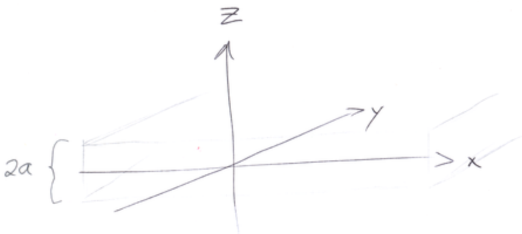
① Tómt 3D-rúm, einsleit plata með þykkt 2a

$$\rho(z) = \left(\frac{z}{a}\right)\rho_0 \qquad |z| \leq a$$



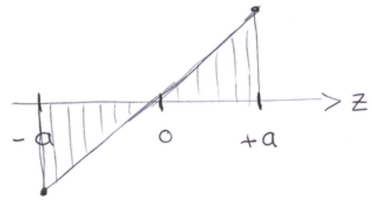
a) Finna rafsviðið $\bar{E}$ í öllu rúminu. Nota heildisform og afleiðuform Gauß lögmáls

Hleðsludreifingin er einsleit í x- og y-stefnum

→ $\bar{E}$ hefur engan þátt í x- og y-stefnum

$$\bar{E} = \hat{a}_z E_z$$

Hleðsludreifing



Gauß-lögmál segir að E_z er fasti á hvoru bili

$$z < -a$$
$$z > a$$

~~þetta Stadd~~ breytir í 2. dæmi.

E_z er breytilegt á bilinu

$$-a \leq z \leq a$$

Heildisform

$$\oint \bar{E} \cdot d\bar{A} = \frac{Q}{\epsilon_0}$$

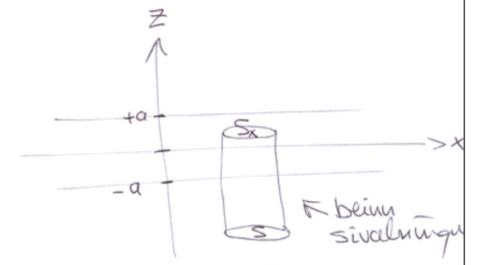
Hleðsludreifingin er andsamhverf um $z=0$-sléttuna. Gerum ráð fyrir að engar hleðslur séu utan plötu (eða hellu). Platan er í heild óhlaðin

Gauß lögmál segir okkur

þá að utan plötu sé

$$\bar{E} = 0$$

Veljum þess vegna Gauß-flöt eins og mynd sýnir



þá er ekkert flæði um bogna flötinn og heldur ekki um botnflötinn S.

Eina flæðið er um S_x, þar er það einsleitt

$$\oint \bar{E} \cdot d\bar{A} = \frac{Q}{\epsilon_0}$$

umskrifast þá sem

$$\bar{A} \cdot \bar{E}_z = \frac{\rho_0}{\epsilon_0} A \int_{-a}^{z} dz' \left(\frac{z'}{a}\right)$$

$$= A \frac{\rho_0}{\epsilon_0 a}\left(\frac{z^2}{2} - \frac{a^2}{2}\right)$$

$$\rightarrow E_z = \frac{\rho_0 a}{2\epsilon_0}\left(\left(\frac{z}{a}\right)^2 - 1\right)$$

sem uppfyllir að

$$E_z(-a) = 0$$
$$E_z(a) = 0$$

Afleiðuform

$$\nabla \cdot \bar{E} = \frac{\rho}{\epsilon_0}$$

samhverfan gefur $E_z \neq 0$ og því

$$\frac{d}{dz} E_z = \frac{1}{\epsilon_0}\left(\frac{z}{a}\right)\rho_0$$

3 svæði

- I $z < -a$
- II $-a \leq z \leq a$
- III $z > a$

Óákveðin heildun á svæði II

$$E_z = \frac{\rho_0}{\epsilon_0 2a} z^2 + C_2$$

Á svæði I fast $E_z = C_1$ og á III $E_z = C_3$

Lausnin á II er samhverf um $z=0$. Sú samhverfa verður að ná út fyrir plötuna → $C_1 = C_3$.

Engar hleðslur fyrir utan plötu → $C_1 = C_3 = 0$.

Rafsviðið er samfelt á jöðrunum $E_z(-a) = E_z(a) = 0$

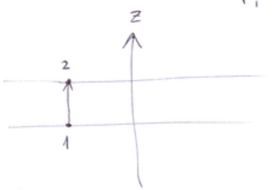
$$\rightarrow E_z(a) = \frac{\rho_0 a}{2\epsilon_0} + C_2 \rightarrow C_2 = -\frac{\rho_0 a}{2\epsilon_0}$$

$$\rightarrow E_z = \frac{\rho_0 a}{2\epsilon_0}\left(\left(\frac{z}{a}\right)^2 - 1\right) \qquad \text{f. } |z| \leq a$$

$$E_z = 0 \qquad \text{f. } |z| > a$$

b) Spennumunur flatanna? (5)

$$V_2 - V_1 = -\int_{P_1}^{P_2} \bar{E}\cdot d\bar{l}$$



$$V(a)-V(-a) = -\int_{-a}^{a} dz\, \frac{\rho_0 a}{2\epsilon_0}\left(\left(\frac{z}{a}\right)^2-1\right)$$

$$= -\frac{\rho_0 a}{2\epsilon_0}\left\{\frac{1}{a^2}\frac{z^3}{3} - z\Big|_{-a}^{a}\right\} = -\frac{\rho_0 a}{2\epsilon_0}\left\{\frac{2}{a^2}\frac{a^3}{3}-2a\right\}$$

$$= -\frac{\rho_0 a^2}{\epsilon_0}\left(-\frac{2}{3}\right) = \frac{2\rho_0 a^2}{3\epsilon_0}$$

(2) Á vissu svæði er rafsviðið einsleitt (6)

-> á svæðinu er $\bar{E}$ fastur vigur

-> $\bar{\nabla}\cdot\bar{E} = 0$

Gauß: $\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon_0}$

-> hleðslan á svæðinu er núll

Rafsviðið er fasti á svæðinu því engar uppsprettur sviðs, hleðslur, eru á svæðinu

þessi röksemdafærslu er ekki hægt að snúa við

(1) Þéttir úr tveimur sammiðja kjörleiðandi kúluflötum (1)

Rafsvárinn breytir ekki „radial samhverfu" uppsetningarinnar. Þetta má skoða betur með $\bar{\nabla}\times\bar{E}=0$ við jaðra eða $\oint_C \bar{E}\cdot d\bar{l} = 0$

-> $\bar{E}$ hefur aðeins radial þátt (útþátt)

Hér er erfitt að nýta sér Gauß-lögmálið

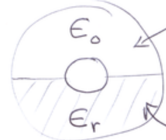
$$\oint_S \bar{E}\cdot d\bar{s} = \frac{Q}{\epsilon_0}$$

Það gildir aðeins fyrir efni, einsleitt með ϵ_0

En almennt gildir

$$\boxed{\oint_S \bar{D}\cdot d\bar{s} = Q}$$

(2)



$\bar{D} = \epsilon_0 \bar{E}$

$\bar{D}' = \epsilon_r \epsilon_0 \bar{E}$

Gerum ráð fyrir að þéttirinn hafi hleðslu Q á flötunum

$$\oint_S \bar{D}\cdot d\bar{s} = Q \quad \to \quad D\cdot 2\pi r^2 + D'\, 2\pi r^2 = Q$$

yfirborð með $a<r<b$

$$\to \epsilon_0 2\pi r^2 E(r) + \epsilon_r\epsilon_0 2\pi r^2 E(r) = Q$$

$$(1+\epsilon_r)\epsilon_0 2\pi r^2 E(r) = Q$$

eða

$$E(r) = \frac{Q}{2\pi(1+\epsilon_r)\epsilon_0 r^2}$$

(3)

Við þurfum spennumuninn milli flatanna

$$V_b - V_a = -\int_a^b \bar{E}\cdot d\bar{l} = -\int_a^b dr\, E(r)$$

$$= -\frac{Q}{2\pi(1+\epsilon_r)\epsilon_0}\int_a^b \frac{dr}{r^2}$$

$$= \frac{Q}{2\pi(1+\epsilon_r)\epsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right) \quad \left\{\text{neikvætt því } b > a\right\}$$

$$\boxed{C = \frac{Q}{|V_b-V_a|} = 2\pi(1+\epsilon_r)\epsilon_0\frac{1}{\left(\frac{1}{a}-\frac{1}{b}\right)} = \frac{2\pi(1+\epsilon_r)\epsilon_0 ab}{b-a}}$$

(4)

Niðurstaðan er því rýmd sem er eins og fyrir tvo samsíðatengda þétta með rýmdir

$$C' = \frac{2\pi\epsilon_r\epsilon_0 ab}{(b-a)} \qquad C'' = \frac{2\pi\epsilon_0 ab}{(b-a)}$$

Hvor um sig væri eins og hálfkúlu þéttir

Þetta mátti ~~auðvitað~~ giska á í upphafi, en ekki rétt að gera ráð fyrir því. Við getum leitt þetta út frá grunnjöfnunum

(5)

(2) ϵ_0

ϵ_r

a) Finna rýmd

Alveg eins og í fyrra dæminu finnum við að $\bar{E}$ mun aðeins geta haft radial þátt

Og aftur verðum við að nýta

$$\boxed{\oint_S \bar{D}\cdot d\bar{S} = Q}$$

Notum kúlusamhverft Gaußyfirborð með $r > a$ og gerum ráð fyrir að kúlan hafi hleðslu Q

$$\rightarrow D\cdot 2\pi r^2 + D' 2\pi r^2 = Q$$

$$(1+\epsilon_r)\epsilon_0 2\pi r^2 E(r) = Q$$

$$\rightarrow E(r) = \frac{Q}{2\pi(1+\epsilon_r)\epsilon_0 r^2}$$

(6)

Við verðum að muna rýmdina við annan flöt í $r \to \infty$

Spennumunurinn er því

$$V_a - V_\infty = -\int_\infty^a \bar{E}\cdot d\bar{l} = -\frac{Q}{2\pi(1+\epsilon_r)\epsilon_0}\int_\infty^a \frac{dr}{r^2}$$

$$= \frac{Q}{2\pi(1+\epsilon_r)\epsilon_0}\frac{1}{a}$$

og rýmdin

$$C = \frac{Q}{|V_a - V_\infty|} = 2\pi(1+\epsilon_r)\epsilon_0 a$$

Rýmdin vex með a og ϵ_r eins og búast má við

(7)

b) Hve stór hluti Q er á hvoru hveli?

$$\hat{a}_{n_2}\cdot(\bar{D}_1-\bar{D}_2)=\rho_s$$

Inni í kúlunni er ekkert rafsvið, setjum $\bar{D}_1=0$

$$\to -\hat{a}_r(-\bar{D}_2)=\rho_s \quad \to \quad \hat{a}_r\cdot\bar{D}_2=\rho_s$$

$D_2(a)=\epsilon_0 E(a)$ á norðurhveli

$$=\frac{Q}{2\pi(1+\epsilon_r)a^2} = \rho_s^N \quad \to \quad Q_N = 2\pi a^2 \rho_s^N = \boxed{\frac{Q}{(1+\epsilon_r)}}$$

Á suðurhveli $\quad D_2(a)=\epsilon_0\epsilon_r E(a)=\rho_s^S$

$$\to \boxed{Q_S = 2\pi a^2\rho_s^S = \frac{Q}{(1+\epsilon_r)}\epsilon_r}$$

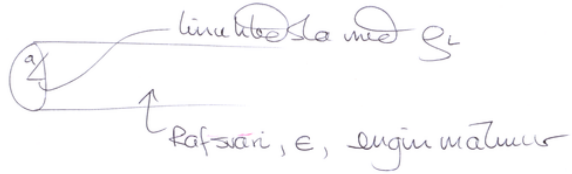
(8)

Og eins og vera ber

$$Q_S+Q_N=\frac{Q\epsilon_r}{1+\epsilon_r}+\frac{Q}{1+\epsilon_r}=Q$$

c) skiptir engu máli, hún er úr kjörleiðara. Þessvegna myndu hleðslur innan í mögulegu holi hennar ekki hafa áhrif á hleðslur á yfirborði hennar.

(1) (1)



a) Finnum rafstöðumættið innan og utan sívalnings. Hér má nota margar aðferðir, t.d. setningu Gauß, ... Vegna sívalningssamhverfunnar er jafna Poissons hér innan sívalnings, $r<a$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right)=-\frac{\rho_L(r)}{\epsilon}$$

vandinn er hér að $\rho_L(r)$ er í raun δ-fall í sívalningshnitum

$$\rho_L(r)=\frac{\rho_L}{2\pi r}\delta(r)$$

(2)

sem við höfum ekki fengist við í jöfnu Poissons, og hér er óþægilegt að hafa δ-fallið á „jöðrunum" $r\sim 0$. Því er heppilegt að beita setningu Gauß með þekkingu á lausn jöfnu Laplace.

Í raun er verkefnið hér allstaðar nema þ. $r\sim 0$ lýst með jöfnu Laplace

$$\frac{d}{dr}\left(r\frac{dV}{dr}\right)=0$$

vegna línuhleðslunnar þurfum við lausnina með sérstöðupunkt í $r=0$

$$V_i(r)=A_i\ln(r)+B_i \qquad \underline{r<a}$$

Gauß gefur okkur $\quad A_i=-\frac{\rho_L}{2\pi\epsilon}$

Lausnirnar eru því

$$V_i(r) = -\frac{\rho_L}{2\pi\epsilon}\ln(r) + B_i \qquad \underline{r<a}$$

$$V_o(r) = -\frac{\rho_L}{2\pi\epsilon_0}\ln(r) + B_o \qquad \underline{r>a}$$

Fyrir sívalning er ekki hægt að velja að $V(r) \to 0$ þegar $r \to \infty$. Við veljum $V(a) = 0$ og notum samfellu $V(r)$ í $r = a$ til að fá:

$$V_i(r) = -\frac{\rho_L}{2\pi\epsilon}\ln\left(\frac{r}{a}\right) \qquad \underline{r<a}$$

$$V_o(r) = -\frac{\rho_L}{2\pi\epsilon_0}\ln\left(\frac{r}{a}\right) \qquad \underline{r>a}$$

því er greinilegt að $E(r)$ er ekki samfelt í $\underline{r=a}$, (berið saman við Ex 3-12 í bók), en $D(r)$ er það.

b) $\bar{E} = -\bar{\nabla} V(r)$ og því

$$\bar{E}_i(r) = \hat{a}_r \frac{\rho_L}{2\pi\epsilon r} \qquad \underline{r<a}$$

$$\bar{E}_o(r) = \hat{a}_r \frac{\rho_L}{2\pi\epsilon_0 r} \qquad \underline{r>0}$$

c) Rafstöðuorkan?

Reiknum fyrst

$$W_e = \frac{1}{2}\int_{V'} dv\, \bar{D}\cdot\bar{E}$$

$$\frac{W_e}{L} = \frac{1}{2}\cdot 2\pi\left[\int_0^a r\,dr\,\frac{\rho_L^2}{4\pi^2\epsilon r^2} + \int_a^\infty r\,dr\,\frac{\rho_L^2}{4\pi^2\epsilon_0 r^2}\right]$$

$$= \frac{\rho_L^2}{4\pi}\left\{\frac{1}{\epsilon}\int_0^a \frac{dr}{r} + \frac{1}{\epsilon_0}\int_a^\infty \frac{dr}{r}\right\}$$

$$= \frac{\rho_L^2}{4\pi}\left\{\frac{1}{\epsilon}\left(\ln(r)\Big|_0^a\right) + \frac{1}{\epsilon_0}\left(\ln(r)\Big|_a^\infty\right)\right\}$$

$$= \frac{\rho_L^2}{4\pi}\left\{\frac{1}{\epsilon}\left(\ln(a) - \lim_{\lambda_1\to 0}\ln(\lambda_1)\right) + \frac{1}{\epsilon_0}\left(-\ln(a) + \lim_{\lambda_2\to\infty}\ln(\lambda_2)\right)\right\}$$

$$= \frac{\rho_L^2}{4\pi}\left\{\ln(a)\left(\frac{1}{\epsilon} - \frac{1}{\epsilon_0}\right) + \lim_{\lambda_2\to\infty}\frac{\ln(\lambda_2)}{\epsilon_0} - \lim_{\lambda_1\to 0}\frac{\ln(\lambda_1)}{\epsilon}\right\}$$

hmm!, lítur ekki gæfulega út, niðurstaðan er ósamleitin.

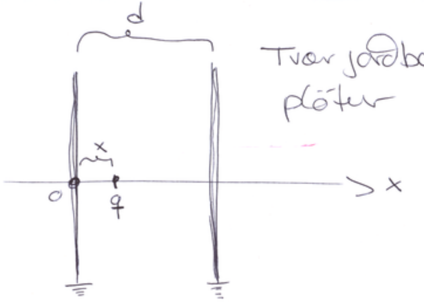
Við lendum ekki í betri málum ef við reiknum

$$W_e = \frac{1}{2}\int_V dv\, \rho V$$

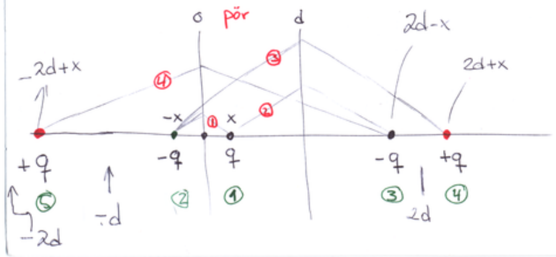
Við neyðumst til að kyngja þeirri staðreynd að rafstöðuorka óendanlegrar línuhleðslu er ósamleitin. Ef við gerum hana endanlega leysist allt.

2

Tvær jarðbundnar kjörleiðandi samsíða plötur



a) Finna kraftinn á hleðsluna. Þetta er mjög sérstakt dæmi. Við getum beitt spegilhleðslum t.þ.a. finna kraftinn.



Það þarf því óendanlega margar spegilhleðslur (Athugið sjálfan þig milli tveggja spegla)

Kraftur vegna (+)-hleðslna

$$F = \frac{q^2}{4\pi\epsilon_0}\left\{\overset{5}{\frac{1}{(2d)}} - \overset{4}{\frac{1}{(2d)}} + \cdots\right\} = 0$$

styttist út í pörum

Kraftur vegna (−)-hleðslna

$$F = \frac{q^2}{4\pi\epsilon_0}\left\{\overset{3}{\frac{1}{(x-(2d-x))^2}} + \frac{1}{(2d+2(d-x))^2} + \frac{1}{(4d+2(d-x))^2}\cdots - \overset{2}{\frac{1}{(2x)^2}} - \frac{1}{(2d+2x)^2} - \frac{1}{(4d+2x)^2}\cdots\right\}$$

− er hér kraftur til vinstri

+ er hér kraftur til hægri

tekið saman

$$F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{4}\left\{\left[\frac{1}{(d-x)^2}+\frac{1}{(2d-x)^2}+\frac{1}{(3d-x)^2}+\cdots\right] - \left[\frac{1}{x^2}+\frac{1}{(d+x)^2}+\frac{1}{(2d+x)^2}+\cdots\right]\right\}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2}{4}\left\{\sum_{n=1}^{\infty}\frac{1}{(nd-x)^2} - \sum_{n=0}^{\infty}\frac{1}{(nd+x)^2}\right\}$$

b) Markgildið þegar $d\to\infty$

$$\lim_{d\to\infty} F = -\frac{q^2}{4\pi\epsilon_0}\frac{1}{4x^2}$$

sem er líka svarið fyrir eina plötu og eina hleðslu

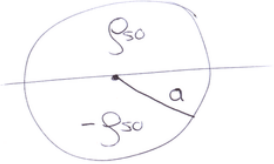
Markgildið þegar $x = \frac{d}{2}$

$$F\left(\frac{d}{2}\right) = \frac{1}{4\pi\epsilon_0}\frac{q^2}{d4}\left\{\underbrace{\sum_{n=1}^{\infty}\frac{1}{(n-1/2)^2} - \sum_{n=0}^{\infty}\frac{1}{(n+1/2)^2}}_{=0}\right\}$$

því $\sum_{n=0}^{\infty}\frac{1}{(n+1/2)^2} = \sum_{n=1}^{\infty}\frac{1}{(n-1/2)^2}$

En, ég spurði ekki um V og yfirborðhleðsluna á plötunum því spegil-hleðslu aðferðin leiðir ekki til samleitinnar raðar fyrir þau. Þar þarf að nota við „reciprocity"-setningu Green's

① Kúluskel með mismunandi hleðslu þéttleika á norður og suður hveli ①



Finnum $V(R,\theta)$ innan og utan skeljar

Höfum séð í fyrirlestri að lausn Laplace jöfnunar er

$$V_n(R,\theta) = \left\{ A_n R^n + B_n R^{-(n+1)} \right\} P_n(\cos\theta)$$

Skoðum tvö svæði

$R<a$: Engin hleðsla innan kúlu $\to B_n = 0$ fyrir öll $n \geq 0$

$$\to V_n^{I}(R,\theta) = A_n R^n P_n(\cos\theta)$$

$R>a$:

$$V_n^{II}(R,\theta) = B_n R^{-(n+1)} P_n(\cos\theta)$$

Engar lausnir sem vaxa þ. $R \to \infty$

②

Við verðum að bæta við að lausnirnar séu settar saman úr þessum

$$V^{I}(R,\theta) = \sum_{n=0}^{\infty} A_n R^n P_n(\cos\theta) \qquad \underline{R<a}$$

$$V^{II}(R,\theta) = \sum_{n=0}^{\infty} B_n R^{-(n+1)} P_n(\cos\theta) \qquad \underline{R>a}$$

Mættið verður að vera samfelt í kúluskelinni $R=a$

$$\to \sum_{n=0}^{\infty} A_n a^n P_n(\cos\theta) = \sum_{n=0}^{\infty} B_n a^{-(n+1)} P_n(\cos\theta)$$

P_n-in eru hornrétt:

(*)

$$\int_0^{\pi} \sin\theta d\theta \, P_n(\cos\theta) P_{n'}(\cos\theta) = \begin{cases} 0 & \text{ef } n' \neq n \\ \frac{2}{2n+1} & \text{ef } n' = n \end{cases}$$

③

því verður að gilda fyrir hvern lið

$$B_n = A_n a^{2n+1}$$

Við erum aðeins búin að tengja A_n og B_n en ekki ákvarða þau.

Vegna yfirborðshleðslunnar verður að gilda

$$\boxed{\hat{a}_{n2} \cdot (\bar{D}_1 - \bar{D}_2) = \rho_s}$$

notum $\bar{E} = -\bar{\nabla} V$, $\bar{D} = \epsilon \bar{E}$

$$\to \left\{ \frac{\partial}{\partial r} V^{II}(R,\theta) - \frac{\partial}{\partial r} V^{I}(R,\theta) \right\}\Bigg|_{R=a} = -\frac{\rho_s(\theta)}{\epsilon_0}$$

$$-\sum_{n=0}^{\infty} (n+1) \frac{B_n}{a^{n+2}} P_n(\cos\theta) - \sum_{n=0}^{\infty} n A_n a^{n-1} P_n(\cos\theta) = -\frac{\rho_s(\theta)}{\epsilon_0}$$

④

notum að $B_n = A_n a^{2n+1}$

$$\to \boxed{\sum_{n=0}^{\infty} (2n+1) A_n a^{n-1} P_n(\cos\theta) = \frac{\rho_s(\theta)}{\epsilon_0}}$$

$$\rho_s(\theta) = \begin{cases} \rho_{s0} & \text{þ. } 0 < \theta < \frac{\pi}{2} \\ -\rho_{s0} & \text{þ. } \frac{\pi}{2} < \theta < \pi \end{cases}$$

notum (*)

$$\frac{2}{(2m+1)} (2m+1) A_m a^{m-1} = \int_0^{\pi} \sin\theta d\theta \, P_m(\cos\theta) \frac{\rho_s(\theta)}{\epsilon_0}$$

$$\to A_m = \frac{1}{2\epsilon_0 a^{m-1}} \int_0^{\pi} \sin\theta d\theta \, P_m(\cos\theta) \rho_s(\theta)$$

5

$$A_m = \frac{1}{2\epsilon_0 a^{m-1}}\left[-\int_{-1}^{0} du\, P_m(u) + \int_{0}^{1} du\, P_m(u)\right]\rho_s$$

$P_m(-u) = (-1)^m P(u) \rightarrow A_m = 0$ ef m = jöfn tala

$\rightarrow A_m = 0$

því er enginn liður utan kúlu sem fellur eins og $\frac{1}{R}$. Kúlan er í heild óhlaðin svo slíkur liður getur ekki verið til

Fyrir m = oddatölu

$$A_m = \frac{\rho_s}{\epsilon_0 a^{m-1}}\int_0^1 du\, P_m(u) = \left(-\frac{1}{2}\right)^{\frac{m-1}{2}} \frac{(m-2)!!}{2\left(\frac{m+1}{2}\right)!}\frac{\rho_s}{\epsilon_0 a^{m-1}}$$

$$(m-2)!! = (m-2)\cdot(m-4)\cdot\ldots\cdot 5\cdot 3\cdot 1$$

6

$$V^{I}(R,\theta) = \frac{\rho_s a}{\epsilon_0}\sum_{l=0}^{\infty}\left(-\frac{1}{2}\right)^{l}\frac{(2l-1)!!}{(l+1)!}\left(\frac{R}{a}\right)^{2l+1}P_{2l+1}(\cos\theta) \qquad R<a$$

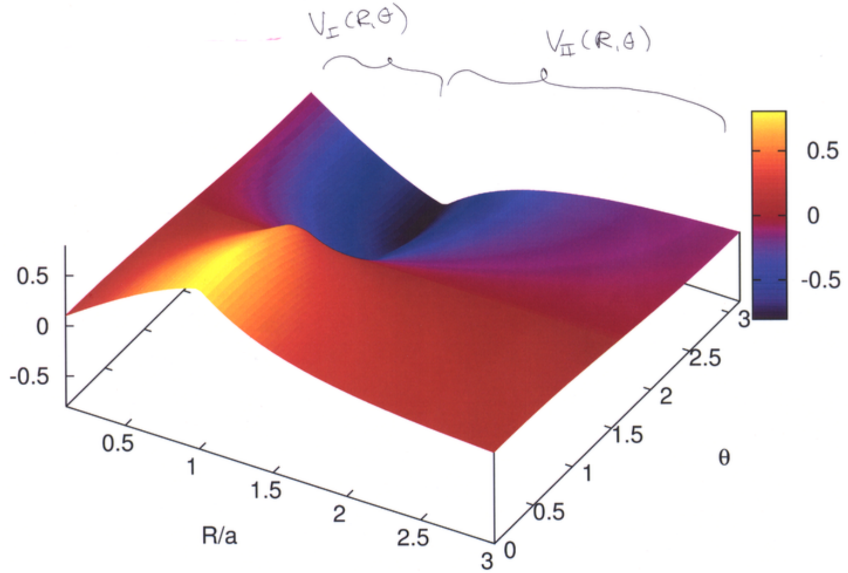
$$V^{II}(R,\theta) = \frac{\rho_s a}{\epsilon_0}\sum_{l=0}^{\infty}\left(-\frac{1}{2}\right)^{l}\frac{(2l-1)!!}{(l+1)!}\left(\frac{a}{R}\right)^{2l+2}P_{2l+1}(\cos\theta) \qquad R>a$$

Ekkert 0-skaut í ytri lausn, en tvískaut og hærri skaut

Rétt vídd því þetta er $\sim \frac{e}{L\cdot\epsilon_0}$

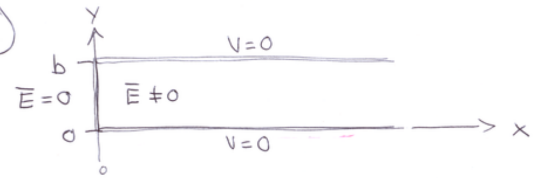
Skoðunin sýnir strax að V er samfellt í $R = a$

6b



7

(2)



lausn var fundin í fyrri bæti

$$V(x,y) = \frac{4V_0}{\pi}\sum_{l=0}^{\infty}\frac{1}{(2l+1)}e^{-k_l x}\sin(k_l y)$$

með $k_l = \frac{(2l+1)\pi}{b}$

notum eftir

$$\hat{a}_{n_2}\cdot(\bar{D}_1 - \bar{D}_2) = \rho_s$$

(8)

$$\hat{a}_x \cdot [\bar{D}] = \rho_s \quad \rightarrow \quad \epsilon \bar{E}\cdot \hat{a}_x = \rho_s$$

eða $\quad \epsilon E_x = \rho_s \quad , \quad \bar{E} = -\bar{\nabla} V$

$$E_x(0,y) = \frac{4V_0}{\pi}\sum_{l=0}^{\infty} \frac{k_l}{(2l+1)} \sin(k_l y)$$

$$= \frac{4V_0}{\pi}\frac{\pi}{b}\sum_{l=0}^{\infty} \sin\left(\frac{(2l+1)\pi y}{b}\right)$$

$$= \frac{4V_0}{b}\sum_{l=0}^{\infty} \sin\left\{\frac{(2l+1)\pi y}{b}\right\}$$

(9)

$$\rightarrow \rho_s = \frac{4\epsilon_0 V}{b}\sum_{l=0}^{\infty} \sin\left\{\frac{(2l+1)\pi y}{b}\right\}$$

$$= \frac{4\epsilon_0 V}{b}\sum_{l=0} \frac{1}{2i}\left\{e^{i(2l+1)\frac{\pi y}{b}} - e^{-i(2l+1)\frac{\pi y}{b}}\right\}$$

$$= \frac{4\epsilon_0 V}{b}\sum_{l=0} \frac{1}{2i}\left\{e^{\frac{i\pi y}{b}}\left(e^{\frac{i2\pi y}{b}}\right)^l - e^{-\frac{i\pi y}{b}}\left(e^{-\frac{i2\pi y}{b}}\right)^l\right\}$$

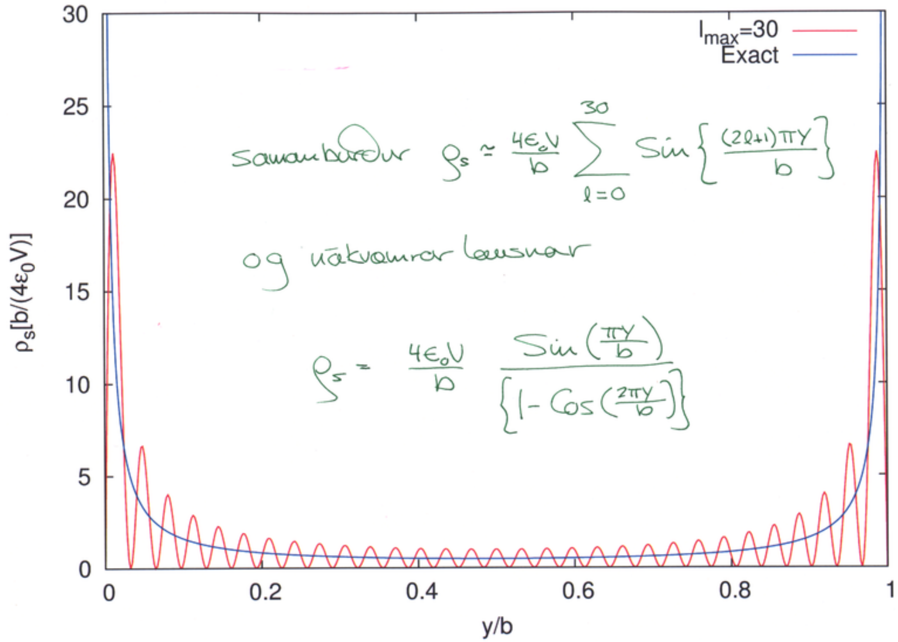
$$= \frac{4\epsilon_0 V}{b}\frac{1}{2i}\left[\frac{e^{i\pi\frac{y}{b}}}{1-e^{i2\pi\frac{y}{b}}} - \frac{e^{-i\pi\frac{y}{b}}}{1-e^{-i2\pi\frac{y}{b}}}\right]$$

(10)

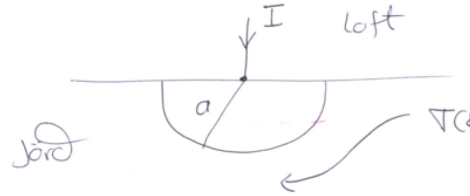
$$\rho_s = +\frac{4\epsilon_0 V}{b}\frac{\sin\left(\frac{\pi y}{b}\right)}{\left\{1-\cos\left(\frac{2\pi y}{b}\right)\right\}}$$

sjá mynd á næstu síðu

(11)



(1)



$\sigma(R) = \sigma_0 \sqrt{\frac{a}{R}}$

$\sigma(R)$ hefur radial samhverfu.

Með því að skoða spegilmynd sést að straumurinn mun einungis hafa $\hat{a}_R$-þátt

straumur um heilkúluna er þá $2I$

$$\rightarrow \bar{J} = \frac{2I}{4\pi R^2}\hat{a}_R \rightarrow \bar{E} = \frac{\bar{J}(R)}{\sigma(R)} \quad \left\{ \text{því } \bar{J} = \sigma \bar{E} \right\}$$

$$\rightarrow \bar{E} = \frac{I}{\sigma_0 2\pi R^2 \sqrt{\frac{a}{R}}}$$

(1)

(2)

$$V_0 - \underset{0}{\underset{\parallel}{V_\infty}} = -\int_{+\infty}^{a} E(R)\,dR = -\frac{I}{2\pi\sigma_0\sqrt{a}}\int_{+\infty}^{a}\frac{dR}{R^{3/2}}$$

$$= -\frac{I}{2\pi\sigma_0\sqrt{a}}\left\{-2\frac{1}{R^{1/2}}\Big|_{+\infty}^{a}\right\} = \frac{I}{\pi\sigma_0 a} = V_0$$

$$\rightarrow R = \frac{V_0}{I} = \frac{1}{\pi\sigma_0 a}$$

b) straumur I

Afl $P = I^2 R = \frac{I^2}{\pi\sigma_0 a}$ eyðist í jörð

c) Afl innan $a < R < 2a$ lags?

(3)

$$V_0 - V_{2a} = -\int_{2a}^{a} E(R)\,dR = -\frac{I}{2\pi\sigma_0\sqrt{a}}\left\{-2\frac{1}{R^{1/2}}\Big|_{2a}^{a}\right\}$$

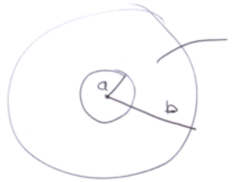
$$= +\frac{I}{\pi\sigma_0\sqrt{a}}\left\{\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{2a}}\right\} = \frac{I}{\pi\sigma_0 a}\left\{1 - \frac{1}{\sqrt{2}}\right\}$$

$$R_{[a-2a]} = \frac{1}{\pi\sigma_0 a}\left\{1 - \frac{1}{\sqrt{2}}\right\} = \left\{1 - \frac{1}{\sqrt{2}}\right\}R \approx 0.293\,R$$

$\rightarrow$ u.þ.b. 29.3 % af aflinu eyðist í fyrsta a-þykka laginu

(4)

(2) Kúluþéttir



$\epsilon(R) = \epsilon_0 \frac{R}{a}$

$\sigma(R) = \sigma_0 \frac{b^2}{R^2}$ $\Big\}$ brýtur ekki radial samhverfu

a) Finna leiðni þéttisins

$$\bar{J} = \frac{I}{4\pi R^2}\hat{a}_R \rightarrow \bar{E} = \frac{\bar{J}}{\sigma} = \frac{I R^2 \hat{a}_R}{4\pi R^2 \sigma_0 b^2} = \frac{I}{4\pi\sigma_0 b^2}\hat{a}_R$$

$$V_0 = -\int_b^a \bar{E}\cdot d\bar{l} = -\int_b^a \frac{I}{4\pi\sigma_0 b^2}\,dR = -\frac{I}{4\pi\sigma_0 b^2}(a-b)$$

$$= \frac{I}{4\pi\sigma_0 b^2}(b-a)$$

$$\left\{ = E(b-a) \right\}$$

$$G = \frac{I}{V_0} = \frac{4\pi\sigma_0 b^2}{b-a}$$

b) Frjálsar hleðslur í þéttinum

Bolhleðslur, $\bar{D} = \epsilon(R)\,\bar{E} = \frac{I}{4\pi\sigma_0 b^2}\,\frac{\epsilon_0}{a}R$

$$\rho = \bar{\nabla}\cdot\bar{D} = \frac{1}{R^2}\frac{d}{dR}\left\{R^2 D(R)\right\} = \frac{I}{4\pi\sigma_0 b^2}\,\frac{\epsilon_0}{a}\,3$$

Yfirborðshleðslur

í raun a^+ ; b^-

$R=a$ $\quad \rho_{sa}(a) = \epsilon(a)\bar{E}(a)\,\hat{a}_r = \epsilon_0 \frac{I}{4\pi\sigma_0 b^2}$

$R=b$ $\quad \rho_{sb}(b) = -\epsilon(b)\bar{E}(b)\,\hat{a}_r = -\epsilon_0\frac{b}{a}\,\frac{I}{4\pi\sigma_0 b^2}$

c) Skautunar hleðslur í efnisrúminu

Munum $\boxed{\bar{D} = \epsilon_0\bar{E} + \bar{P}}$ $\rightarrow$ $\boxed{\bar{P} = \bar{D} - \epsilon_0\bar{E}}$ (3-97)

$\rightarrow$ $\bar{P} = \left\{\epsilon(R) - \epsilon_0\right\}\bar{E}$

og samkvæmt bók er þá (3-88, 3-89)

$\rho_{ps} = \bar{P}\cdot\hat{a}_n$ og $\rho_p = -\bar{\nabla}\cdot\bar{P}$

$$\rho_p = -\bar{\nabla}\cdot\bar{P} = -\frac{1}{R^2}\frac{d}{dR}\left[R^2 P(R)\right] = -\frac{\epsilon_0}{R^2}\frac{d}{dR}\left\{R^2\left(\frac{R}{a}-1\right)\right\}E$$

$$= -\frac{\epsilon_0 E}{R^2}\left\{\frac{3R^2}{a} - 2R\right\} = -\epsilon_0 E\left\{\frac{3}{a} - \frac{2}{R}\right\}$$

$$= -\epsilon_0\frac{I}{4\pi\sigma_0 b^2}\left\{\frac{3}{a} - \frac{2}{R}\right\}$$

$$\rho_{ps}(a^+) = -P(a^+) = -\left\{\epsilon(a) - \epsilon_0\right\}E$$

$$= -\left\{\epsilon_0 - \epsilon_0\right\}\frac{I}{4\pi\sigma_0 b^2} = 0$$

það er ekki stökk í $\epsilon(R)$ í yfirborðinu $R=a$

$$\rho_{ps}(b^-) = P(b^-) = -\left\{\epsilon(b) - \epsilon_0\right\}E$$

$$= -\left\{\epsilon_0\frac{b}{a} - \epsilon_0\right\}\frac{I}{4\pi\sigma_0 b^2} = -\left\{\frac{b}{a} - 1\right\}\frac{\epsilon_0 I}{4\pi\sigma_0 b^2}$$

$\epsilon(R)$ er með stökk í $R=b$

d) Heildar bolhleðsla (Frjálsar hleðslur) — (ekki skautunar hleðslur)

ρ er fasti $\quad \rho = \frac{I}{4\pi\sigma_0 b^2}\,\frac{3\epsilon_0}{a}$

$$\rightarrow Q = \underbrace{\frac{4\pi}{3}\left\{b^3 - a^3\right\}}_{\text{rúmmál milli kúluflata}}\,\frac{I}{4\pi\sigma_0 b^2}\,\frac{3\epsilon_0}{a}$$

Á innra borði $\quad Q_{sa}(a) = 4\pi a^2\cdot\rho_s(a) = \frac{\epsilon_0 I a^2}{\sigma_0 b^2}$

Á ytra borði $\quad Q_{sb}(b) = 4\pi b^2\,\rho_s(b) = -\frac{\epsilon_0 I b}{\sigma_0 a}$

Í heild

$$Q + Q_{sa}(a) + Q_{sb}(b) = \{b^3 - a^3\}\frac{I}{\sigma_0 b^2}\frac{\epsilon_0}{a} + \frac{\epsilon_0 I}{\sigma_0}\left(\frac{a^2}{b^2} - \frac{b}{a}\right)$$

$$= \frac{\epsilon_0 I}{\sigma_0}\left\{\left(\frac{b}{a} - \frac{a^2}{b^2}\right) + \left(\frac{a^2}{b^2} - \frac{b}{a}\right)\right\} = 0$$

Hringskífa með yfirborðshleðslu ρ_s snýst með hornhraða ω (jöfnum)
Finna B á snúningsás hennar.

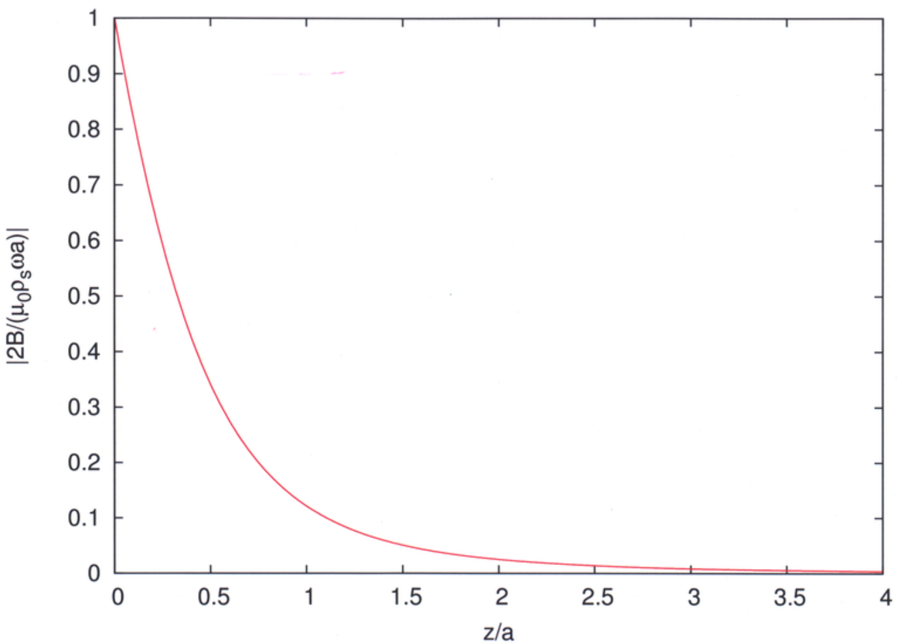
Straumþéttleiki er $j(r) = \rho_s u(r) = \rho_s r\omega$

Hver hringur (straumhringur) á skífunni leiðir til

$$d\bar{B} = \hat{a}_z \frac{\mu_0 r^2 dI}{2(z^2+r^2)^{3/2}} = \hat{a}_z \frac{\mu_0 r^2}{2(z^2+r^2)^{3/2}}\rho_s r\omega dr$$

$$\to \bar{B} = \hat{a}_z\frac{\mu_0\rho_s\omega}{2}\int_0^a \frac{r^3 dr}{(z^2+r^2)^{3/2}} = \frac{\hat{a}_z\mu_0\rho_s\omega}{2}\cdot\left\{\frac{2z^2+a^2}{\sqrt{z^2+a^2}} - 2z\right\}$$

$$= \frac{\hat{a}_z\mu_0\rho_s\omega}{2}\, a\left\{\frac{2\left(\frac{z}{a}\right)^2+1}{\sqrt{\left(\frac{z}{a}\right)^2+1}} - 2\left(\frac{z}{a}\right)\right\}$$



a, μ_f járnseglandi.

μ_0

a) Finna $\bar{B}$, $\bar{H}$ og $\bar{M}$ innan og utan sívalnings

$I = \pi a^2 J$ eða $\bar{J} = \hat{a}_z \frac{I}{\pi a^2}$

Notum lögmál Ampères $\bar{\nabla}\times\bar{H} = \bar{J}$ ← frjáls straumþéttleiki

$\to \oint_C \bar{H}\cdot d\bar{l} = I$

Samhverfa gefur að H sé fasti á hringum með miðju á ás sívalnings

r < a

$$I_{enc} = I\frac{\pi r^2}{\pi a^2} = J\pi r^2$$

(4)

$$\rightarrow \quad H_i \, 2\pi r = J\pi r^2 \quad \rightarrow \quad H_i(r) = \frac{Jr}{2}$$

$$\rightarrow \quad \bar{H}_i(r) = \hat{a}_\phi \frac{Jr}{2} \qquad \underline{r<a}$$

$\underline{r>a}$

$$H_o \, 2\pi r = J\pi a^2 \quad \rightarrow \quad H_o(r) = \frac{Ja^2}{2r}$$

$$\rightarrow \quad \bar{H}_o(r) = \hat{a}_\phi \frac{Ja^2}{2r} \qquad \underline{r>a}$$

Fyrir $\bar{B}$ gildir

$$\bar{B} = \mu_0(1+\chi_m)\bar{H} = \mu_0\mu_r\bar{H} = \mu\bar{H}$$

(5)

$\underline{r<a}$

$$\bar{B}_i = \mu\bar{H} = \hat{a}_\phi \frac{\mu J r}{2} \qquad (*)$$

$\underline{r>a}$

$$\bar{B}_o = \mu_0\bar{H} = \hat{a}_\phi \frac{\mu_0 J a^2}{2r}$$

(*)

Fyrir $\bar{M}$ gildir

$$\bar{H} = \frac{\bar{B}}{\mu_0} - \bar{M} \quad \rightarrow \quad \bar{M} = -\bar{H} + \frac{\bar{B}}{\mu_0}$$

$\underline{r<a}$

$$\bar{M}_i = -\hat{a}_\phi\left\{\frac{Jr}{2} - \frac{\mu}{\mu_0}\frac{Jr}{2}\right\} = \hat{a}_\phi \frac{Jr}{2}\left(\frac{\mu}{\mu_0} - 1\right)$$

$\mu >> \mu_0$

$\underline{r>a}$

$$\bar{M}_o = \hat{a}_\phi\left\{\frac{Ja^2}{2r} - \frac{Ja^2}{2r}\right\} = 0$$

(6)

b) Jafngildir segulvirar straumar?

$$\bar{J}_m = \bar{\nabla}\times\bar{M}, \qquad \bar{J}_{ms} = \bar{M}\times\hat{a}_n$$

$$\bar{J}_m = \hat{a}_z \frac{1}{r}\frac{\partial}{\partial r}(r M_\phi) = \hat{a}_z J\left(\frac{\mu}{\mu_0} - 1\right) \qquad \underline{r<a}$$

$$\bar{J}_{ms} = \bar{M}\times\hat{a}_r\Big|_{r=a} = -\hat{a}_z \frac{Ja}{2}\left(\frac{\mu}{\mu_0} - 1\right) \qquad \text{á yfirborðinu}$$

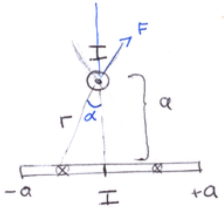
$\mu >> \mu_0$

(*) Við gerðum ráð fyrir því að járnsegullinn hér væri ekki með segulvægi M_0 áður en straumur er settur á hann. Þetta er ekki gefið mál fyrir járnseglandi efni.

(7)

Takið eftir stefnu $\bar{M}$, J_m og J_{ms} fyrir járnseglandi efni þ.e. $\mu >> \mu_0$ borið saman við andseglandi efni með $\mu \leq 1$

①



Finna kraftinn á renningunni á lengdareiningu

Hér má fara margar leiðir. Notum niðurstöður úr Ex 6-21 í bók. Milli tveggja samsíða víra með andstæðar straumstefnur er fráhrindikraftur með styrk

$$F'_{12} = -\hat{a}_y \frac{\mu_0 I_1 I_2}{2\pi d}$$

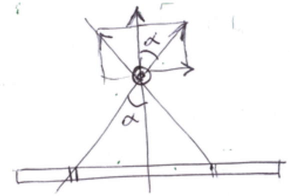
Hér hugsum við renninguna sem settan saman úr mörgum örsmáum samsíða vírum með straum

$$dI_2 = \frac{I}{2a}\, dx$$

②

Við höfum þá

$$\left(dF'_{12}\right)_z = -\frac{\mu_0 I\, dI}{2\pi r}\cos\alpha$$



þar sem $r = \sqrt{a^2 + x^2}$

$$\Rightarrow dF_{12} = -\hat{a}_z \frac{\mu_0 I}{2\pi}\,\frac{Ia}{2a}\,\frac{dx}{a^2+x^2}$$

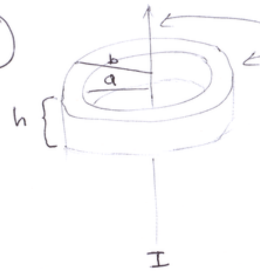
$$\Rightarrow F_{12} = -\hat{a}_z \frac{\mu_0 I^2}{4\pi}\int_{-a}^{a}\frac{dx}{a^2+x^2} = -\hat{a}_z \frac{\mu_0 I^2}{4\pi}\,\frac{\pi}{2a}$$

$$= -\hat{a}_z \frac{\mu_0 I^2}{8a}$$

þegar ég varpa kraftinum á z-ás, þættirnir í y-átt styttast út

$\cos\alpha = \frac{a}{r}$

②



③

finnum víxlspan leiðarans og spólunnar

Amperes lögmál gefur segulsvið vegna spólu innan hennar

$$\bar{B}_s = \hat{a}_\phi \frac{\mu_0 N_s I_s}{2\pi r}$$

En, við viljum víxlspan og þurfum segulsviðið um leiðarann

$$\bar{B}_L = \hat{a}_\phi \frac{\mu_0 I_L}{2\pi r}$$

samkvæmt Ampère. Flæði þess um spóluna er

$$\Phi_{SL} = \int_S d\bar{s}\cdot\bar{B}_L = \frac{\mu_0 I_L}{2\pi}\int_a^b \frac{dr}{r}\int_0^h dz = \frac{\mu_0 I_L h}{2\pi}\ln\left(\frac{b}{a}\right)$$

④

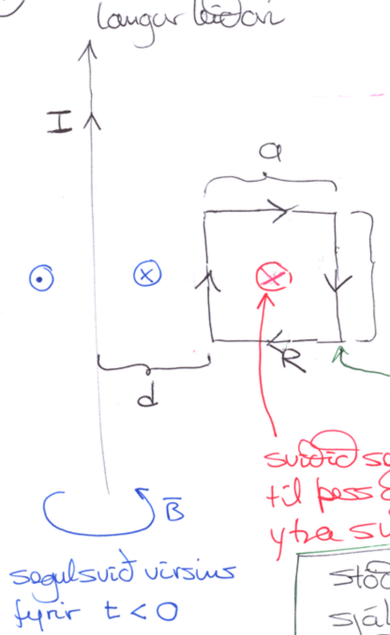
$$N_s \Phi_{SL} = L_{SL} I_L$$

$$\Rightarrow L_{SL} = \frac{N_s \Phi_{SL}}{I_L} = \frac{N_s \mu_0 h}{2\pi}\ln\left(\frac{b}{a}\right)$$

einungis háð lögun og efni

Sjálfspan spólunnar í bók var $L_s = \frac{\mu_0 N_s^2 h}{2\pi}\ln\left(\frac{b}{a}\right)$ þar sem N_s er í öðru veldi, en fyrir víxlspanið fæst bara eins N_s í fyrsta veldi og ekkert N_L eða $N_L = 1$!

①



Straumurinn í leiðaranum er

$$I(t) = I\,\theta(-t)$$

$$\theta(x) = \begin{cases} 0 & \text{ef } x<0 \\ 1 & \text{ef } x>0 \end{cases}$$

a) Straumstefnan í lykkjunni klukkan $t = 0^+$

Stöðluð textabókar lausn sleppir oft sjálfspani, skoðum eftir dæmi 2 hvað gerist ef við tökum það með á síðum (10) – (12)

②

b) Lögmál Ampères gefur $\bar B$ í kringum vír þ.e. $t<0$

$$\bar B = \hat\phi\,\frac{\mu_0 I}{2\pi r}$$

því má finna flæðið um lykkjuna

$$\Phi = \oint_S d\bar s\cdot\bar B = a\int_d^{d+a} dr\,\frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I a}{2\pi}\ln(r)\Big|_d^{d+a}$$

$$= \frac{\mu_0 I a}{2\pi}\left\{\ln(d+a) - \ln(d)\right\} = \frac{\mu_0 I a}{2\pi}\ln\left[\frac{d+a}{d}\right]$$

Í lykkjunni spanast I_{Ly}, íspennan í lykkjunni er

$$\mathcal{E} = I_{Ly}\cdot R = -\frac{d}{dt}\Phi$$

$$\hookrightarrow \quad I_{Ly} = \frac{d}{dt}Q$$

③

$$\rightarrow \quad I_{Ly}\cdot R = -\frac{d}{dt}\Phi$$

$$R\frac{d}{dt}Q = -\frac{d}{dt}\Phi = -\frac{\mu_0 a}{2\pi}\ln\left\{\frac{d+a}{d}\right\}\frac{d}{dt}I$$

$$\frac{d}{dt}Q = -\frac{\mu_0 a}{2\pi R}\ln\left\{\frac{d+a}{d}\right\}\frac{d}{dt}I$$

heildum

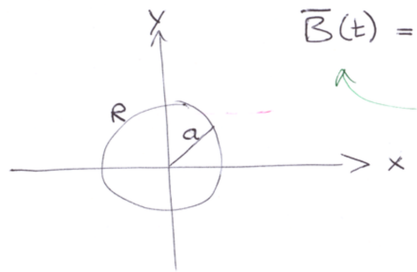
$$\int_0^Q dQ' = -\frac{\mu_0 a}{2\pi R}\ln\left\{\frac{d+a}{d}\right\}\int_I^0 dI'$$

$$\rightarrow \quad Q = \frac{\mu_0 I a}{2\pi R}\ln\left\{\frac{d+a}{d}\right\}$$

hleðslan sem flæðir um hvern punkt lykkjunnar þegar slökkt er á I í langa leiðaranum

④

②



$$\bar B(t) = \frac{B_0}{\sqrt{2}}(\hat a_x + \hat a_z)\left\{1 - e^{-\lambda t}\right\}\theta(t)$$

flæði þessa ytra sviðs í gegnum lykkju

a) Finna $i(t)$ í lykkjunni

$$\Phi_B(t) = \frac{B_0 a^2\pi}{\sqrt{2}}\left\{1 - e^{-\lambda t}\right\}\theta(t)$$

Uppi í gegnum lykkjuna

{Er í raun óþarft því $|B(0)| = 0$}

$$\frac{d\Phi_B}{dt}(t) = \frac{B_0 a^2\pi}{\sqrt{2}}\left[\left\{1 - e^{-\lambda t}\right\}\delta(t) + \theta(t)\lambda e^{-\lambda t}\right]$$

Keðjuregla og $\frac{d\theta(t)}{dt} = \delta(t)$

→ Heaviside step function

→ Dirac delta function

Heildarsegulflæði um lykkjuna er vegna breytinga á þessu "ytra" segulsviði B og breytinga segulflæðis sem streymir um lykkjuna mynda

$$\Phi = \Phi_B + \Phi_L = \Phi_B + Li$$

sjálfspan lykkju

$\Phi = Li$

$$\oint_C \bar{E}\cdot d\bar{l} = -\frac{d\Phi}{dt}$$

$$= Ri$$

því fæst

$$Ri = -\frac{d\Phi_B}{dt} - L\frac{di}{dt}$$

eða

$$L\frac{di}{dt} + Ri = -\frac{d}{dt}\Phi_B$$

$$\boxed{\frac{di}{dt} + \frac{R}{L}i = -\frac{B_0 a^2\pi}{L\sqrt{2}}\left[\{1-e^{-\lambda t}\}\delta(t) + \theta(t)\lambda e^{-\lambda t}\right]}$$

Hér er 1. stigs afleiðujafna sem við verðum að leysa

Jafnan $y' + p(t)y = q(t)$

hefur lausnina

$$y(t) = y(t_0)e^{-P(t)} + e^{-P(t)}\int_{t_0}^{t} ds\, e^{P(s)}q(s)$$

með

$$P(t) = \int_{t_0}^{t} ds\, p(s)$$

Hjá okkur

$$P(t) = \int_0^t ds\,\frac{R}{L} = \frac{Rt}{L}$$

$$\rightarrow i(t) = i(0)e^{-\frac{Rt}{L}} - e^{-\frac{Rt}{L}}\frac{B_0a^2\pi}{L\sqrt{2}}\lambda\int_{0^-}^{t} ds\, e^{\frac{Rs}{L}-\lambda s}$$

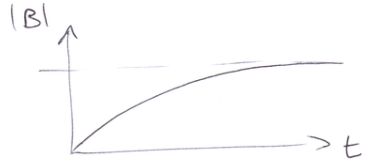
$$= i(0)e^{-\frac{Rt}{L}} - e^{-\frac{Rt}{L}}\lambda\frac{B_0a^2\pi}{L\sqrt{2}}\left[\frac{e^{\frac{R}{L}t-\lambda t}-1}{\left(\frac{R}{L}-\lambda\right)}\right]$$

$$= \underbrace{i(0)}_{0}e^{-\frac{Rt}{L}} - \frac{\lambda B_0a^2\pi}{L\sqrt{2}}\left(\frac{e^{-\lambda t}-e^{-\frac{Rt}{L}}}{\left(\frac{R}{L}-\lambda\right)}\right)$$

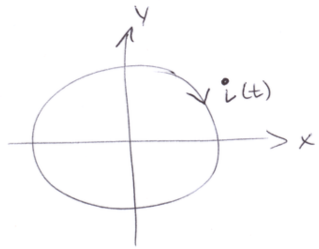
$$= \frac{\lambda B_0a^2\pi}{L\sqrt{2}}\left\{\frac{e^{-\frac{R}{L}t}-e^{-\lambda t}}{\left(\frac{R}{L}-\lambda\right)}\right\}$$

L má reikna, en hér skiptir aðeins máli hvernig hlutfall $\frac{R}{L}$ er miðað við λ svo ég sleppi því

Fall $\bar{B}(t)$ af t er með þeim hætti að kveikt er hægt á $\bar{B}$ sem nær síðan max. gildi

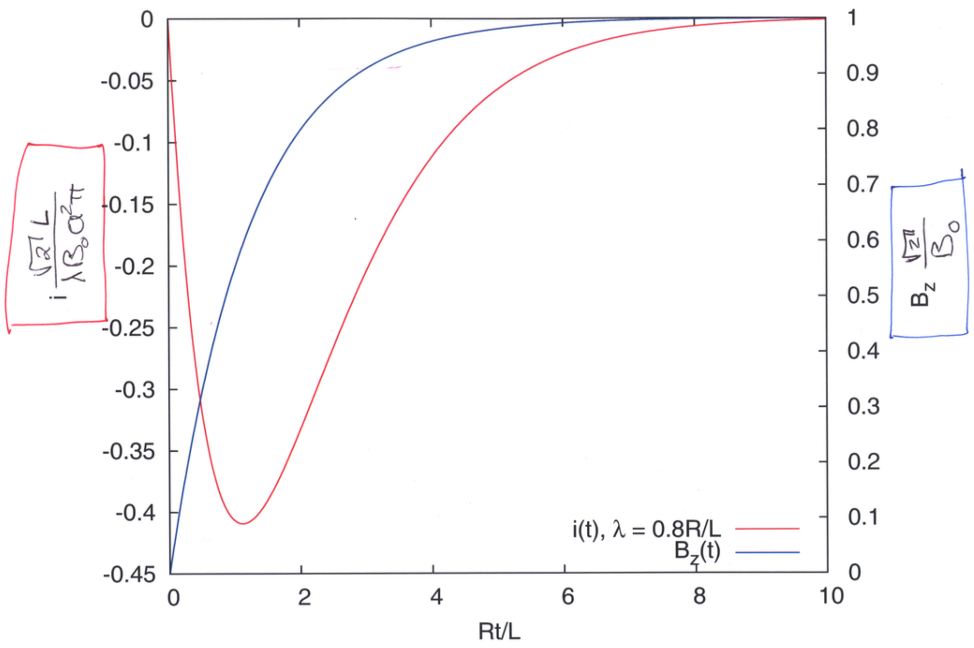


svar lykkjunnar er íspenna sem vinnur á móti þessu sviði



sjá næstu síðu

(9)



(10)

Skoðum aftur 1. dæmið ~~hvað~~ gerist ef sjálfspan lykkju er tekið til greina?

$$\Phi = \Phi_{vír} + \Phi_{lykkja} = \underbrace{\frac{\mu_0 I a}{2\pi} \ln\left\{\frac{d+a}{d}\right\}}_{MI} + L I_{Ly}$$

$$V = -\frac{d}{dt}\Phi$$

$$\longrightarrow \quad I_{Ly} \cdot R = -M \frac{d}{dt} I - L \frac{d}{dt} I_{Ly}$$

$$L \frac{d}{dt} I_{Ly}(t) + I_{Ly}(t) R = -M \frac{d}{dt} I(t)$$

$$I(t) = I\Theta(-t) \longrightarrow \frac{d}{dt} I(t) = -\delta(t) \cdot I$$

(11)

$$L \frac{d}{dt} I_{Ly}(t) + I_{Ly}(t) R = MI\delta(t)$$

$$\frac{d}{dt} I_{Ly}(t) + \frac{R}{L} I_{Ly}(t) = \frac{MI}{L}\delta(t)$$

Lausnin er $$y(t) = y(t_0) e^{-P(t)} + e^{-P(t)} \int_{t_0}^{t} ds\, e^{P(s)} q(s)$$

$$P(t) = \int_{t_0}^{t} ds\, p(s) = \frac{Rt}{L}$$

$$q(s) = MI\delta(t)$$

$$I_{Ly}(t) = I_{Ly}(0) e^{-\frac{R}{L}t} + e^{-\frac{R}{L}t} \frac{MI}{L} \int_{0^-}^{t} ds\, e^{\frac{Rs}{L}} \delta(s)$$

(12)

$$I_{Ly}(t) = e^{-\frac{R}{L}t} \frac{MI}{L}$$

Heildar hleðsla um sérhvern punkt í rás (lykkju)

$$Q = \int_0^\infty dt\, I_{Ly}(t) = \frac{MI}{L} \int_0^\infty dt\, e^{-\frac{R}{L}t}$$

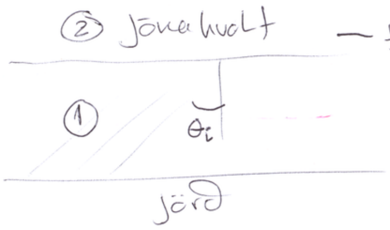
$$= \frac{MI}{L}\left\{ -\frac{e^{-\frac{R}{L}t}}{R/L} \Big|_0^\infty \right\} = \frac{MI}{L}\left\{0 + \frac{L}{R}\right\} = \frac{MI}{R}$$

óháð L!

$$= \frac{\mu_0 a I}{2\pi R} \ln\left\{\frac{d+a}{d}\right\}$$

Sama svar og áður! Hvernig gátum við búist við því!

1 1



— fastur þéttleiki jóna

Reikna $\Gamma_{\parallel}(f)$, $\Gamma_{\perp}(f)$

$\tau_{\parallel}(f)$, $\tau_{\perp}(f)$

fyrir horn á bilinu $60° \leq \theta_i \leq 80°$

og tíðni $\frac{\omega_p}{2} < 2\pi f < \omega_p$

$$\epsilon_p = \epsilon_0\left(1 - \frac{\omega_p^2}{\omega^2}\right) \qquad \omega_p^2 = \frac{Ne^2}{m\epsilon_0} \text{ fasti hér}$$

$$= \epsilon_0\left(1 - \frac{f_p^2}{f^2}\right), \quad \gamma = i\omega\sqrt{\mu\epsilon_0}\sqrt{1 - \frac{f_p^2}{f^2}}$$

$$\eta_p = \frac{\eta_0}{\sqrt{1 - \frac{f_p^2}{f^2}}}$$

2

Getum okkur enga segulvirkni

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\eta_2}{\eta_1}$$

$$\rightarrow \sin\theta_t = \frac{\eta_2 \sin\theta_i}{\eta_1}$$

$$\Gamma_{\perp} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\tau_{\perp} = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

3

Eins

$$\Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\tau_{\parallel} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

notum $\frac{f_p}{2} < f < f_p$ $\qquad \eta_1 = \eta_0$, $\epsilon_1 = \epsilon_0$

$\eta_0 \simeq 120\pi$

$$\rightarrow \epsilon_p < 0$$

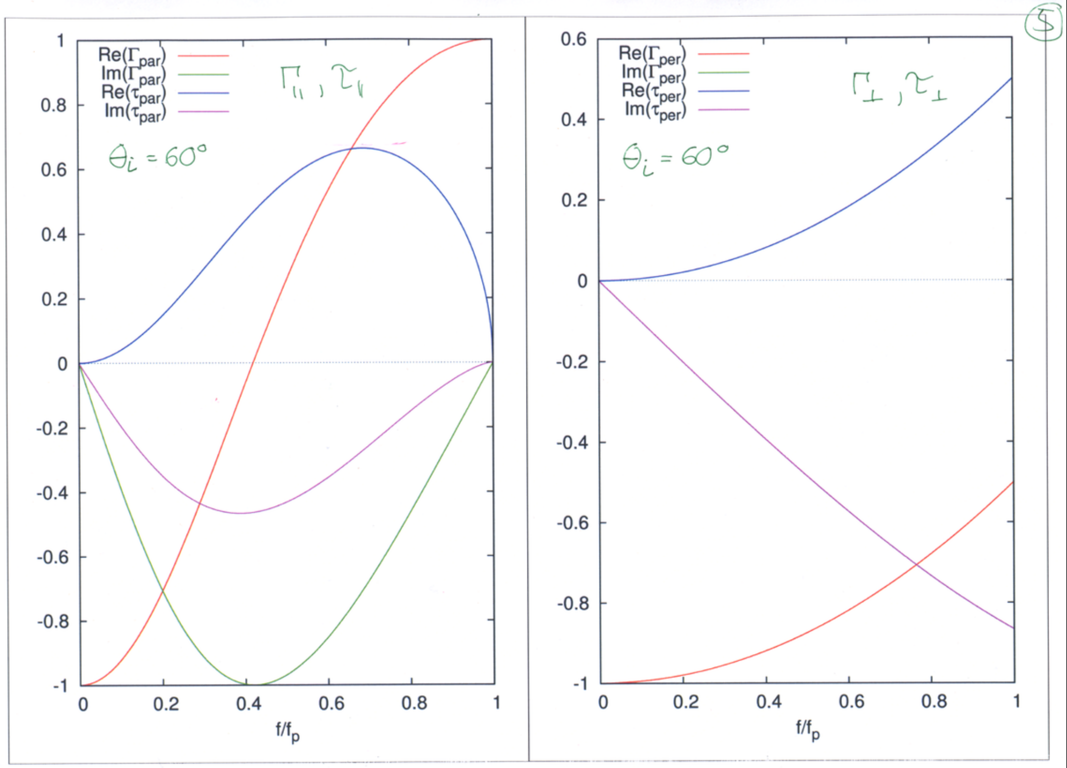
$$\eta_2 = \eta_p = \frac{\eta_0}{\sqrt{1 - \frac{f_p^2}{f^2}}} = -\frac{i\eta_0}{\sqrt{\frac{f_p^2}{f^2} - 1}}$$

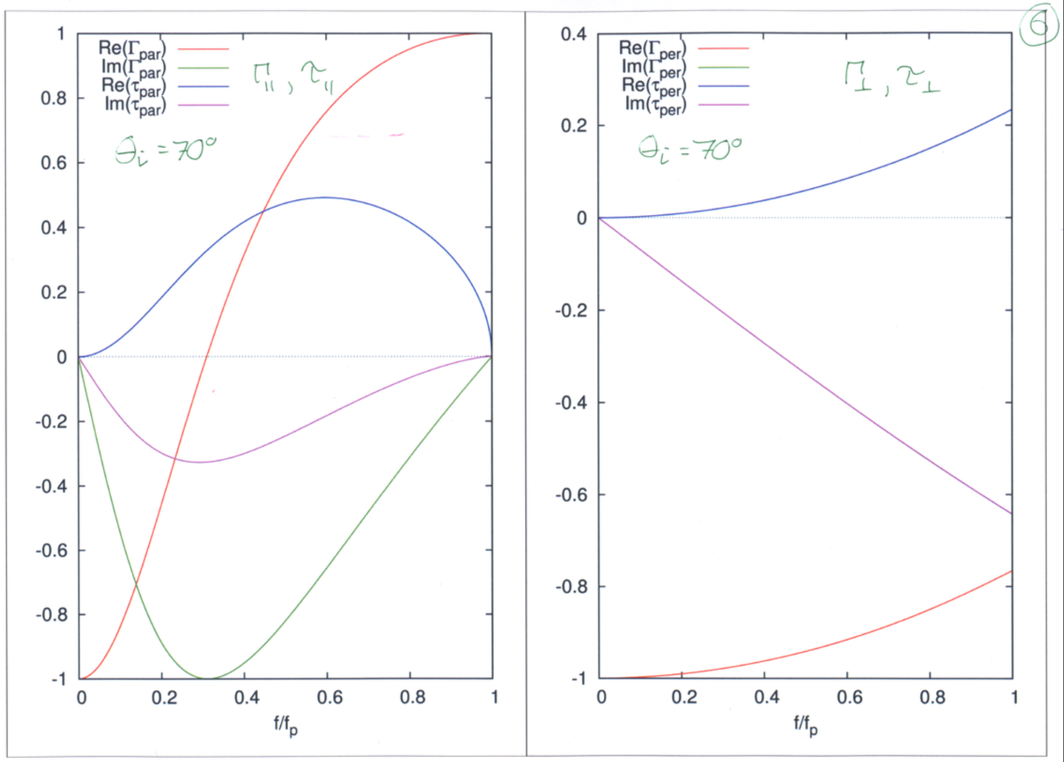
$$\sin\theta_t = \frac{\eta_2\sin\theta_i}{\eta_0} = -\frac{i\sin\theta_i}{\sqrt{\frac{f_p^2}{f^2} - 1}}$$

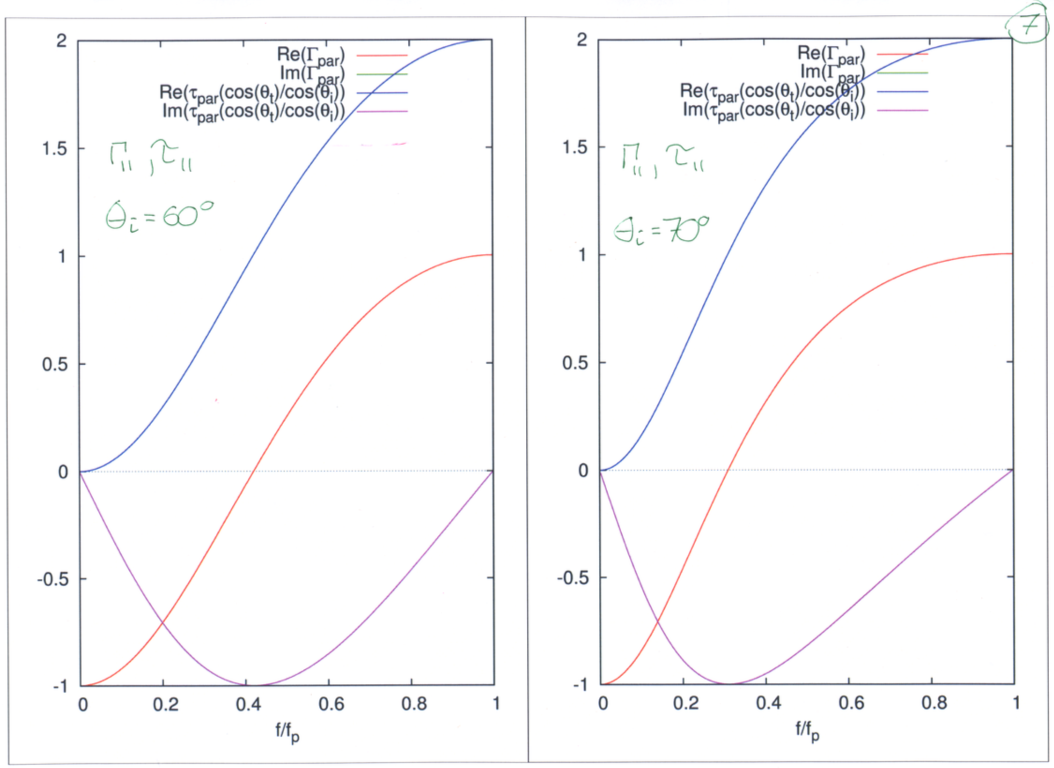
4

$$\cos^2\theta_t = 1 - \sin^2\theta_t = 1 + \frac{\sin^2\theta_i}{\left(\frac{f_p^2}{f^2} - 1\right)}$$

$$\rightarrow \cos\theta_t = \sqrt{1 + \frac{\sin^2\theta_i}{\left(\frac{f_p^2}{f^2} - 1\right)}}$$







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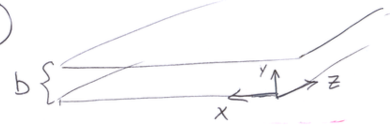
2



samsíða leiðandi plötur

TM$_n$-hættir milli platna (10-63-65) í bók

$$E_z^0(y) = A_n \sin\left(\frac{n\pi y}{b}\right)$$

$$H_x^0(y) = \frac{i\omega\epsilon}{h} A_n \cos\left(\frac{n\pi y}{b}\right)$$

$$E_y^0(y) = -\frac{\gamma}{h} A_n \cos\left(\frac{n\pi y}{b}\right)$$

Hér má ekki gleyma z þætti lausnar $e^{\gamma z}$

$$h^2 = \left(\frac{\pi}{b}\right)^2 \qquad \gamma = i\sqrt{\omega^2\mu\epsilon - \left(\frac{\pi}{b}\right)^2}$$

(9)

Yfirborðshleðslur

neðri $\rho_{sl} = \hat{a}_n \cdot \bar{D}(0) = \epsilon E_y^0(0) = -\frac{\gamma\epsilon}{h} A_n e^{-\gamma z}$

efri $\rho_{su} = \hat{a}_n \cdot \bar{D}(b) = -\epsilon E_y^0(b) = (-1)^n \frac{\gamma\epsilon}{h} A_n e^{-\gamma z}$

$\cos(n\pi)$

Yfirborðsstraumar

neðri $\bar{J}_{sl} = \hat{a}_y \times \bar{H}(0) = -\hat{a}_z \frac{i\omega\epsilon}{h} A_n e^{-\gamma z}$

efri $\bar{J}_{su} = \hat{a}_y \times \bar{H}(b) = \hat{a}_z \frac{i\omega\epsilon}{h} A_n \cos(n\pi) e^{-\gamma z}$

$= \hat{a}_z (-1)^n \frac{i\omega\epsilon}{h} A_n e^{\gamma z}$

(10)

Samfelldnijöfnur

neðri plata

$$\bar{\nabla} \cdot \bar{J}_{sl} = \partial_z J_{sz} = \frac{i\omega\epsilon}{h} \gamma A_n e^{-\gamma z}$$

$$\partial_t \rho_{sl} = i\omega \rho_{sl} = -\frac{i\omega\epsilon}{h} \gamma A_n e^{-\gamma z}$$

$$\rightarrow \quad \partial_t \rho_{sl} + \bar{\nabla} \cdot \bar{J}_{sl} = 0$$

og þetta sést ekki nema við munum eftir lausninni í z-átt, $e^{-\gamma z}$

Sams konar fæst fyrir efri plötu

(11)

En reynum TE_n

úr bók (10-83-85)

$$H_z^0(y,z) = B_n \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_y^0(y,z) = \frac{\gamma}{h} B_n \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_x^0(y,z) = \frac{i\omega\mu}{h} B_n \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

Hér er enginn E-þáttur þvert á plötur $\rightarrow \rho_s = 0$

En straumar

neðri plata $\bar{J}_{sl} = \hat{a}_y \times \bar{H}(0) = \hat{a}_x B_n e^{-\gamma z}$

efri plata $\bar{J}_{su} = -\hat{a}_y \times \bar{H}(b) = \hat{a}_x B_n \cos(n\pi) e^{-\gamma z}$

$= \hat{a}_x \times B_n (-1)^n e^{-\gamma z}$

(12)

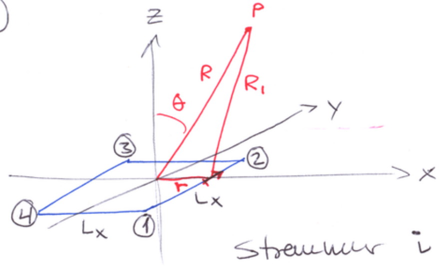
Samfelldnijafnan

$$\bar{\nabla} \cdot \bar{J} = \partial_x J_x = 0$$

$$\rho = 0$$

$\rightarrow$ Samfelldnijafnan er létt uppfyllt hér!

(1) (1)



Ferhyrnings lykkja í miðri x-y-sléttu með hliðarlengdir L_x og L_y

Streymur $i(t) = I_0 \cos\omega t$

lykkjan er lítil

Finna fjarsviðin

a) $\bar{A}$? Hér má búast við að lykkjan geisli eins og segultvískaut, þ.e. fjarsviðið.

Við byrjum því eins og í bókinni fyrir segultvískautið

$$\bar{A} = \frac{\mu_0 I}{4\pi}\oint \frac{e^{-i\beta R_1}}{R_1}\, d\bar{l}'$$

(2)

Nálgun fyrir smáa lykkju

$$e^{-i\beta R_1} = e^{-i\beta R}\, e^{-i\beta(R_1-R)} \simeq e^{i\beta R}\left\{1 - i\beta(R_1-R)+\cdots\right\}$$

og því

$$\bar{A} = \frac{\mu_0 I}{4\pi} e^{-i\beta R}\left\{(1+i\beta R)\oint \frac{d\bar{l}'}{R_1} - i\beta\oint d\bar{l}'\right\}$$

eins og áður fæst $\oint d\bar{l}' = 0$

$$\bar{A} = \frac{\mu_0 I}{4\pi} e^{-i\beta R}\left\{(1+i\beta R)\oint \frac{d\bar{l}'}{R_1}\right\}$$

(3)

Athugum heildið

$$\oint \frac{d\bar{l}'}{R_1} = \left\{\int_{1}^{2} + \int_{2}^{3} + \int_{3}^{4} + \int_{4}^{1}\right\}\frac{d\bar{l}'}{R_1}$$

$$R_1 = \sqrt{R^2 + r^2 - 2\bar{R}\cdot\bar{r}}$$

Tengslin milli kartísku- og kúluhnitanna eru

$$\bar{R} = \hat{a}_x R\sin\theta\cos\phi + \hat{a}_y R\sin\theta\sin\phi + \hat{a}_z R\cos\theta$$

$$\bar{r} = \hat{a}_x\, x + \hat{a}_y\, y$$

Fyrir legginn 1 - 2 fæst $\bar{r} = \hat{a}_x \frac{L_x}{2} + \hat{a}_y y$

$$\rightarrow \bar{R}\cdot\bar{r} = R\frac{L_x}{2}\sin\theta\cos\phi + R y\sin\theta\sin\phi$$

(4)

$$\frac{1}{R_1} = \frac{1}{R}\frac{1}{\sqrt{1+\frac{r^2}{R^2} - \frac{2\bar{R}\cdot\bar{r}}{R^2}}} \simeq \frac{1}{R\left(1-\frac{\bar{R}\cdot\bar{r}}{R^2}\right)}$$

$$\simeq \frac{1}{R}\left\{1+\frac{\bar{R}\cdot\bar{r}}{R^2}\right\} = \frac{1}{R}\left\{1+\frac{L_x}{2R}\sin\theta\cos\phi + \frac{y}{R}\sin\theta\sin\phi\right\}$$

og því

$$\int_{1}^{2} \frac{d\bar{l}'}{R_1} \simeq \frac{\hat{a}_y}{R}\int_{-\frac{L_y}{2}}^{\frac{L_y}{2}} dy\left\{1+\frac{L_x}{2R}\sin\theta\cos\phi + \frac{y}{R}\sin\theta\sin\phi\right\}$$

$$= \frac{\hat{a}_y}{R}\left\{L_y + \frac{L_x L_y}{2R}\sin\theta\cos\phi\right\}$$

Eins fast

$$\int_{3}^{4} \frac{d\bar{l}'}{R_1} \simeq \frac{\hat{a}_y}{R}\left\{-L_y + \frac{L_x L_y}{2R}\sin\theta\cos\phi\right\}$$

$$\to \left\{\int_{1}^{2} + \int_{3}^{4}\right\}\frac{d\bar{l}}{R_1} = \frac{\hat{a}_y}{R} 2\frac{L_x L_y}{2R}\sin\theta\cos\phi$$

og

$$\left\{\int_{2}^{3} + \int_{4}^{1}\right\}\frac{d\bar{l}}{R_1} = -\frac{\hat{a}_x}{R} 2\frac{L_x L_y}{2R}\sin\theta\cos\phi$$

og

$$\bar{A} \simeq L_x L_y \frac{\mu_0 I}{4\pi R^2} e^{-i\beta R}(1+i\beta R)\sin\theta\left\{-\hat{a}_x\sin\phi + \hat{a}_y\cos\phi\right\}$$

$$= \frac{\mu_0 m}{4\pi R^2} e^{-i\beta R}(1+i\beta R)\sin\theta\left\{-\hat{a}_x\sin\phi + \hat{a}_y\cos\phi\right\}$$

ef $m = I L_x L_y = IS$

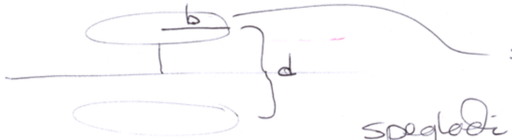
$$= \hat{a}_\phi \frac{\mu_0 m}{4\pi R^2} e^{-i\beta R}(1+i\beta R)\sin\theta$$

Sem er nákvamlega sama fjar-svið $\bar{A}$ og fyrir hringlykkju, sjá (11-25) í bók

$\bar{E}$ og $\bar{H}$ eru samkvæmt (11-26 a-c) í bók

Fjarsviðan fæst með $\beta R >> 1 \to \frac{1}{(i\beta R)^2}$ og $\frac{1}{(i\beta R)^3} \to 0$

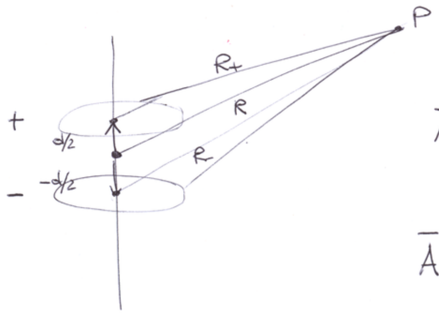
(2) Segul tvískaut yfir leiðandi jörð



streymur $i(t) = I_0 \cos\omega t$

speglaði streymurin er þá

$$i_s(t) = -I_0\cos\omega t$$



$$\bar{A}_+ = \hat{a}_\phi \frac{\mu_0 m}{4\pi R_+^2}(1+i\beta R_+)e^{-i\beta R_+}\sin\theta_+$$

$$\bar{A}_- = -\hat{a}_\phi \frac{\mu_0 m}{4\pi R_-^2}(1+i\beta R_-)e^{-i\beta R_-}\sin\theta_-$$

$b, d << R$

$$R_\pm = \sqrt{R^2 + \left(\frac{d}{2}\right)^2 \mp Rd\cos\theta} \simeq \left(R \mp \frac{d}{2}\cos\theta\right)$$

nýtum at $\theta_+ \sim \theta_-$

$$\bar{A}_\pm \simeq \pm\hat{a}_\phi \frac{\mu_0 m}{4\pi R^2}(1+i\beta R)\sin\theta\, e^{-i\beta(R \mp \frac{d}{2}\cos\theta)}$$

$$= \pm\hat{a}_\phi \frac{\mu_0 m}{4\pi R^2}(1+i\beta R)\sin\theta\, e^{-i\beta R} e^{\pm\frac{i\beta d}{2}\cos\theta}$$

$$\to \bar{A} \simeq \hat{a}_\phi \frac{2i\mu_0 m}{4\pi R^2}(1+i\beta R)\sin\theta\, e^{-i\beta R}\sin\left(\frac{\beta d}{2}\cos\theta\right)$$

(9)

$$\bar{E} \simeq \hat{a}_\phi \frac{\omega \mu_0 m}{2\pi R^2}(1+i\beta R)\sin\theta\, e^{-i\beta R}\sin\left(\frac{\beta d}{2}\cos\theta\right)$$

með fjörsvið

$$\bar{E} \simeq \hat{a}_\phi \frac{\omega \mu_0 m}{2\pi R} i\beta e^{-i\beta R}\sin\theta \sin\left(\frac{\beta d}{2}\cos\theta\right)$$

$$\bar{H} = \frac{1}{\mu_0}\bar{\nabla}\times\bar{A}$$

$$\rightarrow \begin{cases} H_R = \dfrac{1}{\mu_0}\dfrac{1}{R\sin\theta}\dfrac{\partial}{\partial\theta}(A_\phi \sin\theta) \\[2ex] H_\theta = -\dfrac{1}{\mu_0 R}\dfrac{\partial}{\partial R}(R A_\phi) \end{cases}$$

....

(10)

Geislunarmynstrið reiknað af hornþætti $\bar{E}$

$$\rightarrow \bar{E} = \hat{a}_\phi \frac{\omega\mu_0 m}{2\pi R} i\beta e^{-i\beta R} F(\theta)$$

með $F(\theta) = \sin\theta \cdot \sin\left(\frac{\beta d}{2}\cos\theta\right)$

$$\frac{\beta d}{2} = \frac{2\pi d}{2\lambda} << 1$$

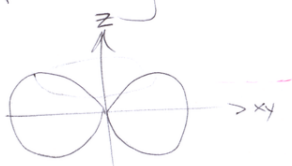
fyri lítið samsett tvístaut

$$\rightarrow F(\theta) \simeq \sin\theta \cdot \frac{\beta d}{2}\cos\theta$$

Soda mynstrið er $\sin\theta \cdot \cos\theta = \frac{1}{2}\sin(2\theta)$

(11)

Einfalda segultvístautið er með mynstur $\sin\theta$



En tvöfalda tvístautið er með $\sin(2\theta)$, sem er mynsturfall fyrir segul fjórstaut