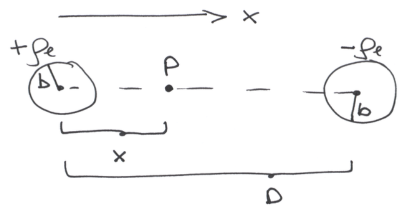
P3-47



Línuhleðsla á hvorum leiðara $\pm\rho_\ell$

$$\bar E(x) = \bar E_1(x) + \bar E_2(x)$$

$$= \hat a_x \left\{ \frac{\rho_\ell}{2\pi\epsilon_0 x} + \frac{\rho_\ell}{2\pi\epsilon_0 (D-x)} \right\}$$

Spennumunur leiðaranna gefinn

$$V_0 = V_1 - V_2 = -\int_{D-b}^{b} \bar E(x)\cdot d\bar x$$

$$= \frac{\rho_\ell}{2\pi\epsilon_0}\int_b^{D-b}\left(\frac{1}{x} + \frac{1}{D-x}\right)dx = \frac{\rho_\ell}{2\pi\epsilon_0}\left\{\ln\left(\frac{D-b}{b}\right) - \ln\left(\frac{b}{D-b}\right)\right\}$$

$$= \frac{\rho_\ell}{\pi\epsilon_0}\ln\left(\frac{D-b}{b}\right) \sim \frac{\rho_\ell}{\pi\epsilon_0}\ln\left(\frac{D}{b}\right) \quad \text{ef } D \gg b$$

Við hugsum vírana sem þétti sem hefur rýmd á lengdareiningu

$$C = \frac{\rho_\ell}{V_0} = \frac{\pi\epsilon_0}{\ln(D/b)}$$

Rafstöðuorkan í kerfinu er (á lengdareiningu)

$$W_e = \frac{1}{2} C V_0^2 = \frac{1}{2} C(D) V_0^2$$

V_0 var gefinn sem fasti, ρ_ℓ getur breyst

$$\bar F_V = \bar\nabla W_e$$

Rafkraftur á lengdareiningu ef gert er ráð fyrir fastri spennu V_0

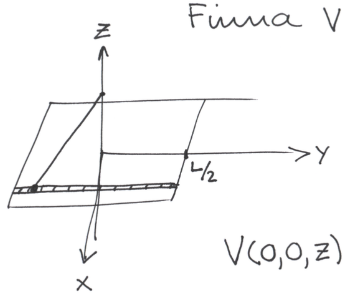
$$= \frac{\partial}{\partial D} W_e \, \hat a_x$$

$$= \hat a_x \frac{V_0^2}{2}\frac{\partial}{\partial D} C(D) = -\hat a_x \frac{V_0^2}{2}\frac{\pi\epsilon_0}{D\{\ln(\frac{D}{b})\}^2}$$

togar vírana saman

P3-18 Hleðsla Q jafndreift á $L\times L$ plötu

Finna V og $\bar E$ yfir miðri plötu



$$V(0,0,z) = \frac{1}{4\pi\epsilon_0}\int_{-L/2}^{L/2} dy' \int_{-L/2}^{L/2} dx' \rho_s \frac{1}{\sqrt{(x')^2 + (y')^2 + z^2}}$$

(GR-2.261)

$$= \frac{\rho_s}{4\pi\epsilon_0}\int_{-L/2}^{L/2} dy' \left\{ \ln\left(2\sqrt{(x')^2+(y')^2+z^2} + 2x'\right)\Big|_{x'=-L/2}^{x'=L/2} \right\}$$

$$= \frac{\rho_s}{4\pi\epsilon_0}\int_{-L/2}^{L/2} dy' \left\{ \ln\left(2\sqrt{\left(\tfrac{L}{2}\right)^2+(y')^2+z^2} + L\right) - \ln\left(2\sqrt{\left(\tfrac{L}{2}\right)^2+(y')^2+z^2} - L\right) \right\}$$

Heildið er samhverft fyrir y' og

$$\ln(2a+L) - \ln(2a-L) = \ln\left(a+\tfrac{L}{2}\right) - \ln\left(a-\tfrac{L}{2}\right)$$

$\longrightarrow$

$$V(0,0,z) = \frac{\rho_s}{2\pi\epsilon_0}\int_0^{L/2} du \left\{ \ln\left(\sqrt{\left(\tfrac{L}{2}\right)^2+u^2+z^2} + \tfrac{L}{2}\right) - \ln\left(\sqrt{\left(\tfrac{L}{2}\right)^2+u^2+z^2} - \tfrac{L}{2}\right) \right\}$$

Setjum $L/2 = a$

$$V(0,0,z) = \frac{\rho_s}{2\pi\epsilon_0}\int_0^{a} du \left\{ \ln\left(\sqrt{a^2+u^2+z^2} + a\right) - \ln\left(\sqrt{a^2+u^2+z^2} - a\right) \right\}$$

$$= \frac{\rho_s a}{2\pi\epsilon_0}\int_0^{1} dt \left\{ \ln\left(\sqrt{1+t^2+\left(\tfrac{z}{a}\right)^2} + 1\right) - \ln\left(\sqrt{1+t^2+\left(\tfrac{z}{a}\right)^2} - 1\right) \right\}$$

$$V(0,0,z) = \frac{\rho_s}{2\pi\epsilon_0}\left\{\left[u\ln\left(\sqrt{a^2+u^2+z^2}+a\right) - u\ln\left(\sqrt{a^2+u^2+z^2}-a\right)\right.\right.$$

integrals.wolfram.com

$$+\, 2a\ln\left(u+\sqrt{a^2+u^2+z^2}\right)$$

$$\left.\left. -\, 2z\arctan\left(\frac{au}{z\sqrt{a^2+u^2+z^2}}\right)\right]\Bigg|_0^a\right\}$$

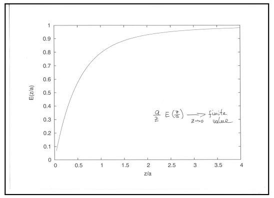
$$= \frac{\rho_s}{2\pi\epsilon_0}\left\{a\ln\left[\frac{\sqrt{2a^2+z^2}+a}{\sqrt{2a^2+z^2}-a}\right] - 2z\arctan\left(\frac{a^2}{z\sqrt{2a^2+z^2}}\right) + \left[2a\ln\left(\frac{a+\sqrt{2a^2+z^2}}{\sqrt{a^2+z^2}}\right)\right]\right\}$$

$$V(0,0,z) = \frac{Q}{4\pi\epsilon_0}\frac{1}{2a}\left\{\ln\left[\frac{\sqrt{2a^2+z^2}+a}{\sqrt{2a^2+z^2}-a}\right] - 2\left(\frac{z}{a}\right)\arctan\left(\frac{a^2}{z\sqrt{2a^2+z^2}}\right) + 2\ln\left(\frac{a+\sqrt{2a^2+z^2}}{\sqrt{a^2+z^2}}\right)\right\}$$

$$= \frac{Q}{4\pi\epsilon_0 z}\left\{\left(\frac{z}{2a}\right)\ln\left[\frac{\sqrt{2+\left(\frac{z}{a}\right)^2}+1}{\sqrt{2+\left(\frac{z}{a}\right)^2}-1}\right] - \frac{z^2}{a^2}\arctan\left(\frac{1}{\frac{z}{a}\sqrt{2+\left(\frac{z}{a}\right)^2}}\right) + \frac{z}{a}\ln\left(\frac{\sqrt{2+\left(\frac{z}{a}\right)^2}+1}{\sqrt{1+\left(\frac{z}{a}\right)^2}}\right)\right\}$$

$$= \frac{Q}{4\pi\epsilon_0 z}\,E\left(\frac{z}{a}\right)$$

we plot this



P3-20

Atómlíkan (gamalt)

Kjarninn: kúlulaga ský jafndreifðrar $+Ne$ hleðslu með geisla R_0

Engir árekstrar við rafeindir eða milli rafeinda

skoða hegðun einnar rafeindar í þessu atómi

a) Kraftur á eina rafeind (til dæmis í H-atómi)

innan atóms: $r < R_0$

$$\rho = \frac{Ne}{\frac{4}{3}\pi R_0^3} = \frac{3Ne}{4\pi R_0^3}$$

Gauss lögmál

hleðsla innan geisla r

$$Q(r) = \rho\cdot\frac{4}{3}\pi r^3 = \frac{3Ne}{4\pi R_0^3}\,\frac{4\pi r^3}{3} = Ne\,\frac{r^3}{R_0^3}$$

$$\oint_S \bar{E}\cdot d\bar{s} = \frac{Q}{\epsilon_0} \;\to\; 4\pi r^2 E = \frac{Ne}{\epsilon_0}\,\frac{r^3}{R_0^3}$$

$$\to\; \bar{E} = \hat{a}_R\,\frac{Ner}{4\pi\epsilon_0 R_0^3}$$

því er kraftarinn línulegur í „r" á rafeind með -e

$$\bar{F} = -\hat{a}_R\,\frac{Ne^2 r}{4\pi\epsilon_0 R_0^3}$$

b) Hreyfing rafeindarinnar

$$ma = F \;\to\; m\frac{d^2r}{dt^2} = -\frac{Ne^2 r}{4\pi\epsilon_0 R_0^3}$$

eða $\boxed{\ddot{r} + \omega_e^2 r = 0}$ með $\omega_e = \sqrt{\dfrac{Ne^2}{4\pi\epsilon_0 m R_0^3}}$

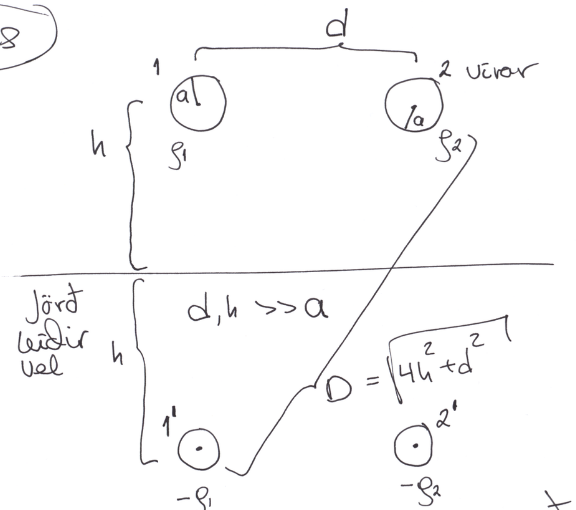
Hreintóna sveifill með tíðni ω_e

c) Klassískt séð (rétt eins og í réttu kjarnmótti) er rafeindinni hraðað -> missir orku og fellur saman

skammtafræði gæfi orkuróf fyrir H-atóm

$E_n = \hbar\omega_e(n+3/2)$, ekki í samræmi við tilraunir

P3-38



Ákvarða

C_{10}, C_{12}, C_{20}

Vírarnir eiga spegilmynd sína í jörð

Heildar rafsviðið er sett saman úr „radial" rafsviði hvers vírs

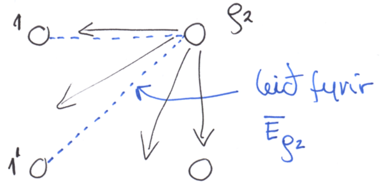
Spennumunur tveggja punkta a og b reiknast sem

$$V_{ab} = -\int_b^a \bar{E}\cdot d\bar{l}$$ óháð leið

því fæst

$$V_{11'} = -\int_{1'}^{1} \left(\overset{1}{\bar{E}_{\rho_1}} + \overset{2}{\bar{E}_{\rho_2}} + \overset{3}{\bar{E}_{\rho_1'}} + \overset{4}{\bar{E}_{\rho_2'}}\right)\cdot d\bar{l}$$

og einfaldast er að velja mismunandi leið fyrir hvern þátt þ.a. leiðin sé ávallt í radial stefnu



$$V_{11'} = +\overbrace{\frac{\rho_1}{2\pi\epsilon_0}\ln\left(\frac{2h-a}{a}\right)}^{1} + \overbrace{\frac{\rho_2}{2\pi\epsilon_0}\ln\left(\frac{D-a}{a}\right) + \frac{\rho_2}{2\pi\epsilon_0}\ln\left(\frac{a}{d-a}\right)}^{2}$$

$$- \overbrace{\frac{\rho_1}{2\pi\epsilon_0}\ln\left(\frac{a}{2h-a}\right)}^{3} - \overbrace{\frac{\rho_2}{2\pi\epsilon_0}\ln\left(\frac{d-a}{a}\right) - \frac{\rho_2}{2\pi\epsilon_0}\ln\left(\frac{a}{D-a}\right)}^{4}$$

$$= 2\left\{\frac{\rho_1}{2\pi\epsilon_0}\ln\left(\frac{2h-a}{a}\right) + \frac{\rho_2}{2\pi\epsilon_0}\ln\left(\frac{D-a}{d-a}\right)\right\}$$

$$\simeq 2\left\{\frac{\rho_1}{2\pi\epsilon_0}\ln\left(\frac{2h}{a}\right) + \frac{\rho_2}{2\pi\epsilon_0}\ln\left(\frac{D}{d}\right)\right\}$$

því $D \gg a$
$d \gg a$

$$D = \sqrt{4h^2 + d^2}$$

Nú gildir að $V_{10} = \frac{1}{2} V_{11'}$ vegna samhverfu

og eftir samskonar leið fæst

$$V_{20} = \frac{1}{2} V_{22'} = \frac{\rho_1}{2\pi\epsilon_0}\ln\left(\frac{D}{d}\right) + \frac{\rho_2}{2\pi\epsilon_0}\ln\left(\frac{2h}{a}\right)$$

Eða

$$\frac{1}{2\pi\epsilon_0}\begin{pmatrix} \ln\left(\frac{2h}{a}\right) & \ln\left(\frac{D}{d}\right) \\ \ln\left(\frac{D}{d}\right) & \ln\left(\frac{2h}{a}\right) \end{pmatrix}\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = \begin{pmatrix} V_{10} \\ V_{20} \end{pmatrix}$$

Leysum fyrir ρ_1 og ρ_2

$$\rho_1 = \Delta_0\left\{V_{10}\ln\left(\frac{2h}{a}\right) - V_{20}\ln\left(\frac{D}{d}\right)\right\} = C_{11}V_{10} + C_{12}V_{20}$$

$$\rho_2 = \Delta_0\left\{-V_{10}\ln\left(\frac{D}{d}\right) + V_{20}\ln\left(\frac{2h}{a}\right)\right\} = C_{21}V_{10} + C_{22}V_{20}$$

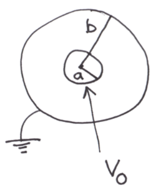
með

$$\Delta_0 = \frac{2\pi\epsilon_0}{\left[\ln\left(\frac{2h}{a}\right)\right]^2 - \left[\ln\left(\frac{D}{a}\right)\right]^2} \qquad (1/\text{ákveða})$$

$$\mathcal{C}_{12} = -C_{12} = \Delta_0 \ln\left(\frac{D}{d}\right)$$ hlut ~~...~~ span víxl

$$\mathcal{C}_{10} = \mathcal{C}_{20} = C_{11} + C_{12} = \frac{2\pi\epsilon_0}{\ln\left(\frac{2h}{a}\right) + \ln\left(\frac{D}{d}\right)}$$ rýmd miðað við jörð (einsleiðara)

P4-06



Langir samása sívalningar

$$\rho = \frac{A}{r} \quad \text{fyrir } a<r<b$$

Finna $V(r)$ á svæði milli sívalninganna

$a<r<b$

$$\nabla^2 V(r) = -\frac{\rho}{\epsilon} = -\frac{A}{\epsilon r}$$

$$\Downarrow$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = -\frac{A}{\epsilon r} \rightarrow \frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = -\frac{A}{\epsilon}$$

Óákveðin heildun gefur

$$r\frac{\partial V}{\partial r} = -\frac{Ar}{\epsilon} + C_1 \rightarrow \frac{\partial V}{\partial r} = -\frac{A}{\epsilon} + \frac{C_1}{r}$$

Enn óákveðin heildun

$$V(r) = -\frac{Ar}{\epsilon} + C_1 \ln r + C_2$$

Jaðar-skilyrðin voru

$$V(a) = -\frac{Aa}{\epsilon} + C_1 \ln a + C_2 = V_0$$

$$V(b) = -\frac{Ab}{\epsilon} + C_1 \ln b + C_2 = 0$$

umskrifum

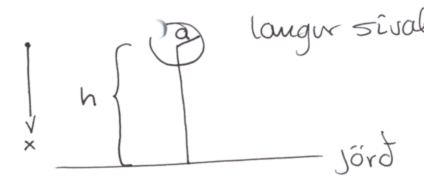
$$C_1 \ln a + C_2 = V_0 + \frac{Aa}{\epsilon}$$

$$C_1 \ln b + C_2 = \frac{Ab}{\epsilon}$$

$$\begin{pmatrix} \ln a & 1 \\ \ln b & 1 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} V_0 + \frac{Aa}{\epsilon} \\ \frac{Ab}{\epsilon} \end{pmatrix}$$

$$C_1 = \frac{1}{\ln(b/a)}\left\{\frac{A}{\epsilon}(b-a) - V_0\right\}, \quad C_2 = \frac{1}{\ln(b/a)}\left\{V_0 \ln b + \frac{A}{\epsilon}(a\ln b - b\ln a)\right\}$$

P4-10



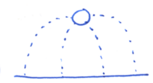
langur sívalur leiðari

finna rýmd leiðarans á lengdarein. til jarðar + kraft á lengdarein.

Skoða Example 4-4 í bók

Þar er jafnspennuflötur með 0-spennu mitt á milli víranna. Því er hér hægt að taka í burtu jörðina og bæta við spegil línuhleðslu

Rýmd vírsins til jarðar er tvöföld rýmd vírsins við spegilmynd sína (sama hleðsla, hálf spennan)



$$\rightarrow C = \frac{2\pi\epsilon_0}{\ln\left\{\frac{h}{a} + \sqrt{\left(\frac{h}{a}\right)^2 - 1}\right\}} = \frac{2\pi\epsilon_0}{\operatorname{Arcosh}\left(\frac{h}{a}\right)}$$

rýmd á lengdarein.

hugsað

{Við lausum dæmið með spegilhleðslu, en innan jarðar er ekkert spennusvið, þaðer einungis til þess að fá rétta lausn utan jarðar}

finna kraftinn á vírinn

Ef spenna vírs til jarðar er haldið fastri notum við sýndar-færslu aðferðina á bls 141-2 í bók

$$W_e = \tfrac{1}{2} C V^2, \quad V \text{ er tvöföld spennan til jarðar}$$

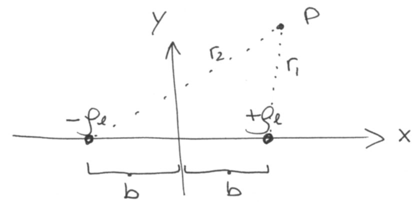
$$\bar{F}_v = \bar{\nabla} W_e = \frac{\partial}{\partial h} W_e \hat{a}_x = -\frac{1}{2} V^2 \frac{2\pi\epsilon_0}{a} \frac{1}{\left\{\operatorname{Arcosh}\left(\frac{h}{a}\right)\right\}^2} \frac{\hat{a}_x}{\sqrt{\left(\frac{h}{a}\right)^2 - 1}}$$

$$V_{v0} = \tfrac{1}{2} V \rightarrow V = 2V_{v0}$$

$$\bar{F}_v = -4\pi\epsilon_0 \frac{V_{v0}^2}{a} \frac{1}{\left\{\operatorname{ArCosh}\left(\frac{h}{a}\right)\right\}^2} \frac{\hat{a}_x}{\sqrt{\left(\frac{h}{a}\right)^2 - 1}}$$

Krafturinn togar í vírinn að jörð

P4-12 Tvær samhliða línuhleðslur



a) finna $\bar{E}$. Jafna (4-48) gefur

$$V_p = \frac{\rho_\ell}{2\pi\epsilon_0} \ln\left\{\frac{\sqrt{(x+b)^2 + y^2}}{\sqrt{(x-b)^2 + y^2}}\right\}$$

$$\bar{E}_p = -\hat{a}_x \frac{\partial V_p}{\partial x} - \hat{a}_y \frac{\partial V_p}{\partial y}$$

$$= -\frac{\rho_\ell}{2\pi\epsilon_0}\left\{\frac{\hat{a}_x 2b(y^2+b^2-x^2) - \hat{a}_y 4bxy}{[(x+b)^2+y^2][(x-b)^2+y^2]}\right\}$$

b) finna jöfnur fyrir fy-sviðslínur

línur samsíða rafsviðinu uppfylla

$$\frac{dy}{dx} = \frac{E_y}{E_x} = -\frac{2xy}{y^2+b^2-x^2}$$

$$\hookrightarrow \frac{d(x^2+y^2)}{(x^2+y^2)-b^2} = \frac{dy}{y}$$

heildun gefur

$$\ln\left[(x^2+y^2) - b^2\right] = \ln y + C_1$$

$$\rightarrow \ln\left[\frac{(x^2+y^2)-b^2}{y}\right] = C_1$$

2

Þetta má umskrifa sem

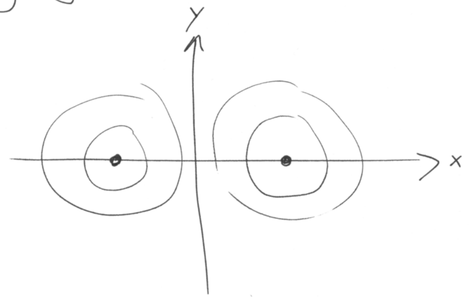
$$x^2+y^2-2Ky=b^2$$

eða

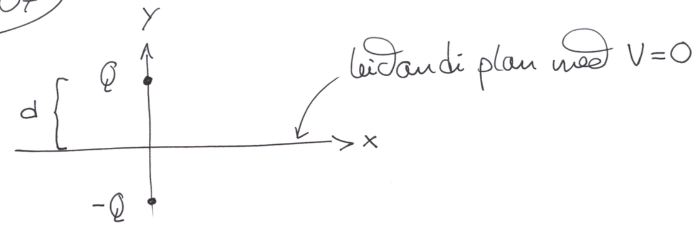
$$x^2+(y-K)^2=b^2+K^2$$

hringir með miðju í $(0,\pm K)$

og geisla b^2+K^2



P4-07



Sama uppsetning og í 4-4.1

$$R_+=\sqrt{x^2+(y-d)^2+z^2}$$

$$V(x,y,z)=\frac{Q}{2\pi\epsilon_0}\left\{\frac{1}{R_+}-\frac{1}{R_-}\right\}$$

$$R_-=\sqrt{x^2+(y+d)^2+z^2}$$

$$\frac{\partial}{\partial y}\frac{1}{R_+}=-\frac{1}{2}\frac{2(y-d)}{(x^2+(y-d)^2+z^2)^{3/2}}\;,\quad \frac{\partial}{\partial y}\frac{1}{R_-}=-\frac{1}{2}\frac{2(y+d)}{(x^2+(y+d)^2+z^2)^{3/2}}$$

$$\bar{E}(x,y=0,z)=\left\{-(\bar{\nabla}V(x,y,z))\cdot\hat{a}_y\right\}_{y=0}\hat{a}_y=-\hat{a}_y\frac{Qd}{2\pi\epsilon_0(x^2+d^2+z^2)^{3/2}}$$

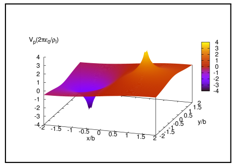
a) $$\rho_s(x,0,z)=\hat{a}_y\cdot\epsilon_0\bar{E}(x,y=0,z)=-\frac{Qd}{2\pi(x^2+d^2+z^2)^{3/2}}$$

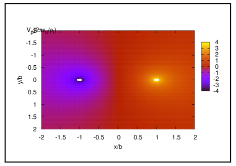
nú er $x^2+z^2=r^2$ í pólhnitum

$$\to\rho_s=-\frac{Qd}{2\pi(r^2+d^2)^{3/2}}$$

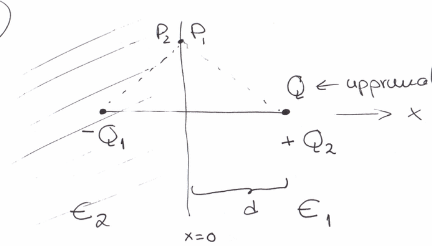
b) Heildarhleðslan (GR 2.264.6)

$$\int_0^\infty 2\pi r\,dr\,\rho_s=\int_0^\infty\frac{Qd\,2\pi r\,dr}{2\pi(r^2+d^2)^{3/2}}=-Q$$





P4-17



a) Sannreyna að rafsviðið í ① fæst frá Q og spegilhleðslu $-Q_1$

b) ....að sviðið í ② fæst frá Q og spegilhleðslunni $+Q_2$

c) finna Q_1 og Q_2

Gilda verður að

① $V_1 = V_2$ á jaðri, $x=0$

og $(\bar{D}_1 - \bar{D}_2)\cdot\hat{a}_{n2} = 0$

$\epsilon_1 \frac{\partial V_1}{\partial x} = \epsilon_2 \frac{\partial V_2}{\partial x}$

②

a) $$V_1(x,y,z) = \frac{Q}{4\pi\epsilon_1\sqrt{(x-d)^2+y^2+z^2}} - \frac{Q_1}{4\pi\epsilon_1\sqrt{(x+d)^2+y^2+z^2}}$$

b) $$V_2(x,y,z) = \frac{Q+Q_2}{4\pi\epsilon_2\sqrt{(x-d)^2+y^2+z^2}}$$

Þetta er greinilega lausnir á jöfnu Poisson's.
Ef þær uppfylla jaðarskilyrðin þá er þetta eina lausnin

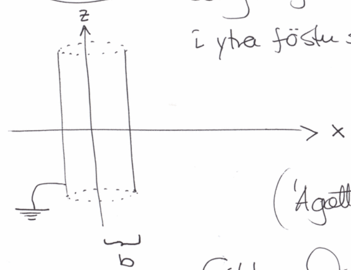
c) ① → $\frac{Q-Q_1}{\epsilon_1} = \frac{Q+Q_2}{\epsilon_2}$

② → $Q+Q_1 = Q+Q_2$

→ $Q_1 = Q_2 = \frac{\epsilon_2-\epsilon_1}{\epsilon_2+\epsilon_1} Q$

P4-25

Langur jarðbundinn sívalningur í ytra föstu sviði $\bar{E}_0 = \hat{a}_x E_0$



finna $V(r,\phi)$ og $E(r,\phi)$ utan sívalnings

(Ágætt að bera saman við Ex 4-10)

Gilda verður

$$\boxed{\begin{aligned} &V(b,\phi) = 0 \quad ① \\ &V(r,\phi) = -E_0 r\cos\phi\ ,\ r >> b \quad ② \end{aligned}}$$

Almenna lausnin er

$$V_n(r,\phi) = r^n\left(A_n\sin(n\phi)+B_n\cos(n\phi)\right)+r^{-n}\left(A_n'\sin(n\phi)+B_n'\cos(n\phi)\right)$$

ef $n \neq 0$

Til þess að uppfylla ② þarf

$A_n = 0$ f. öll n

$B_n = 0$ f $n \neq 1$, $\quad B_1 = -E_0$

$A_n' = 0$ f öll n ← samhverfa um x-z-sléttu

$$\rightarrow V(r,\phi) = -E_0 r \cos\phi + \sum_{n=1}^{\infty} B_n' r^{-n} \cos(n\phi)$$

Nú þarf að uppfylla ①

$$V(b,\phi) = -E_0 b \cos\phi + \sum_{n=1}^{\infty} B_n' \cos(n\phi) b^{-n} = 0$$

$$\rightarrow B_1' = E_0 b^2, \quad B_n' = 0 \text{ ef } n \neq 1$$

Því fæst fyrir $r \geq b$

$$V(r,\phi) = -E_0 r \left(1 - \frac{b^2}{r^2}\right) \cos\phi$$

og sviðið

$$\bar{E}(r,\phi) = -\bar{\nabla} V = -\hat{a}_r \frac{\partial V}{\partial r} + \hat{a}_\phi \frac{\partial V}{r \partial \phi}$$

$$= \hat{a}_r E_0 \left(\frac{b^2}{r^2} + 1\right) \cos\phi + \hat{a}_\phi E_0 \left(\frac{b^2}{r^2} - 1\right) \sin\phi$$

Þegar $r = b$, $\phi = 0, \pi$ fæst $|\bar{E}| = 2E_0$

P4-26

Svipað P4-25, nema nú er sívalningurinn úr rafsvara og finna skal V og $\bar{E}$ bæði innan og utan hans

Notfærum okkur upplýsingar úr P4-25

Nú er ekki hægt að krefjast að $V(b,\phi) = 0$

því er ytri lausnin

$$V_o(r,\phi) = -E_0 r \cos\phi + \sum_{n=1}^{\infty} B_n r^{-n} \cos(n\phi), \quad r \geq b$$

og innri

$$V_i(r,\phi) = \sum_{n=1}^{\infty} A_n r^n \cos(n\phi), \quad r \leq b$$

hér eru $B_n = 0$ f. öll n því lausnin getur ekki haft sérstöðupunkt í $r=0$ (þar er engin línuhleðsla).

Fyrir yfirborðið gildir nú: $\quad V_o(b,\phi) = V_i(b,\phi)$ ①

$\hat{a}_r \left(\bar{D}_o(b,\phi) = \bar{D}_i(b,\phi) \right)$ ②

② jafngildir $\quad -\frac{\partial V_o}{\partial r}\Big|_{r=b} = -\epsilon_r \frac{\partial V_i}{\partial r}\Big|_{r=b}$

---

skoðum ① sem jafngildir

$$-E_0 b \cos\phi + \sum_{n=1}^{\infty} B_n b^{-n} \cos(n\phi) = \sum_{n=1}^{\infty} A_n b^n \cos(n\phi)$$

Þetta þarf að gilda fyrir öll ϕ. Því er nauðsynlegt að flokka stuðla við $\cos(n\phi)$. Því fæst fyrir ①

$$-E_0 b + B_1 b^{-1} = A_1 b \quad (\text{stuðull við } \cos\phi)$$

$$B_n b^{-n} = A_n b^n \quad \text{ef } n \neq 1$$

fyrir ②:

$$E_0 + B_1 b^{-2} = -\epsilon_r A_1$$

$$n B_n b^{-(n+1)} = -\epsilon_r n A_n b^{n-1}, \qquad n \neq 1$$

Seinni tvö skilyrðin ganga aðeins þ. $A_n = B_n = 0 \,,\; n \neq 1$
og fyrri tvö gefa

$$A_1 = -\frac{2E_0}{\epsilon_r + 1}, \qquad B_1 = \frac{\epsilon_r - 1}{\epsilon_r + 1} b^2 E_0$$

sem leiðir til

$$\boxed{\begin{aligned} V_o(r,\phi) &= -\left(1 - \frac{\epsilon_r - 1}{\epsilon_r + 1}\frac{b^2}{r^2}\right) E_0 r \cos\phi \\ V_i(r,\phi) &= -\frac{2}{\epsilon_r + 1} E_0 r \cos\phi \end{aligned}}$$

Rafsviðið

$$\bar{E} = -\bar{\nabla} V = -\hat{a}_r \frac{\partial V}{\partial r} - \hat{a}_\phi \frac{\partial V}{r\,\partial \phi}$$

sem gefur

$$\boxed{\begin{aligned} \bar{E}_o = {} & \hat{a}_r E_0 \left(1 + \frac{\epsilon_r - 1}{\epsilon_r + 1}\frac{b^2}{r^2}\right)\cos\phi \\ & - \hat{a}_\phi E_0 \left(1 - \frac{\epsilon_r - 1}{\epsilon_r + 1}\frac{b^2}{r^2}\right)\sin\phi \end{aligned}}$$

$$\boxed{\bar{E}_i = \frac{2}{\epsilon_r + 1} E_0 \left(\hat{a}_r \cos\phi - \hat{a}_\phi \sin\phi\right)}$$

P4-29

Rafsvarakúla er komið fyrir í föstu ytra rafsviði, reikna V og E innan og utan kúlu

Best gert með samanburði við Ex 4-10 í bók.

$$V_i(R,\theta) = \sum_{n=0}^{\infty} A_n R^n P_n(\cos\theta), \qquad \underline{R \le b}$$

$$V_o(R,\theta) = \sum_{n=0}^{\infty} \left(B_n R^n + C_n R^{-(n+1)}\right) P_n(\cos\theta), \qquad \underline{R \ge b}$$

Lausnum með sérstöðupunkt í $R=0$ er sleppt þar sem þar er engin punkthleðsla til staðar.

Jaðarskilyrði

fyrir $R \gg b$ gildir enn

$$V_o(R,\theta) = -E_0 z = -E_0 R \cos\theta$$

$$\rightarrow B_1 = -E_0, \quad B_n = 0 \quad \text{fyrir } n \neq 1$$

$$V_o(R,\theta) = -E_0 R\cos\theta + C_1 R^{-2}\cos\theta + \sum_{n=2}^{\infty} C_n R^{-(n+1)} P_n(\cos\theta)$$

$n=0$ dettur út því kúlan er óhlaðin í heild (og sá liður gæti ekki uppfyllt jaðarskilyrðin í $R \gg b$)

Einnig þarf að uppfylla

$$V_i(b,\theta) = V_o(b,\theta) \qquad ①$$

$$\hat{a}_R \cdot \left(\bar{D}_i(b,\theta) = \bar{D}_o(b,\theta)\right) \qquad ②$$

Kúlan mun öðlast tvípólsvægi, en ekkert hærra vægi

$\rightarrow \quad C_n = 0$ fyrir $n \neq 1$

① $\rightarrow \quad A_1 b = -E_0 b + C_1 b^{-2}$

② $\rightarrow \quad \epsilon_r \frac{\partial V_i}{\partial R}\Big|_{R=b} = \frac{\partial V_o}{\partial R}\Big|_{R=b} \rightarrow \epsilon_r A_1 = -E_0 - 2C_1 b^{-3}$

Saman gefur þetta

$$A_1 = -\frac{3E_0}{\epsilon_r+2}$$

$$C_1 = \frac{\epsilon_r-1}{\epsilon_r+2} E_0 b^3$$

Mættið verður því

$$V_i(R,\theta) = -\frac{3E_0}{\epsilon_r+2} R\cos\theta$$

$$V_o(R,\theta) = -E_0 R\cos\theta + \frac{(\epsilon_r-1)b^3}{(\epsilon_r+2)R^2} E_0\cos\theta$$

Og rafsviðið

$$\bar{E}_i(R,\theta) = -\nabla V_i = \frac{3E_0}{\epsilon_r+2}\underbrace{(\hat{a}_R\cos\theta - \hat{a}_\theta\sin\theta)}_{\hat{a}_z}$$

$$\bar{E}_o(R,\theta) = -\nabla V_o = \hat{a}_r\left\{1 + \frac{2(\epsilon_r-1)b^3}{(\epsilon_r+2)R^3}\right\}E_0\cos\theta - \hat{a}_\theta\left\{1 - \frac{(\epsilon_r-1)b^3}{(\epsilon_r+2)R^3}\right\}E_0\sin\theta$$

P5-6 Rafsvarandi kúla verður fyrir eldingu í $t=0$

$\epsilon = 1.2\epsilon_0 \quad \sigma = 10$ S/m $\quad b = 0.1$ m

Gert er ráð fyrir að í $t=0$ verði kúlan öll í einu með jafndreifða hleðslu ρ_0, Hleðslan berst síðan hratt út á yfirborðið $\quad Q_0 = 1$ mC

a) Reikna $\bar{E}$ innan og utan kúlu

Samfelldnijafnan $\rightarrow \rho = \rho_0 e^{-(\sigma/\epsilon)t}$

$$\rho_0 = \frac{Q_0}{(\frac{4\pi}{3})b^3} \sim 0.239 \text{ C/m}^3$$

$$\bar{E}_i = \hat{a}_R \frac{(\frac{4\pi}{3})R^3\rho}{4\pi\epsilon R^2} = \hat{a}_R \frac{\rho_0 R}{3\epsilon} e^{-(\frac{\sigma}{\epsilon})t}$$

ef $R<b$

$$\simeq \hat{a}_R\, 7.5\cdot 10^9 R\, e^{-9.42\cdot 10^{11} t} \quad (\text{V/m})$$

$$\bar{E}_o = \hat{a}_R \frac{Q_0}{4\pi\epsilon_0 R^2} \simeq \hat{a}_R \frac{9\cdot 10^6}{R^2} \quad (\text{V/m})$$

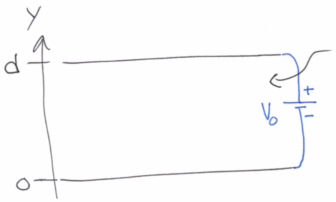
b) Reikna straumþéttleika

$$\bar{J}_i = \sigma\bar{E}_i = \hat{a}_R\, 7.5\cdot 10^{10} R\, e^{-9.42\cdot 10^{11} t} \quad (\text{A/m}^2) \quad \underline{R<b}$$

$\bar{J}_o = 0$ fyrir $R>b$ ($\sigma=0$ hér)

P5-10

þéttir með tvær samsíða plötur með flatarmál S



línuleg breyting á σ frá $\sigma(0)=\sigma_1$ til $\sigma(d)=\sigma_2$

V_0 er sett á milli platna

$$\sigma(y) = \sigma_1 + (\sigma_2-\sigma_1)\frac{y}{d}$$

a) Finna R milli platna

$$\bar{J} = -\hat{a}_y J_0 \rightarrow \bar{E} = \frac{\bar{J}}{\sigma} = -\hat{a}_y \frac{J_0}{\sigma(y)}$$

$$V_0 = -\int_0^d \bar{E}\cdot\hat{a}_y\,dy = \int_0^d \frac{J_0\,dy}{\sigma_1+(\sigma_2-\sigma_1)\frac{y}{d}} = \frac{J_0 d}{\sigma_2-\sigma_1}\ln\left(\frac{\sigma_2}{\sigma_1}\right)$$

$$R = \frac{V_0}{I} = \frac{V_0}{J_0 S} = \frac{d}{(\sigma_2-\sigma_1)S}\ln\left(\frac{\sigma_2}{\sigma_1}\right)$$

b) finna flatarhleðslur platna

$$\rho_s(d) = \epsilon_0 E_y(d) = \epsilon_0\frac{J_0}{\sigma_2} = \frac{\epsilon_0(\sigma_2-\sigma_1)V_0}{\sigma_2 d\ln\left(\frac{\sigma_2}{\sigma_1}\right)}$$

$$\rho_s(0) = -\epsilon_0 E_y(0) = -\epsilon_0\frac{J_0}{\sigma_1} = -\frac{\epsilon_0(\sigma_2-\sigma_1)V_0}{\sigma_1 d\ln\left(\frac{\sigma_2}{\sigma_1}\right)}$$

c) finna heildarhleðslu milli platna og dreifingu hennar.

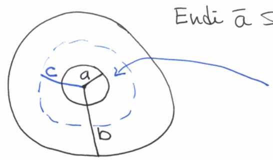
$$\rho(y) = \bar{\nabla}\cdot\bar{D} = \frac{d}{dy}(\epsilon_0 E), \quad E = -\frac{J_0}{\sigma(y)}$$

$$\rightarrow \rho(y) = \frac{d}{dy}(\epsilon_0 E) = -\epsilon_0 J_0\frac{d}{dy}\left(\frac{1}{\sigma(y)}\right)$$

$$= \epsilon_0 J_0\frac{(\sigma_2-\sigma_1)/d}{\left\{\sigma_1+(\sigma_2-\sigma_1)\frac{y}{d}\right\}^2}$$

P5-13

Endi á sívalningsþétti með lengd L



$a<r<c$: ϵ_1 og σ_1

$c<r<b$: ϵ_2 og σ_2

V_0 milli platna

a) finna straumþéttleika á báðum svæðum

Tveir raðtengdir þéttar (með aukaplötur í c)

$$\frac{C_1}{L} = \frac{2\pi\epsilon_1}{\ln(c/a)} \quad \text{og} \quad \frac{C_2}{L} = \frac{2\pi\epsilon_2}{\ln(b/c)}$$

Notum $\frac{C}{G} = \frac{\epsilon}{\sigma}$

raðtengd leiðni

$$\rightarrow G_1 = \frac{2\pi\sigma_1 L}{\ln\left(\frac{c}{a}\right)} \quad \text{og} \quad G_2 = \frac{2\pi\sigma_2 L}{\ln\left(\frac{b}{c}\right)}$$

$$I = V_0 G = V_0 \frac{G_1 G_2}{G_1 + G_2} = \frac{2\pi\sigma_1\sigma_2 L V_0}{\sigma_1 \ln\left(\frac{b}{c}\right) + \sigma_2 \ln\left(\frac{c}{a}\right)}$$

Straumvarðveisla

$$J_1 = J_2 = \frac{I}{2\pi r L} = \frac{\sigma_1\sigma_2 V_0}{r\left\{\sigma_1 \ln\left(\frac{b}{c}\right) + \sigma_2 \ln\left(\frac{c}{a}\right)\right\}}$$

Hér má því lesa E_1 og E_2 með samanburði við $J = \sigma E$

b) Reikna ρ_s á plötunum og á mótum rafsvaranna

$$\rho_s(a) = \epsilon_1 E_1\Big|_{r=a} = \frac{\epsilon_1\sigma_2 V_0}{a\left\{\sigma_1 \ln\left(\frac{b}{c}\right) + \sigma_2 \ln\left(\frac{c}{a}\right)\right\}}$$

$$\rho_s(b) = -\epsilon_2 E_2\Big|_{r=b} = -\frac{\epsilon_2\sigma_1 V_0}{b\left\{\sigma_1 \ln\left(\frac{b}{c}\right) + \sigma_2 \ln\left(\frac{c}{a}\right)\right\}}$$

$$\rho_s(c) = -\left(\epsilon_1 E_1 - \epsilon_2 E_2\right)\Big|_{r=c} \qquad \left(-\hat{a}_r \cdot (\bar{D}_1 - \bar{D}_2) = \rho_s\right)$$

$$= \frac{(\epsilon_2\sigma_1 - \epsilon_1\sigma_2) V_0}{c\left\{\sigma_1 \ln\left(\frac{b}{c}\right) + \sigma_2 \ln\left(\frac{c}{a}\right)\right\}}$$

PS-21

hálfkúla í jörð

$\sigma = 10^{-6}$ (S/m)

$b = 25\,\text{mm}$

Finna R til fjarlægra punkta

Í dæmi PS-20 er sýnt að straumlínur í þessu kerfi eru radial (Hólfurinn er tekinn í burtu og hálfkúlan gerð heil.)

$$\rightarrow \bar{J} = \hat{a}_R \frac{I}{2\pi R^2}, \quad \text{gefum okkur } I$$

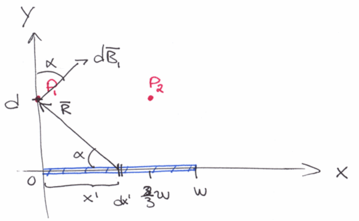
$$\bar{E} = \hat{a}_R \frac{I}{2\pi\sigma R^2} \quad \text{eftir samanburð við } \bar{J} = \sigma\bar{E}$$

$$V_0 = -\int_\infty^b E\,dR = -\frac{I}{2\pi\sigma}\int_\infty^b \frac{dR}{R^2} = \frac{I}{2\pi\sigma b}$$

$$R = \frac{V_0}{I} = \frac{1}{2\pi\sigma b} \approx 6.36 \cdot 10^6\ (\Omega)$$

P6-4

Straumur flýtur eftir löngum flötum leiðara með breidd w. I er inn í blaðið



a) Reikna $\bar{B}_1$ í P_1

b) Nota niðurstöðu úr a) t.þ.a reikna $\bar{B}_2$ í P_2

Hugsum flata leiðarann sem safn samsíða víra með breidd dx' og því straum $I\frac{dx'}{w}$

fyrir einn slíkan leiðara er, (sjá (6-36))

$$|d\bar{B}_1| = \frac{\mu_0 I dx'}{2\pi R w}$$

Stefnan er hornrétt á $\bar{R}$ eins og mynd sýnir

$$d\bar{B}_1 = \hat{a}_x (dB_1)\sin\alpha + \hat{a}_y (dB_1)\cos\alpha \qquad (*)$$

Umformum $\sin\alpha$ og $\cos\alpha$ í föll af x' og getum föstum

$$\sin\alpha = \frac{d}{\sqrt{x'^2+d^2}} \qquad \cos\alpha = \frac{x'}{\sqrt{x'^2+d^2}}$$

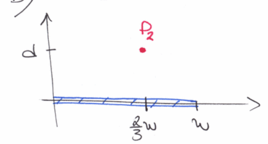
Setjum frá (*)

$$d\bar{B}_1 = \hat{a}_x dB_{x1} + \hat{a}_y dB_{y1}$$

$$B_{x1} = \frac{\mu_0 I d}{2\pi w}\int_0^w \frac{dx'}{x'^2+d^2} = \frac{\mu_0 I}{2\pi w}\arctan\left(\frac{w}{d}\right)$$

$$B_{y1} = \frac{\mu_0 I}{2\pi w}\int_0^w \frac{x'dx'}{x'^2+d^2} = \frac{\mu_0 I}{4\pi w}\ln\left(1+\frac{w^2}{d^2}\right)$$

b)

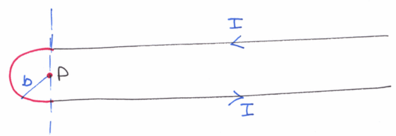


leggja saman segulvigrana fyrir slétturnar vinstra og hægra megin við P_2

Hægra megin $\quad \bar{B}_{2R} = \frac{\mu_0 I}{2\pi w}\left\{\hat{a}_x \arctan\left(\frac{w}{3d}\right) + \hat{a}_y \frac{1}{2}\ln\left(1+\left(\frac{w}{3d}\right)^2\right)\right\}$

Vinstra $\quad \bar{B}_{2L} = \frac{\mu_0 I}{2\pi w}\left\{\hat{a}_x \arctan\left(\frac{2w}{3d}\right) - \hat{a}_y \frac{1}{2}\ln\left(1+\left(\frac{2w}{3d}\right)^2\right)\right\}$

P6-11



Finna $\bar{B}$ í punkti P. Tveir leiðarar í $+\infty$ og hálfhringur.

Berum saman við Ex 6-4 b) þá sést að segulsvið þessara samhliða víra leggst saman og verður eins og segulsvið eins óendanlegs vírs

$$\bar{B}_1 = \hat{a}_z \frac{\mu_0 I}{2\pi b}$$

ef $\hat{a}_z$ er einingarvigurinn út úr blaðinu.

Þá er eftir að finna $\bar{B}_2$ fyrir hálfhringinn

$$\bar{B}_2 = \frac{\mu_0 I}{4\pi}\hat{a}_z \int_{\pi/2}^{3\pi/2} \frac{b^2 d\phi'}{b^3}$$

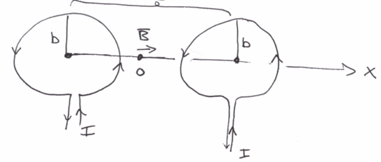
$\hat{a}_r$ - þættirnir koma ekki við sögu þ.a. z og $z'=0$

$$= \hat{a}_z \frac{\mu_0 I}{4b}$$

$$\longrightarrow \bar{B} = \hat{a}_z \frac{\mu_0 I}{2b}\left(\frac{1}{\pi} + \frac{1}{2}\right)$$ fyrir heildarsviðið

P6-12

Helmholtz-spólur



a) finna $\bar{B} = \hat{a}_x B_x$ mitt á milli spólna

Notum niðurstöðuna fyrir eina spólu (6-38)

$$B_x(x) = \frac{N\mu_0 I b^2}{2}\left\{\frac{1}{\left[\left(\frac{d}{2}+x\right)^2 + b^2\right]^{3/2}} + \frac{1}{\left[\left(\frac{d}{2}-x\right)^2 + b^2\right]^{3/2}}\right\}$$

í miðpunkti

$$B_x(0) = \frac{N\mu_0 I b^2}{\left[\left(\frac{d}{2}\right)^2 + b^2\right]^{3/2}}$$

b) Sýna að $\frac{dB_x}{dx} = 0$ í miðpunkti

$$\frac{dB_x}{dx} = \frac{N\mu_0 I b^2}{2}\left\{-\frac{3\left(\frac{d}{2}+x\right)}{\left[\left(\frac{d}{2}+x\right)^2 + b^2\right]^{5/2}} + \frac{3\left(\frac{d}{2}-x\right)}{\left[\left(\frac{d}{2}-x\right)^2 + b^2\right]^{5/2}}\right\}$$

$\longrightarrow 0$ þegar $x \to 0$

c) finna skilyrði á b og d þ.a. $\frac{d^2B_x}{dx^2} = 0$ í miðp.

$$\frac{d^2B_x}{dx^2} = -\frac{3N\mu_0 I b^2}{2}\left\{\frac{1}{\left[\left(\frac{d}{2}+x\right)^2 + b^2\right]^{5/2}} - \frac{5\left(\frac{d}{2}+x\right)^2}{\left[\left(\frac{d}{2}+x\right)^2 + b^2\right]^{7/2}} + \frac{1}{\left[\left(\frac{d}{2}-x\right)^2 + b^2\right]^{5/2}} - \frac{5\left(\frac{d}{2}-x\right)^2}{\left[\left(\frac{d}{2}-x\right)^2 + b^2\right]^{7/2}}\right\}$$

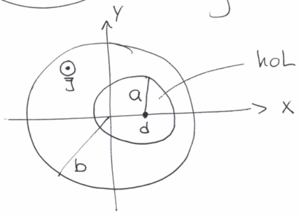
$$\left.\frac{d^2B_x}{dx^2}\right|_{x=0} = -3N\mu_0 I b^2\left\{\frac{b^2 - 4\left(\frac{d}{2}\right)^2}{\left[\left(\frac{d}{2}\right)^2 + b^2\right]^{7/2}}\right\} \longrightarrow 0$$

ef $b = d$

P6-15

Langur sívalningur

$$\boxed{\oint \bar{B}\cdot d\bar{l} = \mu_0 I}$$



$\bar{J} = \hat{a}_z J$

Án hols

$\hookrightarrow 2\pi r_1 B_{\phi 1} = \mu_0 \pi r_1^2 J$

$\to B_{\phi 1} = \frac{\mu_0 r_1}{2} J$

Eða greint í kartesk hnit

$B_{x1} = -\frac{\mu_0 J}{2} y_1$

$B_{y1} = +\frac{\mu_0 J}{2} x_1$

Hugsum holið sem hlut með $-J$

þar fæst þá

$B_{\phi 2} = -\frac{\mu_0 r_2}{2} \to$ $B_{x2} = +\frac{\mu_0 J}{2} y_2$, $B_{y2} = -\frac{\mu_0 J}{2} x_2$

leggjum saman í hnit og notum

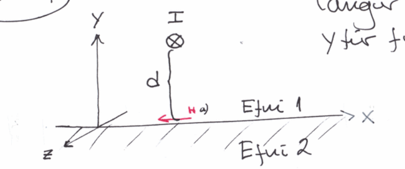
$y_2 = y_1$, en $x_1 = x_2 + d$

$\to B_x = B_{x1} + B_{x2} = 0$

$B_y = B_{y1} + B_{y2} = \frac{\mu_0 J}{2} d$ fasti

P6-34

Langur leiðari með straum I yfir fylltu hálfrúmi



a) Hugleiða hver og samsíða þættir $\bar{B}$ og $\bar{H}$ við skilflötinn

i) Ef $\sigma_2 \to \infty$

Innan 2 gildir

$\bar{\nabla}\times\bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$

$\sigma_2 \to \infty$ því er $\bar{E} = 0$ innan 2

$\to \frac{\partial}{\partial t}\bar{H} = 0$

Ekkert segulsvið í upphafi áður en kveikt er á leiðara $\to$ ekkert $\bar{H}$ eftir að kveikt er

$\bar{H}_2 = 0$, $\bar{B}_2 = 0$

$B_{n1} = B_{n2}$, $\mu_1 H_{n1} = \mu_2 H_{n2}$ $\to$ $B_{1n} = 0$, $H_{1n} = 0$

$\hat{a}_y \times \bar{H}_1 = \bar{J}_s \to \bar{J}_s = -\hat{a}_z H_{1x}$ $\quad H_{1x} < 0$

Spegilmynd vírs þyrfti að hafa gagnstæðan straum

ii) $\mu_2 \to \infty \to \bar{H}_2 = 0$ (Annars væri B „mjög stórt")

$\hookrightarrow \bar{J}_s = 0$

$\hookrightarrow H_{1t} = H_{2t} = 0$

$B_{1n} = B_{2n}$

$\to$ Spegilmynd vírs hefur straum í sömu átt

b) Finna $\bar H$ Hugsum vír í hæð d á y-ás

i) $\bar H(x,y) = \bar H_1 + \bar H_2$

$$\bar H_1 = \frac{I}{2\pi}\left\{\frac{\hat a_x (y-d)}{x^2+(y-d)^2} - \frac{\hat a_y\, x}{x^2+(y-d)^2}\right\}$$

$$\bar H_2 = \frac{I}{2\pi}\left\{-\frac{\hat a_x (y+d)}{x^2+(y+d)^2} + \frac{\hat a_y\, x}{x^2+(y+d)^2}\right\}$$

ii) $\bar H(x,y) = \bar H_1 - \bar H_2$

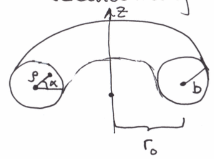
c) yfirborðstraumur

(ii). $J_s = 0$

(i) $J_s = -\hat a_z H_{1x} = \hat a_z\left(\frac{Id}{x^2+d^2}\right)$

P6-35

Kleinuhringur N-vafningar



$b \ll r_0$

finna sjálfspan

Bera saman við Ex 6-2 þá sést að góð stikun á kleinuhringnum gefur

$$\bar B = \hat a_\phi \frac{\mu_0 NI}{2\pi r}, \quad r = r_0 - \rho\cos\alpha$$

því fæst flæðið

$$\Phi = \frac{\mu_0 NI}{2\pi}\int_0^b \rho\, d\rho \int_0^{2\pi} \frac{d\varphi}{r_0 - \rho\cos\alpha}$$

(GR 3.613.1)

$$\Phi = \frac{\mu_0 NI}{2\pi}\int_0^b \rho\, d\rho\, \frac{2\pi}{\sqrt{r_0^2-\rho^2}} = -\mu_0 NI\sqrt{r_0^2-\rho^2}\,\Big|_0^b$$

$$= -\mu_0 NI\left(\sqrt{r_0^2-b^2} - r_0\right) = \mu_0 NI\left(r - \sqrt{r_0^2-b^2}\right)$$

$$L = \frac{N\Phi}{I} = \mu_0 N^2\left(r_0 - \sqrt{r_0^2-b^2}\right)$$

Ef $r_0 \gg b \longrightarrow B_\phi \simeq \frac{\mu_0 NI}{2\pi r_0}$ ← fasti

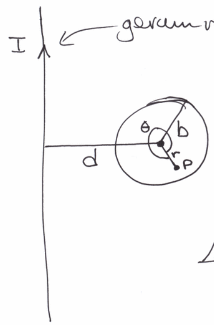
$$\Phi \simeq B_\phi(\pi b^2) = \frac{\mu_0 N b^2 I}{2r_0} \longrightarrow L \simeq \frac{\mu_0 N^2 b^2}{2r_0}$$

eða $L = \frac{\mu_0 N^2}{r_0}\left(1 - \sqrt{1-\frac{b^2}{r_0^2}}\right) \simeq \frac{\mu_0 N^2}{r_0}\left(1 - 1 + \frac{b^2}{2r_0^2} + \cdots\right) \simeq \frac{\mu_0 N^2 b^2}{2r_0}$

P6-39

Finna víxlspanið

gerum ráð fyrir I



$\bar B$ í $P(r,\theta)$ er

$$\bar B = \hat a_\phi \frac{\mu_0 I}{2\pi(d + r\cos\theta)}$$

$$\Lambda_{12} = \frac{\mu_0 I}{2\pi}\int_0^b r\,dr \int_0^{2\pi} d\theta\, \frac{1}{d + r\cos\theta}$$

(GR 3.613.1)

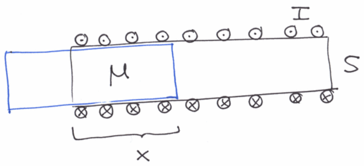
$$= \frac{\mu_0 I}{2\pi}\int_0^b \frac{2\pi r\,dr}{\sqrt{d^2-r^2}} = \mu_0 I\left(d - \sqrt{d^2-b^2}\right) = L_{12} I$$

$$\longrightarrow L_{12} = \mu_0\left(d - \sqrt{d^2-b^2}\right)$$

P6-53

Járnkjarni í spólu

Finna kraftinn á kjarnann



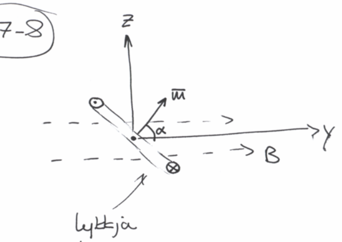
$W_m = \frac{1}{2}\int \mu H^2 dv$ Gerum ráð fyrir sýndarfærslu Δx

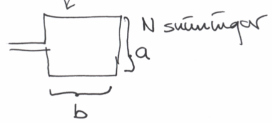
$$W_m(x+\Delta x) = W_m(x) + \frac{1}{2}\int_{S\cdot\Delta x} (\mu-\mu_0) H^2 dv$$

$$= W_m(x) + \frac{1}{2}(\mu-\mu_0)\, S\cdot\Delta x \,\frac{(\mu n I)^2}{\mu^2} = W_m(x) + \frac{\mu_0(\mu_r-1)n^2I^2S\Delta x}{2}$$

$$(F)_x = \frac{\partial W_m}{\partial x} = \frac{\mu_0}{2}(\mu_r-1)n^2I^2S \longrightarrow \bar{F}$$

P7-8





Vægið á lykkjuna er

$$\bar{T} = \bar{m}\times\bar{B} = -\hat{a}_x NabIB\sin\alpha$$

Mekanísk vinna við að snúa lykkjunni um $-\Delta\alpha$ er

$$W_m = T(-\Delta\alpha) = NabIB(\Delta\alpha)\sin\alpha$$

Flæðitengsl við lykkju

$$\Lambda = N\Phi = Nab\cos\alpha$$

íspennan spönuð í rásinni

$$V = -N\frac{d\Phi}{dt} = NabB\left(\frac{\Delta\alpha}{\Delta t}\right)\sin\alpha$$

Raf vinnan sem þarf til þess að senda strauminn I á móti þessari íspennu á tímabilinu Δt

$$W_e = VI\Delta t = INabB(\Delta\alpha)\sin\alpha = W_m$$

P7-12

Sýna að Lorentz-kvarðinn

$$\bar{\nabla}\cdot\bar{A} + \mu\epsilon\frac{\partial}{\partial t}V = 0$$

sé í samræmi við samfelldnijöfnuna

$$\frac{\partial}{\partial t}\rho + \bar{\nabla}\cdot\bar{J} = 0$$

Skilgreinum $\Box^2 = \nabla^2 - \mu\epsilon\frac{\partial^2}{\partial t^2}$ (d'Alembertian)

þá eru bylgjujöfnurnar

$$\Box^2\bar{A} = -\mu\bar{J} \quad \text{og} \quad \Box^2 V = -\frac{\rho}{\epsilon}$$

Skoðum nú

$$\Box^2\left(\bar\nabla\cdot\bar A+\mu\epsilon\frac{\partial}{\partial t}V\right)=0$$

Lorentz skilyrðið er að þetta sé jafnt 0

$$=\bar\nabla\cdot\left(\Box^2\bar A\right)+\mu\epsilon\frac{\partial}{\partial t}\left(\Box^2 V\right)=0$$

$$=\bar\nabla\cdot\left(-\mu\bar J\right)+\mu\epsilon\frac{\partial}{\partial t}\left(-\frac{\rho}{\epsilon}\right)=0$$

$$\boxed{\bar\nabla\cdot\bar J+\frac{\partial}{\partial t}\rho=0}$$

Bylgjujöfnurnar + Lorentz-skilyrðin gefa samfelldni jöfnuna. (Innihalda hana)

P7-21

Sýna að

$$V(R,t)=\frac{1}{4\pi\epsilon}\int_{V'}\frac{\rho(t-R/u)}{R}\,dv'$$

uppfylli

$$\nabla^2V-\mu\epsilon\frac{\partial^2V}{\partial t^2}=-\frac{\rho}{\epsilon}$$

Veljum ρ kúlusamhverft

Getum notað

$$\nabla^2(fg)=\bar\nabla\cdot\bar\nabla(fg)=g\nabla^2f+f\nabla^2g+2(\bar\nabla f)\cdot(\bar\nabla g)$$

til þess að fá

$$\nabla^2\left(\frac{\rho}{R}\right)=\frac{1}{R}\nabla^2\rho+\rho\nabla^2\left(\frac{1}{R}\right)+2(\bar\nabla\rho)\cdot\bar\nabla\left(\frac{1}{R}\right)$$

Skilgreinum nýja breytu $s=t-\frac{R}{u}$

$$\nabla^2\rho(s)=\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial\rho}{\partial R}\right)=\frac{1}{u^2}\frac{d^2\rho}{ds^2}-\frac{2}{uR}\frac{d\rho}{ds}$$

Notum síðan að

$$\nabla^2\left(\frac{1}{R}\right)=-4\pi\delta(R)$$

Dirac δ-fallið

og

$$\bar\nabla\rho\cdot\bar\nabla\left(\frac{1}{R}\right)=\frac{\partial\rho}{\partial R}\left(-\frac{1}{R^2}\right)=\frac{1}{uR^2}\frac{d\rho}{ds}$$

því fæst að

$$\nabla^2\left(\frac{\rho}{R}\right)=\frac{1}{u^2R}\frac{d^2\rho}{ds^2}-4\pi\rho\delta(R)$$

Reynum nú lausnina

$$\nabla^2V=\frac{1}{4\pi\epsilon}\nabla^2\int_{V'}\frac{\rho}{R}\,dv'=\frac{1}{4\pi\epsilon}\int_{V'}\left\{\frac{1}{u^2R}\frac{d^2\rho}{ds^2}-4\pi\rho\delta(R)\right\}dv'$$

$$\frac{\partial^2V}{\partial t^2}=\frac{1}{4\pi\epsilon}\int_{V'}\frac{1}{R}\frac{d^2\rho}{ds^2}\,dv'$$

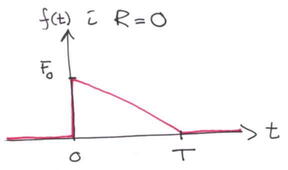
Setjum saman í bylgjujöfnuna

$$\nabla^2V-\frac{1}{u^2}\frac{\partial^2V}{\partial t^2}=\frac{1}{4\pi\epsilon}\int_{V'}\left\{\frac{1}{u^2R}\frac{d^2\rho}{ds^2}-4\pi\rho\delta(R)-\frac{1}{u^2R}\frac{d^2\rho}{ds^2}\right\}dv'$$

$$=-\int_{V'}dv'\frac{\rho}{\epsilon}\delta(R)=-\frac{\rho}{\epsilon}$$

$\delta(\bar x-\bar x')$ því er $\rho(\bar x)$ eins og á að vera

P7-22 fyrir $f(t)$ í $R=0$



Teikna

a) $f(t-\frac{R}{u})$ vs t fyrir fast R

b) $f(t-\frac{R}{u})$ vs R fyrir $t>T$

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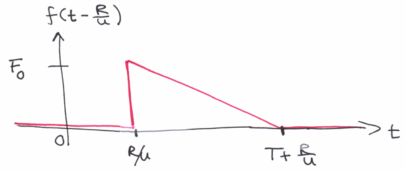
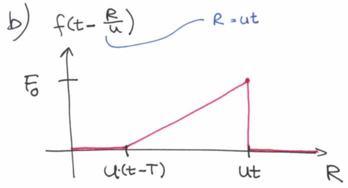
a)



b)



P7-29 Efni með

$\rho=0,\ \bar{J}=0,\ \mu=\mu_0$

en $\bar{P}\neq 0$. Þar er til vigurmátti: $\bar{\Pi}_e$ þ.a.

$$\bar{H} = i\omega\epsilon_0\nabla\times\bar{\Pi}_e \qquad (*)$$

$\bar{\Pi}_e$ er þekkt sem vigur Hertz eða Hertz-mátti

a) finna $\bar{E}$ táknað við $\bar{\Pi}_e$ og $\bar{P}$

Faraday $\quad \nabla\times\bar{E} = -i\omega\mu_0\bar{H} = \omega^2\mu_0\epsilon_0\nabla\times\bar{\Pi}_e = k_0^2\nabla\times\bar{\Pi}_e$

$\hookrightarrow \nabla\times(\bar{E}-k_0^2\bar{\Pi}_e)=0$

Hægt að skilgreina V_e (skalarmátti) þ.a.

$$\bar{E}-k_0^2\bar{\Pi}_e = \nabla V_e \qquad (**)$$

Ampère-Maxwell

$$\nabla\times\bar{H} = i\omega\bar{D} = i\omega(\epsilon_0\bar{E}+\bar{P}) = i\omega\epsilon_0\left(\bar{E}+\frac{\bar{P}}{\epsilon_0}\right)$$

Nýtum (*) og (**) hér

$$i\omega\epsilon_0\nabla\times\nabla\times\bar{\Pi}_e = i\omega\epsilon_0\left(k_0^2\bar{\Pi}_e+\nabla V_e+\frac{\bar{P}}{\epsilon_0}\right)$$

$$= i\omega\epsilon_0\left\{\nabla(\nabla\cdot\bar{\Pi}_e)-\nabla^2\bar{\Pi}_e\right\}$$

Veljum $\quad \nabla\cdot\bar{\Pi}_e = V_e \quad$ „Lorentz-skilyrði"

Jafnan verður þá

$$\boxed{\nabla^2\bar{\Pi}_e + k_0^2\bar{\Pi}_e = \frac{\bar{P}}{\epsilon_0}} \qquad (***)$$

Sem er svar við b-lið

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a) $\quad \bar{E} = k_0^2\bar{\Pi}_e + \nabla V_e = k_0^2\bar{\Pi}_e + \nabla(\nabla\cdot\bar{\Pi}_e)$

„Lorentz skilyrði" notað

$$= k_0^2\bar{\Pi}_e + \left(\nabla^2\bar{\Pi}_e + \nabla\times\nabla\times\bar{\Pi}_e\right)$$

notum (***) t.þ.a. fá þetta á formið

$$\boxed{\bar{E} = \nabla\times\nabla\times\bar{\Pi}_e - \frac{\bar{P}}{\epsilon_0}}$$

P8-4

Lotubundið í tíma
Einsleitt rúm

Lotubinding í tíma $e^{i\omega t} \Rightarrow \frac{\partial}{\partial t} \to i\omega$

fasorar $\bar{E} = \bar{E}_0 e^{-i\bar{k}\cdot\bar{R}}$ $\quad \bar{H} = \bar{H}_0 e^{-i\bar{k}\cdot\bar{R}}$

$$\Rightarrow \bar{\nabla}\cdot\bar{E} \to -i\bar{k}\cdot\bar{E}, \quad \bar{\nabla}\times\bar{E} \to -i\bar{k}\times\bar{E}$$

$$\bar{\nabla}V \to -i\bar{k}V$$

Þá eru jöfnur Maxwells

$$\bar{k}\times\bar{E} = \omega\mu\bar{H}, \quad \bar{k}\times\bar{H} = -\omega\epsilon\bar{E}$$

$$\bar{k}\cdot\bar{E} = 0, \quad \bar{k}\cdot\bar{H} = 0$$

P8-8

a) Ellipskt skautuð bylgja er samsett úr hringskautuðum bylgjum með hægri og vinstri skautun

Almenn ellipsk bylgja

$$\bar{E} = \hat{a}_x E_1 \pm \hat{a}_y E_2 e^{i\alpha},$$

E_1, E_2 og α eru fastar. Hér er stuðli e^{-ikz} sleppt

Hægri hringb. $\bar{E}_{rc} = E_{rc}(\hat{a}_x - i\hat{a}_y)$

Vinstri hringb. $\bar{E}_{\ell c} = E_{\ell c}(\hat{a}_x + i\hat{a}_y)$

Veljum nú $E_{rc} = \frac{1}{2}(E_1 \pm iE_2 e^{i\alpha})$ og $E_{\ell c} = \frac{1}{2}(E_1 \mp iE_2 e^{i\alpha})$

Þá fæst

$$\bar{E} = \bar{E}_{rc} + \bar{E}_{\ell c}$$

b) Hringskautuð bylgja er sett saman úr tveimur ellipsuskautuðum í sitthvora áttina

$$\bar{E}_{rc} = E_{rc}(\hat{a}_x - i\hat{a}_y)$$

$$= E_{rc}\left(\tfrac{1}{2}\hat{a}_x - 2i\hat{a}_y\right) + E_{rc}\left(\tfrac{1}{2}\hat{a}_x + i\hat{a}_y\right) = \bar{E}_{\ell+} + \bar{E}_{\ell-}$$

Ellipskar bylgjur

$$\bar{E}_{\ell c} = E_{rc}(\hat{a}_x + i\hat{a}_y)$$

$$= E_{rc}\left(\tfrac{1}{2}\hat{a}_x + 2i\hat{a}_y\right) + E_{rc}\left(\tfrac{1}{2}\hat{a}_x - i\hat{a}_y\right) = \bar{E}_{\ell-} + \bar{E}_{\ell+}$$

P8-16

a) US-staðall setur öryggismörk fyrir EM geislun

10 $\frac{mW}{cm^2}$

$$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathrm{Re}(\bar{E}\times\bar{H}^*), \quad |H| = \frac{1}{\eta_0}E$$

$$\mathcal{P}_{av} = \frac{|E|^2}{2\eta_0} = 10^{-2}\ \frac{W}{cm^2}$$

$$\to |E| = \sqrt{2\cdot 10^{-2}\eta_0} = 275\ V/m$$

$$|H| = |E|/\eta_0 = 0{,}728\ A/m$$

b) Sólin gefur 1,3 kW/m² (Ef einlitt)

$|E| = 990$ V/m $\quad |H| = 2.63$ A/m

P8-26

Lóðrétt innfall sléttrarbylgju á flöt milli tveggja rofsvara — taplaust efni

finna skilyrði þess að $|\Gamma| = |\tau|$

$(8\text{-}142) \rightarrow 1 + \Gamma = \tau$

Vitum líka frá (8-140)

$|\Gamma| \leq 1$

Ef $|\tau| = |\Gamma|$ $\rightarrow |\eta_2 - \eta_1| = 2|\eta_2|$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Ef $\Gamma < 0$

$\eta_1 - \eta_2 = 2\eta_2$

$\rightarrow \eta_1 = 3\eta_2$

$$\sqrt{\frac{\mu_1}{\epsilon_1}} = 3\sqrt{\frac{\mu_2}{\epsilon_2}}$$

$\rightarrow |\Gamma| = \frac{1}{2}$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = 3$$

$$S_{db} = 20 \log 3 = 9.54\,\text{dB}$$

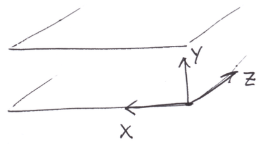
P10-7

u_{en} fyrir TE_n í taplausum stokki milli samsíða platna

$\gamma = i\beta$

Bera saman við Ex. 10-6 fyrir TM_n

Sviðin eru gefin í (10-83, 85)



$$H_z^0(y) = B_n \cos\left(\frac{n\pi y}{b}\right)$$

$$H_y^0(y) = \frac{\gamma}{h} B_n \sin\left(\frac{n\pi y}{b}\right)$$

$$E_x^0(y) = \frac{i\omega\mu}{h} B_n \sin\left(\frac{n\pi y}{b}\right)$$

$$\bar{S}_{ave} = \frac{1}{2}\Re(\bar{E} \times \bar{H}^*) = \frac{1}{2}\Re\left(\hat{a}_z E_x^0 H_y^{0*} - \hat{a}_y E_x H_z^{0*}\right)$$

$$\bar{S}_{ave} \cdot \hat{a}_z = \frac{1}{2}\Re\left(E_x^0 H_y^{0*}\right) = \frac{\omega\mu\beta}{2h^2} B_n^2 \sin^2\left(\frac{n\pi y}{b}\right)$$

$$(P_z)_{ave} = \int_0^b \bar{S}_{ave} \cdot \hat{a}_z \, dy = \frac{\omega\mu\beta b}{4h^2} B_n^2$$

á lengdareiningu í x-átt

Orkuþéttleiki

$$(w_e)_{ave} = \frac{\epsilon}{4}\Re\left(\bar{E}^0 \cdot \bar{E}^{0*}\right) = \frac{\epsilon\omega^2\mu^2}{4h^2} B_n^2 \sin^2\left(\frac{n\pi y}{b}\right)$$

$$(W_e)_{ave} = \int_0^b (w_e)_{ave} \, dy = \frac{\epsilon\omega^2\mu^2 b}{8h^2} B_n^2 \leftarrow$$

og sama fæst fyrir $(W_m)_{ave}$

$$u_{en} = \frac{(P_z)_{ave}}{(W_e)_{ave} + (W_m)_{ave}} = \frac{\omega\mu\beta b}{\epsilon\omega^2\mu^2 b} = \frac{\beta}{\epsilon\mu\omega} = \frac{\omega\beta}{\epsilon\mu\omega^2}$$

$$= \frac{\omega\beta}{k^2} = u\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

sama og fyrir TM

$$k^2 = \omega^2\mu\epsilon$$

P10-2

Hornréttur bylgjustokkur

a) Teikna $\frac{u_g}{u}$ og $\frac{\beta}{k}$ vs $\frac{f}{f_c}$

Eq. (10-38)

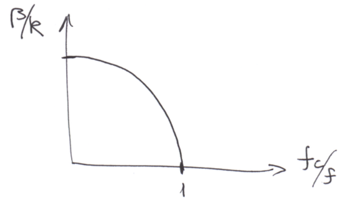
$$\beta = k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

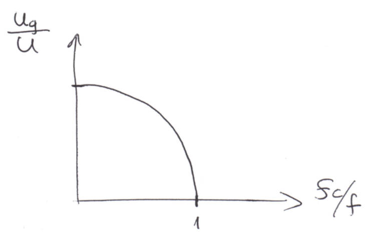
Eq. (10-43)

$$u_g = u\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\left(\frac{\beta}{k}\right)^2 + \left(\frac{f_c}{f}\right)^2 = 1$$

$$\left(\frac{u_g}{u}\right)^2 + \left(\frac{f_c}{f}\right)^2 = 1$$





b) Teikna $\frac{u}{u_p}$, $\frac{\beta}{k}$, og $\frac{\lambda_g}{\lambda}$ vs f/f_c

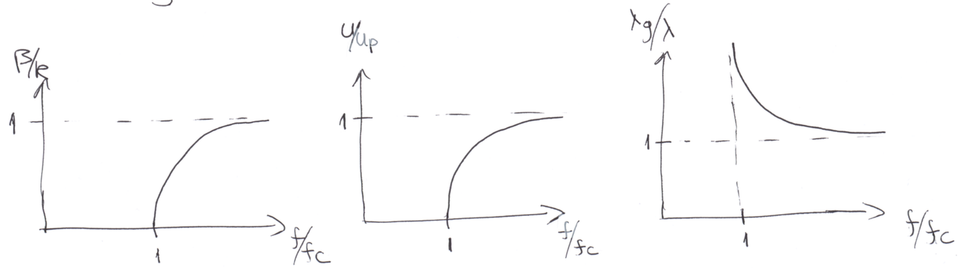
Eq. (10-42)

$$\frac{u_p}{u} = \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \rightarrow \left(\frac{u}{u_p}\right)^2 = 1-\left(\frac{f_c}{f}\right)^2 = 1 - \frac{1}{\left(\frac{f}{f_c}\right)^2}$$

$$\left(\frac{\beta}{k}\right)^2 = 1-\left(\frac{f_c}{f}\right)^2 = 1-\frac{1}{\left(\frac{f}{f_c}\right)^2}$$

Eq. (10-43)

$$\left(\frac{\lambda}{\lambda_g}\right)^2 = 1-\left(\frac{f_c}{f}\right)^2 \rightarrow \left(\frac{\lambda_g}{\lambda}\right)^2 = \frac{1}{1-\left(\frac{f_c}{f}\right)^2} = \frac{\left(f/f_c\right)^2}{\left(\frac{f}{f_c}\right)^2-1}$$



c) $\frac{u_p}{u}$, $\frac{u_g}{u}$, $\frac{\beta}{k}$, og $\frac{\lambda_g}{\lambda}$ við $f = 1.25 f_c$

$\rightarrow$ $u_p/u = 1.67$

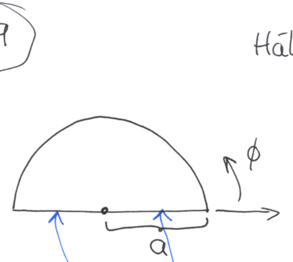
$u_g/u \simeq 0{,}60$

$\beta/k = 0{,}60$

$\lambda_g/\lambda = 1{,}67$

P10-29

Hálfsívalningur sem bylgjuleiðari



a) finna E_z^0 fyrir TM

Lausnin er $E_z^0(r\phi) = A_n J_n(hr) \sin(n\phi)$

því þá er jaðarstilyrðið $E_z^0 = 0$

líka uppfyllt fyrir $\phi = 0, \pi$

$J_n(ha) = 0$

ákvarðar h

J_0 er ekki möguleg lausn því þá fæst aðeins 0-lausn

$\rightarrow$ Engin TM$_{0p}$-háttur

b) finna H_z^0 fyrir TE

$$H_z^0 = A_n J_n(hr)\cos(n\phi)$$

$$\hookrightarrow E_r^0 = \frac{i\omega\mu n}{h^2 r} A_n J_n(hr) \sin(n\phi)$$

$$E_\phi^0 = \frac{i\omega\mu}{h} A_n J_n'(hr) \cos(n\phi)$$

$J_n'(ha) = 0$

$E_r^0 = 0$ fyrir $\phi = 0, \pi$

Hér er $n=0$ möguleg lausn

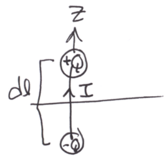
Engin þáttur $\bar{E}$ samsíða leiðara

c) Eigingildi katta

TM: $J_n(ha)=0 \;\rightarrow\; (h)_{TMnp} = X_{np}/a\,,\quad n=1,2,3$

TE: $J_n'(ha)=0 \;\rightarrow\; (h)_{TEnp} = X'_{np}/a\,,\quad n=0,1,2,\dots$

P11-2 Hertz-tvískaut



$$\bar{A} = \frac{\mu_0}{4\pi}\int_{V'} \frac{\bar{J}e^{-ikR}}{R}\,dv'\,, \qquad V = \frac{1}{4\pi\epsilon_0}\int_{V'}\frac{\rho e^{-ikR}}{R}\,dv'$$

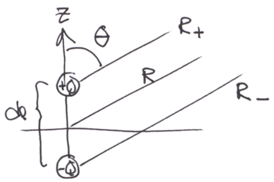
$$\bar{E} = -\bar{\nabla}V - i\omega\bar{A}$$

Úr bók $\bar{A} = \hat{a}_z\frac{\mu_0 I dl}{4\pi}\left(\frac{e^{-i\beta R}}{R}\right)$, $\hat{a}_z = \hat{a}_R\cos\theta - \hat{a}_\theta\sin\theta$

$$\rightarrow \quad A_R = \frac{\mu_0 I dl}{4\pi}\left(\frac{e^{-i\beta R}}{R}\right)\cos\theta$$

$$A_\theta = -\frac{\mu_0 I dl}{4\pi}\left(\frac{e^{-i\beta R}}{R}\right)\sin\theta$$

$$A_\phi = 0$$



$$\rightarrow V = \frac{Q}{4\pi\epsilon_0}\left\{\frac{e^{-i\beta R_+}}{R_+} - \frac{e^{-i\beta R_-}}{R_-}\right\}$$

$$R_\pm = \left(R^2 + \frac{dl^2}{4} \mp R\,dl\cos\theta\right)^{1/2} \simeq R \mp \tfrac{1}{2}dl\cdot\cos\theta$$

$(11\text{-}10) \rightarrow Q = \frac{I}{i\omega}$, gefið $(dl)^2 \ll R^2$

$$V \simeq \frac{Ie^{-i\beta R}}{4\pi\epsilon_0 i\omega}\frac{1}{R^2}\left\{\left(R+\frac{dl}{2}\cos\theta\right)e^{i\beta\frac{dl}{2}\cos\theta} - \left(R-\frac{dl}{2}\cos\theta\right)e^{-i\beta\frac{dl}{2}\cos\theta}\right\}$$

$$= \frac{Ie^{-i\beta R}}{4\pi\epsilon_0 i\omega R^2}\left\{2iR\sin\underbrace{\left(\frac{\beta dl\cos\theta}{2}\right)}_{1 \leq \frac{\beta dl}{2}} + 2\left(\frac{dl}{2}\cos\theta\right)\cos\underbrace{\left(\frac{\beta dl}{2}\cos\theta\right)}_{1 \leq \frac{\beta dl}{2}}\right\}$$

$$V \simeq \frac{Ie^{-i\beta R}}{4\pi\epsilon_0 i\omega R^2}\left\{2iR\cdot\frac{\beta dl}{2}\cos\theta + dl\cdot\cos\theta\right\}$$

$$= \frac{I dl\cos\theta}{4\pi R^2}\,\eta_0\left(R + \frac{1}{i\beta}\right)e^{-i\beta R}$$

$$\left.\begin{aligned}\beta = k_0 &= \frac{\omega}{c} = \omega\sqrt{\epsilon_0\mu_0}\\ \eta_0 &= \frac{\omega\mu_0}{k_0} = \sqrt{\frac{\mu_0}{\epsilon_0}}\end{aligned}\right\} \frac{\beta}{\epsilon_0\omega} = \sqrt{\epsilon_0\mu_0}\,\frac{1}{\epsilon_0} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \eta_0$$

Rafsviðið kemur frá

$$\bar{E} = -\bar{\nabla}V - i\omega\bar{A}$$

$$\hookrightarrow \quad E_R = -\frac{\partial V}{\partial R} - i\omega A_R\,, \qquad E_\phi = -\frac{\partial V}{R\sin\theta\,\partial\phi} - i\omega A_\phi$$

$$E_\theta = -\frac{\partial V}{R\,\partial\theta} - i\omega A_\theta$$

$$\to E_R = -\frac{Idl}{4\pi}\eta_0\beta^2 2\cos\theta\left\{\frac{1}{(i\beta R)^2}+\frac{1}{(i\beta R)^3}\right\}e^{-i\beta R}$$

$$E_\theta = -\frac{Idl}{4\pi}\eta_0\beta^2\sin\theta\left\{\frac{1}{i\beta R}+\frac{1}{(i\beta R)^2}+\frac{1}{(i\beta R)^3}\right\}e^{-i\beta R}$$

$$E_\phi = 0$$

Eins og í bók (11-16a-c)

og því ljóst að hin aðferðin er styttri

P11-4

Hertz-tvískaut með lengd L á z-ás

Segultvískaut með flöt S í x-y-sléttu

Sama I_0 og ω

fjærsvið

EP: $$E_\theta(R) = i\frac{I_0 L}{4\pi}\left(\frac{e^{-i\beta R}}{R}\right)\eta_0\beta\sin\theta$$

$$\to E_\theta(Rt) = -\frac{I_0\eta_0\beta\sin\theta}{4\pi R}L\cdot\sin(\omega t-\beta R)$$

MP: $$E_\phi(R) = \frac{\omega\mu_0 m}{4\pi}\left(\frac{e^{-i\beta R}}{R}\right)\beta\sin\theta,$$

$m = I_0 S$, $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$, $\lambda = \frac{2\pi}{\beta}$, $\beta = \frac{\omega}{c} = \omega\sqrt{\epsilon_0\mu_0}$

$$\to E_\phi(Rt) = \frac{I_0\eta_0\beta\sin\theta}{4\pi R}\left(\frac{2\pi S}{\lambda}\right)\cos(\omega t-\beta R)$$

því fæst

$$\frac{E_\theta^2(Rt)}{\left(\frac{I_0\eta_0\beta\sin\theta}{4\pi R}\right)^2 L^2} + \frac{E_\phi^2(Rt)}{\left(\frac{I_0\eta_0\beta\sin\theta}{4\pi R}\right)^2\left(\frac{2\pi S}{\lambda}\right)^2} = 1$$

$\to$ Ellipsu skautun

og hringskautun ef $L = \frac{2\pi S}{\lambda}$

P11-7

Miðfætt loftnet með lengd $2h$, $h \ll \lambda$

$$I(z) = I_0\left(1-\frac{|z|}{h}\right)$$

a) finna fjær E og H-svið

Í samanburði við (11-55) fæst

$R_1 \simeq R - z\cos\theta$

$$E_\theta = i\frac{I_0\eta_0\beta\sin\theta}{4\pi R}e^{-i\beta R}\int_{-h}^{h}\left(1-\frac{|z|}{h}\right)e^{i\beta z\cos\theta}dz$$

$$= i\frac{I_0\eta_0\beta\sin\theta}{2\pi R}e^{-i\beta R}\int_0^h\left(1-\frac{z}{h}\right)\cos(\beta z\cos\theta)dz$$

$$\simeq \frac{i\,60 I_0}{(\beta h)R}e^{-i\beta R}F(\theta), \quad F(\theta) = \frac{\sin\theta\left[1-\cos(\beta h\cos\theta)\right]}{\cos^2\theta}$$

$$H_\phi = \frac{E_\theta}{\eta_0} = \frac{i I_0}{(\beta h)\, 2\pi R} e^{-i\beta R} F(\theta)$$

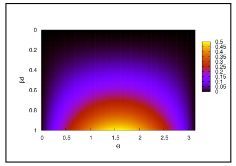
þegar $h << \lambda \;\to\; \beta h << 1$

$$\cos(\beta h \cos\theta) \approx 1 - \frac{1}{2!}(\beta h \cos\theta)^2 + \cdots$$

$$\to F(\theta) \simeq \frac{1}{2}(\beta h)^2 \sin\theta$$

$$E_\theta = \frac{i\, 30\, \beta h}{R} I_0 e^{-i\beta R} \sin\theta$$

$$H_\phi = \frac{i \beta h}{4\pi R} I_0 e^{-i\beta R} \sin\theta$$



b) finna R_r

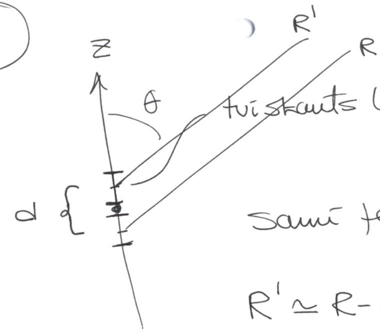
$$P_r = \frac{1}{2}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta\, R^2 E_\theta H_\phi^* = \frac{I_0^2}{2}\left[80\pi^2 \left(\frac{h}{\lambda}\right)^2\right]$$

$$R_r = \frac{P_r}{\frac{1}{2} I_0^2} = 20\pi^2 \left(\frac{2h}{\lambda}\right)^2$$

c)

$$D = \frac{4\pi |E_{max}|^2}{\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta\, |E_\theta(\theta)|^2} = \frac{2}{\int_0^{\pi} \sin^3\theta} = 1{,}5$$

P11-15



tvískauts loftnet $\quad 2h << \lambda$

$d > 2h$

sami fasi og styrkur

$R' \simeq R - d\cos\theta$

a)

$$E_\theta = \frac{i I (2h)}{4\pi} \eta_0 \beta \sin\theta\, e^{i\beta R} \left\{ \underbrace{\frac{1}{R} + \frac{e^{i\beta d \cos\theta}}{R'}} \right\}$$

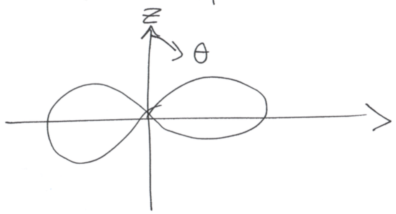
$\frac{1}{R} \sim \frac{1}{R'}$ í fjarsviði

$$= \frac{i\, 60\, I h}{R} 2\beta\, e^{-i\beta\left(R - \frac{d}{2}\cos\theta\right)} F(\theta)$$

með

$$F(\theta) = \sin\theta \cos\left(\frac{\beta d}{2} \cos\theta\right)$$

b) $|F(\theta)| = |\sin\theta \cos(\frac{\pi}{2}\cos\theta)|$



c) $|F(\theta)| = |\sin\theta \cos(\pi\cos\theta)|$