\sqrt{\mu_0\epsilon_0}\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$ hrein þvertala, bylgja berst um jónahvolfið

Jónahvolfið lokar því bylgjur með $\omega < \omega_p$ inni í gufuhvolfinu

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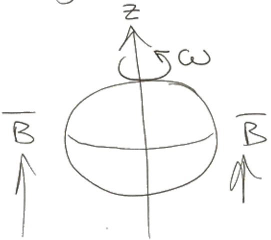
(4) Kjörleiðandi kúluskel snýst með ω um z-ás í föstu ytra segulflæðisviði $\bar{B} = B_0 \hat{a}_z$
Geisli a



a) Finna spennumun á N-pól og miðbaug

Faradays lögmálið er

$$\oint_C \bar{E}' \cdot d\bar{l} = \int_S ds\, \partial_t \bar{B} + \oint_C (\bar{u} \times \bar{B}) \cdot d\vec{l}$$

þar sem $\bar{E}'$ er spanaða rafsviðið miðað við kúluna

$$\partial_t \bar{B} = 0$$

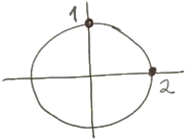
$$\rightarrow \oint_C \bar{E}' \cdot d\bar{l} = \oint_C (\bar{u} \times \bar{B}) \cdot d\bar{l}$$ hér

$\bar{u} = \omega a \sin\theta \cdot \hat{a}_\phi$, ég vel stefnuna

$$\bar{u} \times \bar{B} = \omega a B_0 \sin\theta \cdot \hat{a}_\phi \times \hat{a}_z$$

$$\hat{a}_z = \hat{a}_R \cos\theta - \hat{a}_\theta \sin\theta$$

Hér þurfum við að velja heildisveginn sem opinn bút á milli punkta 1 og 2



$$\int_1^2 \bar{E}' \cdot d\bar{\ell} = \mathcal{E}_{21} = \int_1^2 (\bar{u} \times \bar{B}) \cdot d\bar{\ell} \quad , \quad \mathcal{E}_{21} = V_2 - V_1$$

$$\rightarrow \mathcal{E}_{21} = \int_0^{\pi/2} \omega a B_0 \sin\theta \left\{ \hat{a}_\phi \times \hat{a}_z \right\} \cdot a\, d\theta\, \hat{a}_\theta$$

$$= \omega a^2 B_0 \int_0^{\pi/2} d\theta \sin\theta \left\{ \hat{a}_\phi \times \left(\hat{a}_R \cos\theta - \hat{a}_\theta \sin\theta \right) \cdot \hat{a}_\theta \right\}$$

$\hat{a}_\theta$ $\qquad$ $-\hat{a}_R$

$$= \omega a^2 B_0 \int_0^{\pi/2} d\theta \sin\theta \cos\theta = \frac{\omega a^2 B_0}{2}$$

b) Sé heildið gert milli skautanna fæst

$$\mathcal{E}_{SN} = \omega a^2 B_0 \int_0^{\pi} d\theta \, \sin\theta \cos\theta = 0$$

skautin eru eins því leyti þar er krafturinn $\bar{U} \times \bar{B} = 0$, því er enginn spennumunur á þeim.

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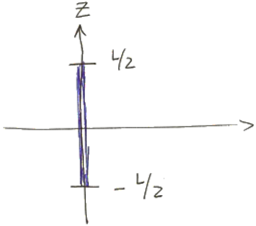
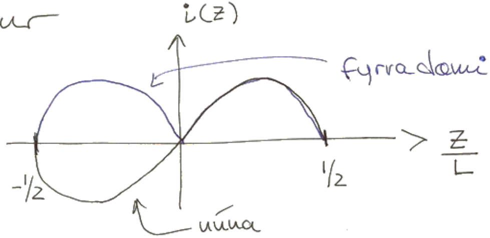
(5) Loftnet

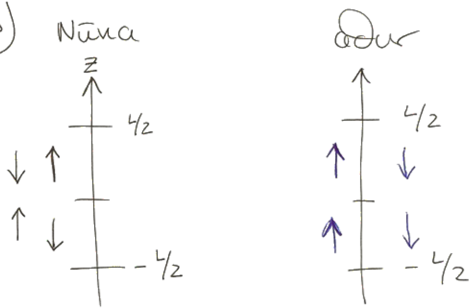
$$i(z) = I_0 \sin\left(\frac{2\pi z}{L}\right) \qquad |z| < \frac{L}{2}$$



a) Straumur



b)



← tvístautshreyfingar

fjórstauts hreyfingar, því má búast við að mynsturfallið sé fjórstaut þegar $\beta L \to 0$ fyrir fjarsviðið

c) fjarsvið $R >> L$

$$R' = \left\{ R^2 + z^2 - 2Rz\cos\theta \right\}^{1/2} \simeq R - z\cos\theta$$

~~notað~~ í fasa þátt, en $\frac{1}{R'} \sim \frac{1}{R}$

$$\rightarrow dE_\theta = \eta_0 dH_\phi = i\,\frac{i(z)dz}{4\pi}\left\{\frac{e^{-i\beta R'}}{R'}\right\}\eta_0\beta\sin\theta$$

$$\rightarrow E_\theta = \eta_0 H = i\,\frac{I_0\eta_0\beta\sin\theta}{4\pi R}e^{-i\beta R}\int_{-L/2}^{L/2} dz\, e^{i\beta z\cos\theta}\sin\left(\frac{2\pi z}{L}\right)$$

$$= i\,\frac{I_0\eta_0\beta L}{4\pi R}e^{-i\beta R}\sin\theta\int_{-1/2}^{1/2} du\, e^{i\beta L u\cos\theta}\sin(2\pi u)$$

eftir breytuskipti $u = \frac{z}{L}$

$$E_\theta = i\,\frac{I_0 \eta_0 \beta L}{R}\, e^{-i\beta R}\; F(\theta)$$

por sem

$$F(\theta) = \frac{\sin\theta}{4\pi}\int_{-1/2}^{1/2} du\, e^{i\beta L u \cos\theta}\, \sin(2\pi u)$$

$$= \frac{\sin\theta}{4\pi}\left\{ \frac{2\pi\, e^{+\frac{i\beta L\cos\theta}{2}}}{4\pi^2 - (\beta L\cos\theta)^2} - \frac{2\pi\, e^{-\frac{i\beta L\cos\theta}{2}}}{4\pi^2 - (\beta L\cos\theta)^2} \right\}$$

$$= \frac{\sin\theta}{4\pi}\left\{ 2i\, \frac{2\pi \sin\left(\beta L\, \frac{\cos\theta}{2}\right)}{4\pi^2 - (\beta L\cos\theta)^2} \right\}$$

$$F(\theta) = i\,\frac{\sin\left(\beta L \frac{\cos\theta}{2}\right)}{4\pi^2 - (\beta L \cos\theta)^2}\sin\theta \quad , \quad \beta L = \frac{2\pi}{\lambda} L$$

ef $\beta L \to 0$

$$F(\theta) \to \frac{i}{4\pi^2}\,\beta L\,\frac{\cos\theta\,\sin\theta}{2} = \frac{i}{4\pi^2}\,\beta L\,\underbrace{\sin(2\theta)}_{\text{mynstrið fyrir fjórskaut}}$$