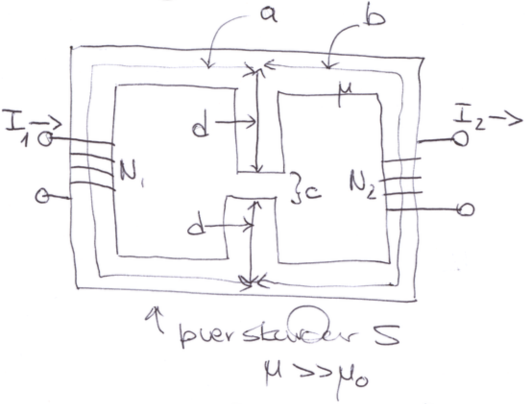
1 Segulrās



jojustraumsrās

segulflodirās

$$R_c = \frac{c}{\mu_0 S}$$

$$R_d = \frac{d}{\mu S}$$

jojugizdrās meo segul-vidnamumn

$$R_a = \frac{a}{\mu S}$$

$$R_b = \frac{b}{\mu S}$$

skoðum tvær lykkjur

1: $$N_1 I_1 = R_a(\Phi_1 + \Phi_2) + 2R_d \Phi_1 + R_c \Phi_1$$

2: $$N_1 I_1 + N_2 I_2 = R_a(\Phi_1 + \Phi_2) + R_b \Phi_2$$

ókendu stødddirnar eru Φ_1 og Φ_2, vit þurfum Φ_1 til þess at avgera

$$B = \mu_0 H_c = \mu_0 \frac{\Phi_1}{\mu_0 S} = \frac{\Phi_1}{S}$$

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2 $\rightarrow$ $$\Phi_2 (R_a + R_b) = N_1 I_1 + N_2 I_2 - R_a \Phi_1$$

$$\rightarrow \Phi_2 = \frac{N_1 I_1 + N_2 I_2 - R_a \Phi_1}{R_a + R_b}$$

Notam i 1

$$N_1 I_1 = R_a \Phi_1 + \frac{R_a \left(N_1 I_1 + N_2 I_2 - R_a \Phi_1 \right)}{R_a + R_b} + \Phi_1 \left(2R_d + R_c \right)$$

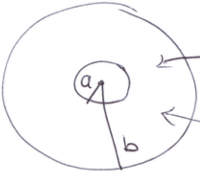
$$= \Phi_1 \left(R_a + 2R_d + R_c \right) + \frac{R_a}{R_a + R_b} \left(N_1 I_1 + N_2 I_2 \right)$$

$$- \frac{R_a^2}{R_a + R_b} \Phi_1$$

$$\rightarrow \Phi_1 \left\{ R_a + 2R_d + R_c - \frac{R_a^2}{R_a + R_b} \right\} = N_1 I_1 - \frac{R_a}{R_a + R_b} \left(N_1 I_1 + N_2 I_2 \right)$$

$$\rightarrow \Phi_1 = \left\{ \frac{N_1 I_1 - \frac{R_a}{R_a + R_b} \left(N_1 I_1 + N_2 I_2 \right)}{R_a + 2R_d + R_c - \frac{R_a^2}{R_a + R_b}} \right\}$$

2 Sívalningsþéttir, fastur straumur I milli skauta



$\epsilon(r) = \epsilon_0 \frac{r}{b}$

$\sigma(r) = \sigma_0 \frac{a}{r}$ lekur þéttir

brýtur ekki horn og z-samhverfu.

a) Finna leiðni hans G.

Straumþéttni finnst frá $\bar{\nabla} \cdot \bar{J} = 0$ rafstöðufræði

$$\bar{J} = \frac{1}{2\pi r}\left(\frac{I}{L}\right)\hat{a}_r$$

mjög langur, $L \to \infty$ köllum $\left(\frac{I}{L}\right) = I'$

$$\bar{E} = \frac{\bar{J}(r)}{\sigma(r)} = \frac{I'}{2\pi r}\frac{r}{\sigma_0 a}\hat{a}_r$$

$$= \frac{I'}{2\pi \sigma_0 a}\hat{a}_r$$

sem er óháð með fasta ytri straumnum I

Finnum spennunum skauta

$$V_b - V_a = -\int_a^b \bar{E}\cdot d\bar{l} = -\int_a^b dr\,\frac{I'}{2\pi\sigma_0 a}$$

$$= -\frac{I'}{2\pi\sigma_0 a}(b-a)$$

$$\rightarrow \quad G = \frac{I'}{|V_b - V_a|} = \frac{2\pi\sigma_0 a}{b-a}$$

b) Frjálsar hleðslur. Þar tengjast $\bar{D}(r) = \epsilon(r)\,\bar{E}(\bar{r})$

$$\bar{D}(r) = \frac{I'\epsilon_0 r}{2\pi\sigma_0 ba}\,\hat{a}_r$$

Bolhleðslur (í rafsvara) ← sívalningshnit

$$\rho = \bar{\nabla} \cdot \bar{D} = \frac{1}{r}\frac{d}{dr}\left\{ r\, D(r) \right\} = \frac{I' \epsilon_0}{\pi \sigma_0 a b}$$

Yfirborðshleðslur á skautum þéttis

r=a frá $\hat{a}_{n2} \cdot (\bar{D}_1 - \bar{D}_2) = \rho_s$

$$\rho_s(a^+) = \hat{a}_r \cdot \bar{D}_1(a^+) = \epsilon(a^+)\bar{E}(a^+) \cdot \hat{a}_r$$

$$= \frac{I' \epsilon_0}{2\pi \sigma_0 b}$$

$$\rho_s(b^-) = -\hat{a}_r \cdot \bar{D}_1(b^-) = -\epsilon(b^-)\bar{E}(b^-) \cdot \hat{a}_r$$

$$= -\frac{I' \epsilon_0}{2\pi \sigma_0 a}$$

c) Skautumerthlæðslur í og á föðrum rafsvora ⑦

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P} \rightarrow \bar{P} = \bar{D} - \epsilon_0 \bar{E}$$

$$\rightarrow \bar{P} = \left\{ \epsilon(r) - \epsilon_0 \right\} \bar{E} = \epsilon_0 \left[\frac{r}{b} - 1 \right] \frac{I' \hat{a}_r}{2\pi \sigma_0 a}$$

bolskautun er þá

$$\rho_p = -\bar{\nabla} \cdot \bar{P}$$

$$\rho_p = -\frac{1}{r} \frac{d}{dr} \left\{ \epsilon_0 \left(\frac{r}{b} - 1 \right) \frac{I' r}{2\pi \sigma_0 a} \right\}$$

$$= -\frac{1}{r} \frac{I' \epsilon_0}{2\pi \sigma_0 a} \left(\frac{2r}{b} - 1 \right)$$

$$= -\frac{I' \epsilon_0}{2\pi \sigma_0 a} \left(\frac{2}{b} - \frac{1}{r} \right)$$

Yfirborðs skautunarhleðslur

$$\rho_{ps} = \overline{P} \cdot \hat{a}_n$$

$$\rho_{ps}(a^+) = -\overline{P}(a^+) \cdot \hat{a}_r = -\frac{I'\epsilon_0}{2\pi\sigma_0 a}\left(\frac{a}{b} - 1\right)$$

$$\rho_{ps}(b^-) = \overline{P}(b^-) \cdot \hat{a}_r = \frac{I'\epsilon_0}{2\pi\sigma_0 a}\left(\frac{b}{b} - 1\right) = 0$$

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d) Heildar frjálsa hleðslan á lengdareiningu

$$\frac{Q}{L} = 2\pi a \rho_s(a^+) + 2\pi b \rho_s(b^-) + \int_a^b \rho(r)\, 2\pi r\, dr$$

$$= \frac{I'\epsilon_0 a}{\sigma_0 b} - \frac{I'\epsilon_0 b}{\sigma_0 a} + \frac{I'\epsilon_0}{\sigma_0} \frac{1}{ab}\left(b^2 - a^2\right)$$

$$= \underline{0}$$

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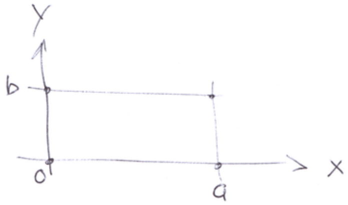

Rétthyrndur bylgjustokkur

TE_{10} -háttur

Lausnir Helmholtz jöfnu eru þá

$$H_z^0(x,y) = H_0 \cos\left(\frac{\pi x}{a}\right)$$

$$E_y^0(x,y) = -\frac{i\omega\mu}{h^2} H_0 \sin\left(\frac{\pi x}{a}\right)\cdot\frac{\pi}{a}$$

$$H_x^0(x,y) = \frac{\gamma}{h^2}\frac{\pi}{a} H_0 \sin\left(\frac{\pi x}{a}\right)$$

$$h^2 = \left(\frac{\pi}{a}\right)^2$$

$$\gamma = i\sqrt{\omega^2\mu\epsilon - \left(\frac{\pi}{a}\right)^2}$$

munum líka að z-þáttur lausnarinnar er alltaf $e^{-\gamma z}$

Því höfum við

$$H_z^0(x,y) = H_0 \cos\left(\frac{\pi x}{a}\right)$$

$$E_y^0(x,y) = -\frac{i\omega\mu a}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right)$$

$$H_x^0(x,y) = \frac{\gamma a}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right)$$

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a) Yfirborðsstraumar á inni veggjum?

Þeir koma til vegna

$$\hat{a}_{n2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s$$

Við erum aðeins með leiðandi fleti samsíða

x og y - ás, við erum með $H_{x,z} \rightarrow$

straumar er å botn og topp plåte

$$\bar{J}_s\Big|_{\substack{botn\\ y=0}} = \hat{a}_y \times \left(H_x^0\,\hat{a}_x + H_z^0\,\hat{a}_z\right)\Big|_{\substack{botn\\ y=0}}$$

$$= -H_x^0\Big|_{\substack{botn\\ y=0}}\hat{a}_z + H_z^0\Big|_{\substack{botn\\ y=0}}\hat{a}_x$$

$$= -\frac{\gamma a}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right)\hat{a}_z + H_0 \cos\left(\frac{\pi x}{a}\right)\hat{a}_x$$

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b) ~~yfirborðshleðslur ráðast af~~

$$\cancel{\hat{a}_{n2}\cdot\left(\bar{D}_1 - \bar{D}_2\right) = \rho_s}$$

Á topplötu

$$\bar{J}_s\Big|_{\substack{\text{toppur}\\ y=b}} = -\hat{a}_y \times \left(H_x^0 \hat{a}_x + H_z^0 \hat{a}_z\right)\Big|_{\substack{\text{toppur}\\ y=0}}$$

$$= + \frac{\gamma a}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right) \hat{a}_z - H_0 \cos\left(\frac{\pi x}{a}\right) \hat{a}_x$$

sem sé öfug átt miðað við botnplötu

Hliðar. Hér þarf að muna að normallinn á hliðar

$\pm \hat{a}_x$ er hornréttur á $\hat{a}_z$

$$\bar{J}_s\Big|_{\substack{\text{vinstri}\\ x=0}} = \hat{a}_x \times H_z^0 \hat{a}_z = -H_0 \hat{a}_y$$

og

$$J_s\Big|_{\substack{\text{hægri}\\ x=a}} = -\hat{a}_x \times H_z^o\,\hat{a}_z = +H_o\,\hat{a}_y$$

í öfuga stefnu við $\bar{J}_s\Big|_{\text{vinstri}}$

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b) yfirborðshleðslur ráðast af

$$\hat{a}_{n2}\cdot(\bar{D}_1 - \bar{D}_2) = \rho_s$$

Við erum aðeins með E_y - þátt, því eru aðeins hleðslur á topp og botn plötu

$$\rho_s \Big|_{\substack{botn \\ y=0}} = \epsilon \hat{a}_y \cdot E_y^\circ \hat{a}_y \Big|_{y=0} = - \frac{i\omega\mu\epsilon a}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right)$$

$$\rho_s \Big|_{\substack{toppur \\ y=b}} = -\epsilon \hat{a}_y \cdot E_y^\circ \hat{a}_y \Big|_{y=b} = \frac{i\omega\mu\epsilon a}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right)$$

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c) Sýna að samfeldnijöfnur séu uppfylltar

Nú þarf að muna að öll sviðin eru með $e^{-\gamma z}$ þátt -> líka straumar og hleðslur

<u>skoðum botn</u> plötu

$$\bar{\nabla} \cdot \bar{J}_{botn} = \partial_x J_{sx} + \partial_z J_{sz}$$

$$= -H_0 \frac{\pi}{a} \operatorname{Sin}\left(\frac{\pi x}{a}\right) e^{-\gamma z} + \frac{\gamma^2 a}{\pi} H_0 \operatorname{Sin}\left(\frac{\pi x}{a}\right) e^{-\gamma z}$$

$$= + \frac{a}{\pi} H_0 \operatorname{Sin}\left(\frac{\pi x}{a}\right) e^{-\gamma z} \left[\underbrace{\gamma^2 - \frac{\pi^2}{a^2}}_{-\left(\omega^2\mu\epsilon - \frac{\pi^2}{a^2}\right) - \frac{\pi^2}{a^2}} \right]$$

$$= -\omega^2 \mu\epsilon \cdot \frac{a}{\pi} H_0 \operatorname{Sin}\left(\frac{\pi x}{a}\right) e^{-\gamma z}$$

$$= -i\omega \left[-i\omega \frac{\mu\epsilon a}{\pi} H_0 \operatorname{Sin}\left(\frac{\pi x}{a}\right) e^{-\gamma z} \right] = -i\omega\, \rho_s \Big|_{botn}$$

eins og þarf að vera

Samskonar fæst
fyrir toppplötu fyrst það hefur einungis
formerki víxlast.

Fyrir hliðar

$$\bar{\nabla} \cdot \bar{J}_s \Big|_{vinstri} = \partial_y J_{sy} = -\partial_y H_0 = 0$$

þar var heldur engin hleðsla

Samskonar fyrir hægri hlið

→ samfelldnijafnan $\partial_t \rho + \bar{\nabla} \cdot \bar{J} = 0$
er uppfyllt á veggjum stokksins

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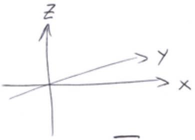
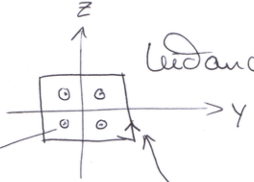


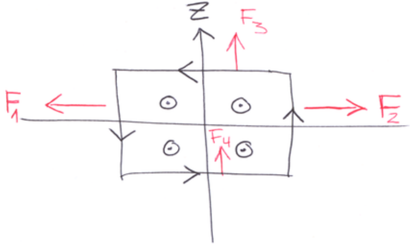


Leiðandi lykkja

$\bar{B} = B_0 z \hat{a}_x$

gerum ráð fyrir þessari straum átt

Notum $\bar{F} = I \int d\bar{l} \times \bar{B}$



Athugið segulsviðið er **ekki** fasti

$\bar{F}_1$ og $\bar{F}_2$ eru jafn stórir, en í gagnstæða átt

$\bar{F}_3$ og $\bar{F}_4$ eru í sömu átt því segulsviðið er í gagnstæða átt fyrir þá búta

$$\overline{F}_3 = I\,\frac{a}{2}\,B_0\,a = \frac{I B_0 a^2}{2}\,\hat{a}_z$$

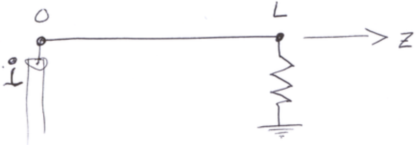
$$\overline{F}_4 = -I\left(-\frac{a}{2}\right)B_0\,a = \frac{I B_0 a^2}{2}\,\hat{a}_z$$

$$\overline{F}_{Heild} = I B_0 a^2\,\hat{a}_z$$

athugið að B_0 er hér með vídd segulsviðs á lengd vegna $\overline{B} = B_0 z\,\hat{a}_x$

það er heildarkraftur á lykkjuna vegna þess að $\overline{B}$ er <u>ekki</u> fasti

5 Hlaupabylgju loftnet



$$i(z,t) = I\cos(\omega t)\, e^{-i\beta z}$$

$$\beta = \frac{\omega}{u}$$

$$dA_z = \frac{\mu_0 e^{-i\beta R'}}{4\pi R'}\, dI \,,$$

$$R' = \sqrt{R^2 + z^2 - 2Rz\cos\theta} \simeq R - z\cos\theta$$

fyrir fjarsvið

$$A_z \simeq \frac{\mu_0 e^{-i\beta R}}{4\pi R} \int_0^L dz\, I e^{-i\beta z + i\beta z\cos\theta}$$

nota hér

og hér $\frac{1}{R'} \sim \frac{1}{R}$ í fjarsviði

$$A_z \simeq \frac{\mu_0 I e^{-i\beta R}}{4\pi R} \int_0^L dz \, \exp\{-i\beta(1-\cos\theta) z\}$$

$$= \frac{i \mu_0 I e^{-i\beta R}}{4\pi \beta (1-\cos\theta) R} \left[e^{-i\beta(1-\cos\theta)L} - 1 \right]$$

$$= \frac{\mu_0 I e^{-i\beta R} e^{-i\beta(1-\cos\theta)L/2}}{2\pi \beta (1-\cos\theta)} \sin\left(\frac{\beta(1-\cos\theta)L}{2}\right)$$

Tengjum við kúluhnit

$$\bar{A} = A_z \cos\theta \, \hat{a}_R - A_z \sin \hat{a}_\theta$$

$$= A_R \hat{a}_R + A_\theta \hat{a}_\theta$$

$$E_\theta = -i\omega A_\theta \quad , \qquad \omega\mu_0 = \eta\beta$$

$$\rightarrow \quad E_\theta = \omega A_z \sin\theta = \frac{\eta I \sin\theta}{2\pi(1-\cos\theta)R}\left\{\sin\left[\frac{\beta L(1-\cos\theta)}{2}\right]\right\}$$