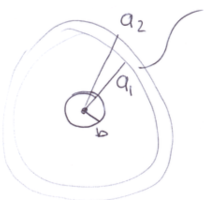
1 spherical $\rho(R,\theta,\phi) = \rho_0 \exp\left\{-\frac{R}{b}\right\}$ with radius b



Conductor

Define regions

- I $R<b$
- II $b<R<a_1$
- III $a_1<R<a_2$
- IV $R>a_2$

Total spherical symmetry

$\rightarrow$ $\bar{E}$ is radial

I Enclosed charge is

$$Q_{enc}(R) = \rho_0 4\pi \int_0^R r^2 dr\, e^{-r/b} = 4\pi\rho_0\left\{2b^3 - (bR^2 + 2b^2R + 2b^3)e^{-R/b}\right\}$$

For later the total charge is $Q_{enc}(b) = 4\pi\rho_0\left\{2b^3 - 5b^3e^{-1}\right\}$

or $Q_{enc}(b) = Q = 4\pi b^3\rho_0\left\{2 - \frac{5}{e}\right\}$

The electric field

$$\oint_S \bar{E}\cdot d\bar{S} = \frac{Q_{enc}(R)}{\epsilon_0}$$

$$4\pi R^2 E_R = \frac{Q_{enc}(R)}{\epsilon_0}$$

$$\rightarrow E_R = \frac{\rho_0 b^3}{R^2\epsilon_0}\left\{2 - \left(\frac{R^2}{b^2} + \frac{2R}{b} + 2\right)e^{-R/b}\right\}$$

and the electrical field is then

$$\bar{E}(R) = \frac{\rho_0 b^3}{R^2\epsilon_0}\hat{a}_R\left\{2 - \left(\frac{R^2}{b^2} + \frac{2R}{b} + 2\right)e^{-R/b}\right\}$$

II Gauss again

$$\oint_S \bar{E}\cdot d\bar{S} = \frac{Q}{\epsilon_0}$$

$$\bar{E}(R) = \frac{\rho_0 b^3}{\epsilon_0 R^2}\left\{2 - \frac{5}{e}\right\}\hat{a}_R$$

III Perfect conductor (inside)

$\bar{E} = 0$ (Equilibrium electro-statics)

The shell is uncharged initially, but the charge on the sphere inside induces charges on both surfaces of the shell. (The conductor)

Radial symmetry of $\bar{E}$ everywhere $\rightarrow$ The total charge on the inside sphere $+Q$ induces surface total charge $-Q$ on the inner surface of the shell $\rightarrow$ $+Q$ on the outer surface

IV spherical symmetry

$\rightarrow$ Gauss gives for $R>a_2$ the same solution as in II

$$\bar{E}(R) = \frac{\rho_0 b^3}{R^2\epsilon_0}\hat{a}_R\left\{2 - \frac{5}{e}\right\}$$

see a graph on page 8 of the $|E|$ for all the regions in the special case $a_1 = 2b$ and $a_2 = 3b$

b) Skelin er ekki ~~með~~ heildar~~hleðslu~~hleðslu ef hún var 0 í upphafi, en þéttleiki yfirborðs~~hleðslu~~hleðslunar er ekki sá sami á innra og ytra borði

It is uncharged in total if it was so initially. The surface charge densities are not the same on the inner and outer surface

$$\sigma_1 = -\frac{Q}{4\pi a_1^2}, \quad \sigma_2 = +\frac{Q}{4\pi a_2^2}$$

c) The total force on the outer shell

We can use energy consideration, how would the energy change if we vary "the radius" of the shell?

The field energy density is

$$w_e = \frac{1}{2}\epsilon_0 E^2$$

In region I the total energy $\int dv\, w_e$ does not change as the shell is varied. Nothing in E depends on a_1 and a_2!

(5)

This could be different for some force field that did not obey the static Maxwell equations.

$$W_e^{(II)} = \frac{1}{2}\epsilon_0 \int_b^{a_1} 4\pi r^2 dr \, \frac{\rho_0^2 b^6}{\epsilon_0^2 r^4} \left\{2 - \frac{5}{e}\right\}^2$$

þægilegt til að víddargreina
Good for dimensional analysis

$$= \frac{2\pi}{\epsilon_0} \rho_0^2 b^6 \left\{2 - \frac{5}{e}\right\}^2 \int_b^{a_1} \frac{dr}{r^2}$$

$$= \quad -||- \quad \times \left\{ -\frac{1}{r}\Big|_b^{a_1} \right\} = -||- \times \left\{ \frac{1}{b} - \frac{1}{a_1} \right\}$$

$$= \frac{2\pi}{\epsilon_0} \rho_0^2 \left\{2 - \frac{5}{e}\right\}^2 b^5 \left\{1 - \frac{b}{a_1}\right\} = \frac{2\pi}{\epsilon_0} \frac{(\rho_0 b^3)^2}{b} \left\{2 - \frac{5}{e}\right\}^2 \left\{1 - \frac{b}{a_1}\right\}$$

(6)

$$W_e^{(IV)} = \frac{1}{2}\epsilon_0 \int_{a_2}^{\infty} 4\pi r^2 dr \, \frac{\rho_0^2 b^6}{\epsilon_0^2 r^4} \left\{2 - \frac{5}{e}\right\}^2$$

$$= \frac{2\pi}{\epsilon_0} \frac{(\rho_0 b^3)^2}{b} \left\{2 - \frac{5}{e}\right\}^2 \frac{b}{a_2}$$

Now we need to consider

$$(F_Q)_R = -\frac{\partial W_e}{\partial R} \qquad \text{only radial force}$$

Lets consider the shell thickness to be constant

$$a_1 = x$$

$$a_2 = x + (a_2 - a_1) = x + \Delta a$$

(7)

the total field energy

$$W_e^{(II)} + W_e^{(IV)} = W_e = C_1 \left\{ \frac{1}{a_2} - \frac{1}{a_1} \right\} + C_1$$

Where C_1 and C_2 are constants

$$(F_Q)_R = -C_1 \frac{\partial}{\partial x} \left\{ \frac{1}{x + \Delta a} - \frac{1}{x} \right\}$$

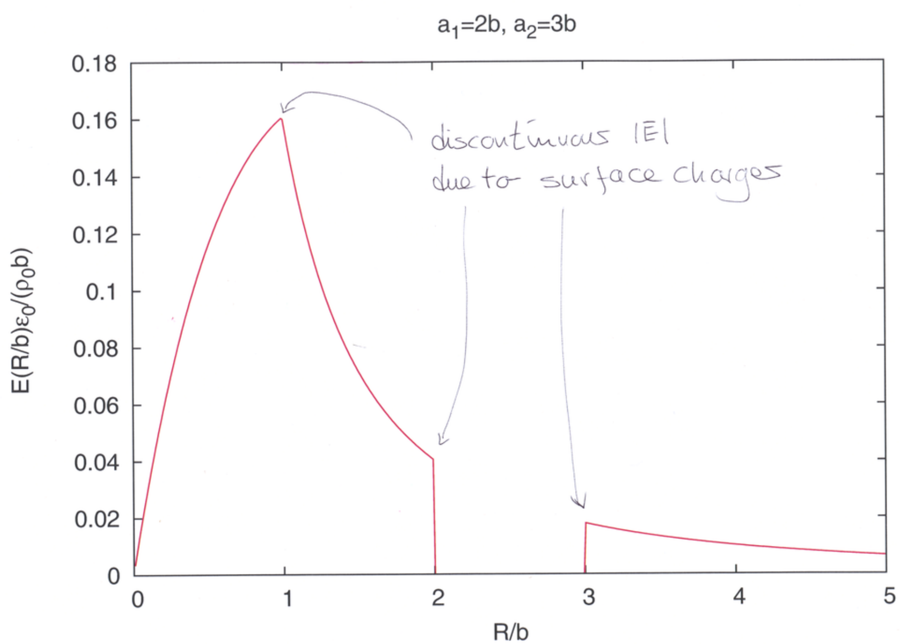
$$= -C_1 \left\{ \frac{1}{x^2} - \frac{1}{(x + \Delta a)^2} \right\} = -C_1 \left\{ \frac{1}{a_1^2} - \frac{1}{a_2^2} \right\}$$

$$= -\underbrace{\frac{2\pi}{\epsilon_0} \frac{(\rho_0 b^3)^2}{b^2} \left\{2 - \frac{5}{e}\right\} \left\{ \frac{1}{a_1^2} - \frac{1}{a_2^2} \right\}}_{>0} < 0$$

The force is directed inward

The total force

(8)



(1)

(1) We have a sphere with radius a and $\rho(R) = \rho_0 \exp\left(-\frac{R}{a}\right)$.

We will use two methods to calculate potential energy of the charge

$$W_e = \frac{1}{2}\int dv' \rho V$$

$$W_e = \frac{1}{2}\int dv' \bar{D}\cdot\bar{E}$$

The energy is present in V or E inside and outside the sphere

We need solutions in both regions

$$\bar{E} = -\bar{\nabla} V$$

$R < a$

Use the Poisson equation

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

$$-\frac{1}{R^2}\frac{d}{dR}\left(R^2 \frac{dV}{dR}\right) = \rho_0 \frac{e^{-R/a}}{\epsilon_0}$$

$$\downarrow$$

$$\frac{d}{dR}\left(R^2\frac{dV}{dR}\right) = -\rho_0 \frac{R^2 e^{-R/a}}{\epsilon_0}$$

undetermined integration

$$R^2 \frac{dV}{dR} = +\frac{\rho_0}{\epsilon_0}\left\{aR^2 + 2a^2R + 2a^3\right\}e^{-R/a} + C_1$$

$$\downarrow$$

$$\frac{dV}{dR} = \frac{\rho_0}{\epsilon_0}\left\{a + \frac{2a^2}{R} + \frac{2a^3}{R^2}\right\}e^{-R/a} + \frac{C_1}{R^2}$$

(2)

Here we have to determine C_1

$\frac{dV}{dR} \sim E_R$ which should not be singular at $R=0$, since there is no point charge there

When we take the limit $R \to 0^+$ we have to remember

$$e^{-\frac{R}{a}} \approx 1 - \frac{R}{a} + \frac{1}{2}\left(\frac{R}{a}\right)^2 + \cdots$$

and the $\frac{1}{R}$ terms cancel automatically, but we need to set

$$C_1 = -2a^3\frac{\rho_0}{\epsilon_0}$$

to cancel the $\frac{1}{R^2}$ term

We thus get

$$\frac{dV}{dR} = \frac{\rho_0}{\epsilon_0}\left\{-\frac{2a^3}{R^2} + \left(a + \frac{2a^2}{R} + \frac{2a^3}{R^2}\right)e^{-R/a}\right\}$$

(If we rearrange this we see the same result as we got with the Gauss theorem last week)

$$E_R = -\frac{dV}{dR}$$

$$= \frac{\rho_0 a^3}{R^2\epsilon_0}\left\{2 - \left(\frac{R^2}{a^2} + \frac{2R}{a} + 2\right)e^{-R/a}\right\}$$

we also need V in this region

One more indetermined integration gives

(3)

$$V(R) = \frac{\rho_0 a^2}{\epsilon_0}\left\{\frac{2a}{R} - \left(1 + \frac{2a}{R}\right)e^{-\frac{R}{a}}\right\} + C_2$$ (Wolfram Alpha)

we wait with the determination of C_2, First we find the external solution

$R > a$

In this region there is no charge

$$\to \nabla^2 V = 0$$

we have to solve laplace equation

(Or we could have used Gauss theorem to find E and then integrate)

$$\frac{1}{R^2}\frac{d}{dR}\left(R^2\frac{dV}{dR}\right) = 0$$

$$\frac{d}{dR}\left(R^2\frac{dV}{dR}\right) = 0$$

$$\to R^2\frac{dV}{dR} = C_3$$

$$\frac{dV}{dR} = \frac{C_3}{R^2}$$

$$\to V = -\frac{C_3}{R^2} + C_4$$

(4)

We can use the convention here that $V(R) \to 0$ as $R \to \infty$

$$\to C_4 = 0$$

Then we use that

$$V^{I}(a) = V^{II}(a)$$

Continuity at the surface of the charged sphere

$$\frac{\rho_0 a^2}{\epsilon_0}\left\{2 - (1+2)e^{-1}\right\} + C_2$$

$$= -\frac{C_3}{a}$$

och

$$\frac{\rho_0 a^2}{\epsilon_0}\left\{2 - \frac{3}{e}\right\} + C_2 = -\frac{C_3}{a}$$

One way to proceed is to remember that Gauss theorem gives for the outer solution that

$$V(R) = \frac{Q}{4\pi\epsilon_0 R}$$

where Q is the total charge and we could determine C_3

this was done last week

(5)

With the solution

$$Q = 4\pi a^3 \rho_0 \left\{2 - \frac{5}{e}\right\}$$

$$\to \quad -C_3 = \frac{a^3 \rho_0}{\epsilon_0}\left(2 - \frac{5}{e}\right)$$

so

$$\frac{\rho_0 a^2}{\epsilon_0}\left\{2 - \frac{3}{e}\right\} + C_2 = \frac{\rho_0 a^2}{\epsilon_0}\left\{2 - \frac{5}{e}\right\}$$

$$C_2 = \frac{\rho_0 a^2}{\epsilon_0} e^{-1}(-2)$$

and we have for $R<a$

$$V(R) = \frac{\rho_0 a^2}{\epsilon_0}\left\{\frac{2a}{R} - \frac{2}{e} - \left(1 + \frac{2a}{R}\right)e^{-\frac{R}{a}}\right\}$$

We start with

$$W_e = \frac{1}{2}\int dv' \rho V$$

as ρ is 0 outside the sphere we only need V inside

But you saw though how the internal solution was determined by B.C. cond. at ∞

(6)

$$W_e = \frac{1}{2}\int_0^a 4\pi R^2 dR\, \rho(R) V(R)$$

$$= 2\pi \frac{\rho_0^2 a^2}{\epsilon_0}\int_0^a dR \left\{2aRe^{-R/a} - \frac{2R^2 e^{-R/a}}{e} - (R^2 + 2aR)e^{-2R/a}\right\}$$

$$= 2\pi \frac{\rho_0^2 a^2}{\epsilon_0}\left\{2a^3\left(1 + \frac{2}{e}\right) - \frac{2a^3}{e}\left(2 - \frac{5}{e}\right) + \frac{a^3}{4}\left(11e^{-2} - 3\right)\right\}$$

$$= 2\pi \frac{\rho_0^2 a^5}{\epsilon_0}\left\{\left(2 - \frac{3}{4}\right) - \frac{4}{e}(1+1) + \frac{1}{e^2}(10 + 11)\right\}$$

$$= \frac{\rho_0^2 a^5}{\epsilon_0} 2\pi\left(\frac{5}{4} - 8e^{-1} + \frac{51}{4}e^{-2}\right)$$

(7)

Now we try and compare

No dielectric $\bar{D} = \epsilon_0 \bar{E}$

$$W_e = \frac{1}{2}\int dv' \bar{D}\cdot\bar{E}$$

Have already

$$E_R = \frac{\rho_0 a^3}{R^2 \epsilon_0}\left\{2 - \left(\frac{R^2}{a^2} + \frac{2R}{a} + 2\right)e^{-R/a}\right\} \qquad R<a$$

$$E_R = \frac{\rho_0 a^3}{R^2 \epsilon_0}\left\{2 - \frac{5}{e}\right\} \qquad R>a$$

$$W_e = \frac{1}{2}\int_0^a 4\pi R^2 dR \frac{\rho_0^2 a^6}{R^4 \epsilon_0}\left\{2 - \left(\frac{R^2}{a^2} + \frac{2R}{a} + 2\right)e^{-R/a}\right\}^2 + \frac{1}{2}\int_a^\infty 4\pi R^2 dR \frac{\rho_0^2 a^6}{R^4 \epsilon_0}\left\{2 - \frac{5}{e}\right\}$$

(8)

$$W_e = 2\pi \frac{\rho_0^2 a^6}{\epsilon_0}\left[\frac{(48 - 11e)e - 49}{4ae^2} + \left\{2 - \frac{5}{e}\right\}^2 \frac{1}{a}\right]$$

$$= \frac{\rho_0^2 a^5}{\epsilon_0} 2\pi\left\{\frac{5}{4} - 8e^{-1} + \frac{51}{4}e^{-2}\right\}$$

same swer! same answer!

9

(2)

$$V(\bar{R}) = A\frac{e^{-\lambda R}}{R}$$ fyrir einhverja hleðsludreifingu

Þannig þarf að leysa í skrefum og fyrstu niðurstöður getur þurft að endurbæta!

Coulomb-lögmálið segir okkur að heildarhleðslan hljóti að hverfa $Q=0$, ef ekki á yrði að vera þáttur $\frac{1}{R}$ í mættinu, ánstýlingar.

Aðfellu formið þegar $R \to 0$, $V(R) \xrightarrow[R\to 0]{} \frac{A}{R}$, bendir til þess að inni í samfelldri hleðsludreifingu sé punkthleðsla með hleðslu $q = 4\pi\epsilon_0 A$

10

fyrir $R \neq 0$ er rótsviðið

$$\bar{E} = -\bar{\nabla} V(R)$$

sem hér

$$E_R = -\frac{\partial V}{\partial R} = -A\left\{-\frac{1}{R^2} - \frac{\lambda}{R}\right\} e^{-\lambda R}$$

$$\to \bar{E} = -A\,\hat{a}_R\left\{-\frac{1}{R^2} - \frac{\lambda}{R}\right\} e^{-\lambda R}$$

Til þess að finna hleðsludreifinguna alla (fyrir utan punkt-þáttinn) notum við

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

Í kúluhnitum hér

$$-\epsilon_0\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial}{\partial R}V\right) = \rho(R) \quad \text{ef } R \neq 0$$

11

$$\to \rho(R) = -A\epsilon_0\frac{\lambda^2 e^{-\lambda R}}{R} \qquad \text{þ. } R \neq 0$$

skoðum heildarhleðsluna frá þessum þætti

$$\int dv\, \rho(R) = \left(4\pi\int_0^\infty R^2 dR\, \frac{e^{-\lambda R}}{R}\right)\cdot(-A\epsilon_0\lambda^2)$$

$$= -4\pi\epsilon_0 A\int_0^\infty x\,dx\, e^{-x} = -4\pi\epsilon_0 A$$

$$= -q$$

því þurfum við í viðbót við $\rho(R) = -A\epsilon_0\frac{\lambda^2 e^{-\lambda R}}{R}$ þ. $R \neq 0$ eina punkthleðslu í $R=0$ með stærð $q = 4\pi\epsilon_0 A$

12

Heildarhleðslan er $Q=0$. Það er líka gaman að sjá að $\oint \bar{E}\cdot d\bar{s} \to 0$ þegar notað er kúluyfib. með vaxandi $R \to \infty$ og E sem við fundum.

Auka

Punkthleðsluna má tákna við þrívítt δ-fall

$$\rho_p(\bar{r}) = \epsilon_0 A\, 4\pi\, \delta^{(3)}(\bar{r}) = \epsilon_0 A\, 4\pi \frac{1}{R^2}\delta(R)\frac{1}{4\pi}$$

1

Sphere with surface potential

$$V_0(a,\theta,\phi) = k\cos(2\theta)$$

No charge is specified $\rightarrow$ need laplace equation

$$\nabla^2 V = 0 \quad \text{or}$$

$$\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial}{\partial R}V\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) = 0$$

We assume separability

$$V(R,\theta) = \Gamma(R)\Theta(\theta)$$

No dependence on ϕ, azimuthal symmetry

General solution with no singularity in angular coordinates is

$$V_n(R,\theta) = \left\{A_n R^n + B_n R^{-(n+1)}\right\} P_n(\cos\theta)$$

Inside sphere, $R<a$

No charge

$$V_n^{\mathrm{I}}(R,\theta) = A_n R^n P_n(\cos\theta)$$

i.e. here $B_n = 0$

Outside sphere, $R>a$

$$V_n^{\mathrm{II}}(R,\theta) = B_n R^{-(n+1)} P_n(\cos\theta)$$

2

Since $A_n = 0$ to fulfill

$$V^{\mathrm{II}}(R,\theta) \underset{R\to\infty}{\longrightarrow} 0$$

The common B.C. as $R\to\infty$ for spherical coordinates

Boundary condition, $R=a$

We have $V_0(a,\theta,\phi) = k\cos(2\theta)$

$$\cos(2\theta) = 2\cos^2(\theta) - 1$$

And

$$P_0(\theta) = 1$$

$$P_1(\theta) = \cos\theta$$

$$P_2(\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$

$\rightarrow$

$$\frac{4}{3}P_2(\theta) - \frac{1}{3}P_0(\theta)$$

$$= 2\cos^2(\theta) - 1 = \cos(2\theta)$$

$$\rightarrow V_0(a,\theta) = \frac{k}{3}\left\{4P_2(\theta) - P_0(\theta)\right\}$$

To fulfill the B.C. the solution inside and outside the sphere will only have these two components with $n=2$ and $n=0$

The surface charge distribution is a combination of a monopole ($n=0$) and quadrupole ($n=2$)

3

I $R<a$

$$A_2 R^2 P_2(\cos\theta) + A_0 R^0 P_0(\cos\theta)$$

II $R>a$

$$B_2 R^{-3} P_2(\cos\theta) + B_0 R^{-1} P_0(\cos\theta)$$

$$V^{\mathrm{I}}(a,\theta) = V^{\mathrm{II}}(a,\theta) = \frac{k}{3}\left\{4P_2(\theta) - P_0(\theta)\right\}$$

$$\rightarrow A_2 a^2 = \frac{4k}{3}, \quad B_2 a^{-3} = \frac{4k}{3}$$

$$A_0 = -\frac{k}{3}, \quad B_0 a^{-1} = -\frac{k}{3}$$

or

$$A_2 = \frac{4k}{3a^2}$$

$$A_0 = -\frac{k}{3}$$

$$B_2 = \frac{4ka^3}{3}$$

$$B_0 = -\frac{ka}{3}$$

notice different dimensions

4

The solutions are thus

I $R<a$:

$$V^{\mathrm{I}}(R,\theta) = \frac{k}{3}\left\{4\left(\frac{R}{a}\right)^2 P_2(\cos\theta) - \left(\frac{R}{a}\right)^0 P_0(\cos\theta)\right\}$$

II $R>a$:

$$V^{\mathrm{II}}(R,\theta) = \frac{k}{3}\left\{4\left(\frac{a}{R}\right)^3 P_2(\cos\theta) - \left(\frac{a}{R}\right) P_0(\cos\theta)\right\}$$

quadrupole

With the characteristic $\frac{1}{R^3}$ decay as $R\to\infty$

monopole with the usual $\frac{1}{R}$ asymptotics as $R\to\infty$

Here is no dipole moment, that would decay as $\frac{1}{R^2}$

5

The surface charge can be derived from the electric field

$$\bar{E} = -\bar{\nabla} V(R,\theta)$$

$$= -\hat{a}_R \partial_R V(R,\theta) - \hat{a}_\theta \frac{1}{R}\partial_\theta V(R,\theta)$$

We need only the normal part to the surface $\Rightarrow$ the radial part

$$E_R = -\partial_R V(R,\theta)$$

and as $\epsilon^{(I)} = \epsilon^{(II)} = \epsilon_0$ we have here

$$\boxed{\epsilon_0 \hat{a}_R \cdot \left(\bar{E}^{(II)}(a) - \bar{E}^{(I)}(a)\right) = \rho_s(a,\theta)}$$

$$\rho_s(\theta) = \epsilon_0\left(E_R^{(II)}(a,\theta) - E_R^{(I)}(a,\theta)\right)$$

$$= -\epsilon_0 \partial_R V^{(II)}(R,\theta)\Big|_{R=a^+}$$

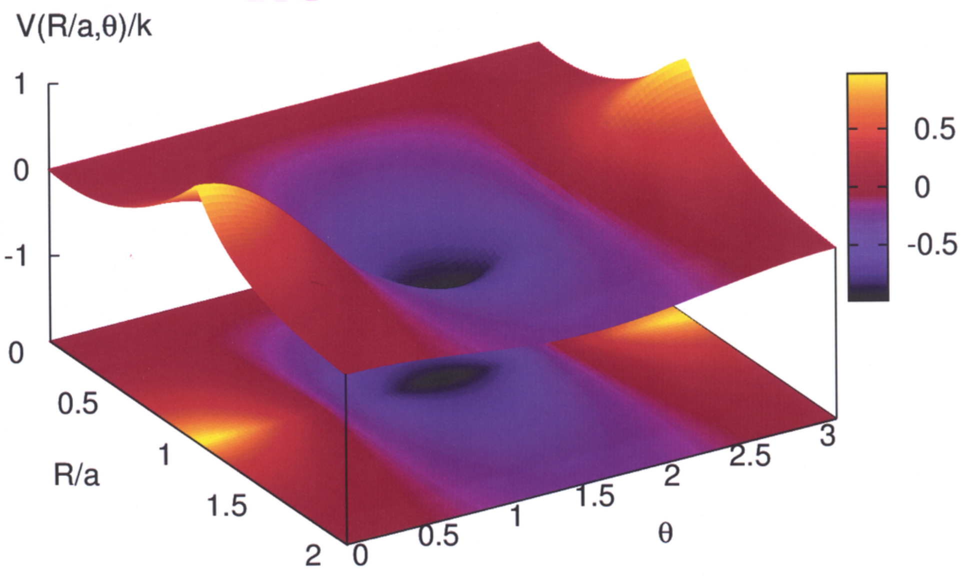
$$+\epsilon_0 \partial_R V^{(I)}(R,\theta)\Big|_{R=a^-}$$

$$= \epsilon_0 \frac{4k}{a} P_2(\cos\theta) - \epsilon_0 \frac{k}{3a} P_0(\cos\theta)$$

$$+ \frac{\epsilon_0 8k}{3a} P_2(\cos\theta) - \frac{\epsilon_0 k}{3a} P_0(\cos\theta)$$

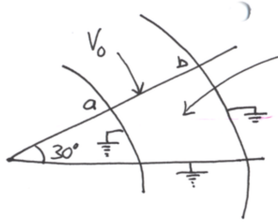
$$= \frac{\epsilon_0 k}{a}\left[\frac{20}{3} P_2(\cos\theta) - \frac{2}{3} P_0(\cos\theta)\right]$$

6



4

(2)



Finna hér V, $\bar{E}$ og $\bar{D}$

Sivalnings samhverfa

$$\frac{1}{r}\partial_r\left(r\partial_r V(r,\varphi)\right) + \frac{1}{r^2}\partial_\varphi^2 V(r,\varphi) = 0$$

Jafnan er aðgreinanleg

$$V(r,\varphi) = R(r)\Phi(\varphi)$$

$$\frac{d^2\Phi}{d\varphi^2} + k^2 \Phi(\varphi) = 0$$

$$\frac{d^2}{dr^2}R + \frac{1}{r}\frac{d}{dr}R - \frac{k^2}{r^2}R = 0$$

$k \neq 0$, því loysnin

$R(r) \sim C_1 \ln(r) + C_2$

getur ekki uppfylt jaðarst. $R(a) = 0$ og $R(b) = 0$

5

Loysnin fyrir $k^2 > 0$ er ekki góð því við getum ekki uppfylt jaðarst. með $R(r) = A_n r^n + B_n r^{-n}$

Skoðum $k^2 < 0$

Setjum $k^2 = -\kappa^2$ með $\kappa \in \mathbb{R}$ þá fæst

$$\Phi(\varphi) = A_\kappa \cosh(\kappa\varphi) + B_\kappa \sinh(\kappa\varphi)$$

$$R(r) = C_\kappa r^{i\kappa} + D_\kappa r^{-i\kappa}$$

en $r^{i\kappa} = e^{i\kappa \ln r}$

$$\rightarrow R(r) = C_\kappa \cos(\kappa \ln(r)) + D_\kappa \sin(\kappa \ln(r))$$

fall með sveiflum, því átti að vera hægt að uppfylla jaðarst. með því

(6)

Reynum $C_K = 0$, og lausn

$$V_K(r,\varphi) = B_K \sin\left(K \ln\left(\frac{r}{a}\right)\right) \sinh(K\varphi)$$

$$V_K(b,\varphi) = 0 \quad \longrightarrow \quad \sin\left(K \ln\left(\frac{b}{a}\right)\right) = 0$$

eða

$$K \ln\left(\frac{b}{a}\right) = n\pi, \quad n = 1, 2, 3, \ldots$$

$$K = \frac{n\pi}{\ln\left(\frac{b}{a}\right)}$$

Verðum að uppfylla líka

$$V_K(r, 30^\circ) = V_0 \quad \text{fyrir öll } r$$

$$= V_K\left(r, \frac{\pi}{6}\right)$$

(7)

$$V(r,\varphi) = \sum_{n=1}^{\infty} B_{K_n} \sin\left(K_n \ln\left(\frac{r}{a}\right)\right) \sinh(K_n\varphi)$$

$$V\left(r, \frac{\pi}{6}\right) = \sum_{n=1}^{\infty} B_{K_n} \sin\left(K_n \ln\left(\frac{r}{a}\right)\right) \sinh\left(K_n \frac{\pi}{6}\right) = V_0$$

$$V_0 \int_a^b r\,dr \sin\left(K_p \ln\left(\frac{r}{a}\right)\right) = \sum_{n=1}^{\infty} \sinh\left(K_n \frac{\pi}{6}\right) B_{K_n} \int_a^b r\,dr \sin\left(K_n \ln\left(\frac{r}{a}\right)\right) \cdot \sin\left(K_p \ln\left(\frac{r}{a}\right)\right)$$

$$= \sinh\left(K_p \frac{\pi}{6}\right) B_{K_p} \int_a^b r\,dr \sin^2\left(K_p \ln\left(\frac{r}{a}\right)\right)$$

(8)

$$\longrightarrow B_{K_p} = \frac{V_0 \int_a^b r\,dr \sin\left(K_p \ln\left(\frac{r}{a}\right)\right)}{\sinh\left(K_p \frac{\pi}{6}\right) \int_a^b r\,dr \sin^2\left(K_p \ln\left(\frac{r}{a}\right)\right)}$$

$$V(r,\varphi) = \sum_{n=1}^{\infty} B_{K_n} \sin\left(K_n \ln\left(\frac{r}{a}\right)\right) \sinh(K_n\varphi)$$

$$\bar{E} = -\bar{\nabla} V(r,\varphi) = \hat{a}_r \frac{\partial V}{\partial r} + \hat{a}_\varphi \frac{\partial V}{r\partial\varphi}$$

$$= -\sum_{n=1}^{\infty} B_{K_n} \frac{K_n \cos\left(K_n \ln\left(\frac{r}{a}\right)\right)}{r} \sinh(K_n\varphi) \cdot \hat{a}_r$$

$$- \sum_{n=1}^{\infty} B_{K_n} \sin\left(K_n \ln\left(\frac{r}{a}\right)\right) \cosh(K_n\varphi) K_n \frac{\hat{a}_\varphi}{r}$$

$$\bar{D} = \epsilon_0 \bar{E}$$

(10)

Þetta dæmi er nokkuð óvenjulegt, því hér þarf „lotubundin"-lausnar föll í r-áttina í stað φ-áttar. Þetta er ekki heppilegt prófdæmi, en það er gott til þess að minna okkur á að k-in í $k_r^2 + k_\varphi^2 = 0$ geta hvort fyrir sig verið þver- eða raun tölur, en ekki samtímis sömu tegundar. Það þarf að sannreyna að $\sin(K_n \ln(\frac{r}{a}))$ sé hornréttur fullkominn grunnur. Ég athugaði heildin, en sá að töflugildi fyrir heildin getur ekki verið alls kostar rétt.

(11)

Evaluation

$$\int_a^b r\,dr\,\sin\left(k_p \ln\left(\frac{r}{a}\right)\right) = \left. \frac{r^2\left[2\sin\left\{k_p\ln\left(\frac{r}{a}\right)\right\} - k_p\cos\left\{k_p\ln\left(\frac{r}{a}\right)\right\}\right]}{k_p^2+4}\right|_a^b$$

$$= \frac{b^2\left[2\sin\left\{k_p\ln\left(\frac{b}{a}\right)\right\} - k_p\cos\left\{k_p\ln\left(\frac{b}{a}\right)\right\}\right]}{k_p^2+4}$$

$$+ \frac{a^2\, k_p \cos\{0\}}{k_p^2+4}$$

$$= \frac{b^2\left[2\sin\{p\pi\} - \frac{p\pi}{\ln\left(\frac{b}{a}\right)}\cos\{p\pi\}\right] + a^2\frac{p\pi}{\ln\left(\frac{b}{a}\right)}}{\frac{p^2\pi^2}{(\ln(b/a))^2}+4} =$$

(12)

$$= \frac{a^2k_p - b^2k_p\cos(p\pi)}{k_p^2+4} = \frac{k_p\left\{a^2-b^2(-1)^p\right\}}{k_p^2+4}$$

$$\int_a^b r\,dr\,\sin^2\left(k_p\ln\left(\frac{r}{a}\right)\right) = \left.\frac{r^2\left[-k_p\sin\left\{2k_p\ln\left(\frac{r}{a}\right)\right\} - \cos\left\{2k_p\ln\left(\frac{r}{a}\right)\right\} + k_p^2+1\right]}{4(k_p^2+1)}\right|_a^b$$

$$= \frac{b^2\left[-k_p\sin\{2p\pi\} - \cos\{2p\pi\} + k_p^2+1\right]}{4(k_p^2+1)} + \frac{a^2 - a^2k_p^2 - a^2}{4(k_p^2+1)}$$

$$= \frac{b^2\left[k_p - 1 + k_p^2+1\right] + a^2\left[1 - k_p^2 - 1\right]}{4(k_p^2+1)} = \frac{b^2k_p(1+k_p) - a^2k_p^2}{4(k_p^2+1)}$$

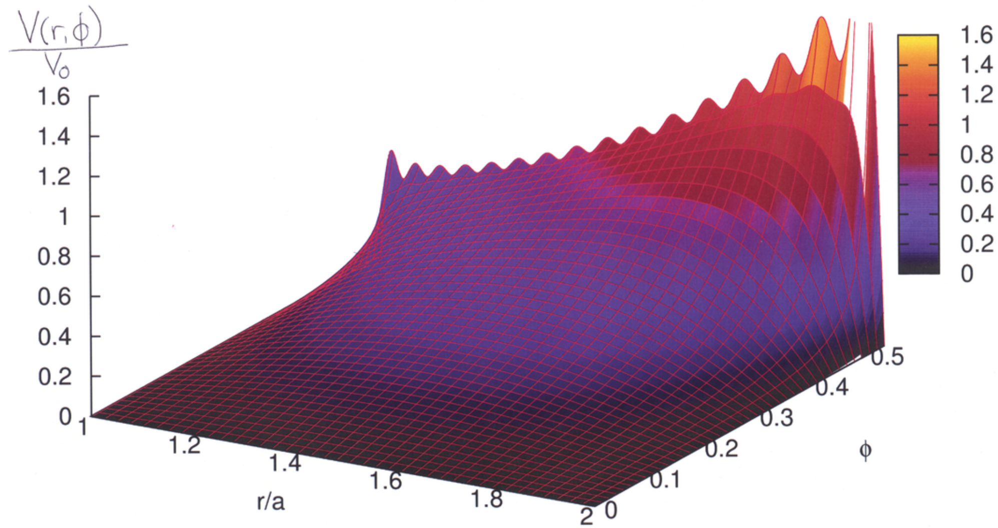
(13)

$$B_{k_p} = V_0\left\{\frac{k_p\left[a^2-b^2(-1)^p\right]\cdot 4(k_p^2+1)}{\left[b^2k_p(1+k_p)-a^2k_p^2\right]\cdot(k_p^2+4)\cdot\sinh\left(k_p\frac{\pi}{6}\right)}\right\}$$

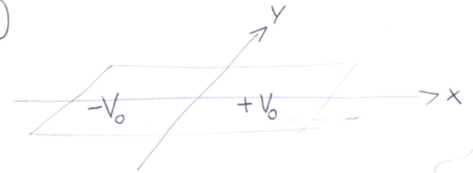
(14)

p_{max}=32

$b = 2a$



①



Infinite half planes at different potentials

The problem is almost solved as soon as we realize that this problem is best described in Cylinder coordinates. We measure r from the y-axis and ϕ from the positive x-half plane

Nothing breakes the r-symmetry (1)

just think what changes as you move the observation point away from the plane

Only ϕ- dependence

$$\nabla^2 V(\phi) = 0$$

$$\hookrightarrow \quad \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} = 0$$

with solution

$$V(\phi) = A\phi + B$$

②

Above plane, $z>0$ Ⓘ $0<\phi<\pi$

$$\left.\begin{array}{l} V^{I}(0) = +V_0 \\ V^{I}(\pi) = -V_0 \end{array}\right\} \to \quad \begin{array}{l} B = +V_0 \\ A\pi + V_0 = -V_0 \;, \quad A = -\frac{2V_0}{\pi} \end{array}$$

$$\to V^{I}(\phi) = -\frac{2V_0}{\pi}\phi + V_0 = V_0\left\{1 - \frac{2}{\pi}\phi\right\}$$

Under plane, $z<0$ Ⓘⓘ $\pi < \phi < 2\pi$

$$\left.\begin{array}{l} V^{II}(\pi) = -V_0 \\ V^{II}(2\pi) = +V_0 \end{array}\right\} \to \quad \left.\begin{array}{l} A\pi + B = -V_0 \\ A2\pi + B = +V_0 \end{array}\right\} \to \begin{array}{l} A = \frac{2V_0}{\pi} \\ B = -3V_0 \end{array}$$

$$V^{II}(\phi) = \frac{2V_0}{\pi}\phi - 3V_0 = V_0\left\{\frac{2}{\pi}\phi - 3\right\}$$

③

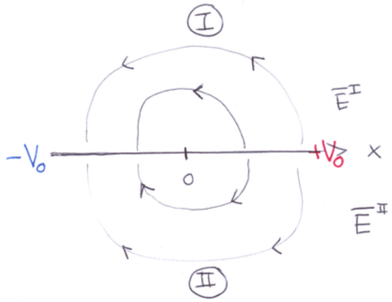
b) Determine the surface charge

We need the electric field

$$\bar{E} = -\bar{\nabla} V(\phi) = -\hat{a}_\phi \frac{1}{r}\frac{\partial}{\partial \phi}$$

$$\to \quad \bar{E}^{I}(r) = \hat{a}_\phi \frac{2V_0}{\pi r}$$

$$\bar{E}^{II}(r) = -\hat{a}_\phi \frac{2V_0}{\pi r}$$



Only normal $\bar{E}$ at the planes

$|\bar{E}|$ depends on r

At the conducting surface

$$\rho_s = \epsilon_0 E_n$$

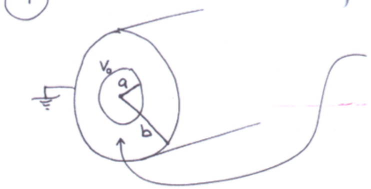
Right plane, $x>0$

$$\rho_s(r) = \frac{2V_0\epsilon_0}{\pi r}$$

left plane, $x<0$

$$\rho_s(r) = -\frac{2V_0\epsilon_0}{\pi r}$$

(1) ①



$$\sigma = \frac{m}{r} + k$$

Spennumunur V_0 milli sívalninga

Finna R, $\bar{J}$ og $\bar{E}$

Ef heildarstreymurin er I (enn ópekt) þá er

$$\bar{J} = \frac{I}{2\pi r L}\hat{a}_r$$

Við hövum $\bar{J} = \sigma \bar{E}$ ella $\bar{E} = \frac{\bar{J}}{\sigma} = \frac{I}{2\pi L(m+kr)}\hat{a}_r$

$$V_0 = -\int_b^a \bar{E}\cdot d\bar{l} = -\int_b^a \frac{I\,dr}{2\pi L(m+kr)} = -\frac{I}{2\pi L k}\ln\left\{\frac{m+ka}{m+kb}\right\}$$

og því

$$R = \frac{V_0}{I} = \frac{1}{2\pi L k}\ln\left\{\frac{m+kb}{m+ka}\right\}$$

(2)

Eins til det før leidar-straumen

$$I = \frac{V_0}{R} = 2\pi L k V_0 \left\{ \ln\left(\frac{m+kb}{m+ka}\right) \right\}^{-1}$$

og þui

$$\bar{E} = \frac{I}{2\pi L(m+kr)}\hat{a}_r = \frac{k V_0 \hat{a}_r}{(m+kr)\ln\left(\frac{m+kb}{m+ka}\right)}$$

og

$$\bar{J} = \sigma \bar{E} = \frac{k V_0 \hat{a}_r}{r \ln\left(\frac{m+kb}{m+ka}\right)}$$

Surface charge

$$\bar{E} = \frac{k V_0 \hat{a}_r}{(m+kr)\ln\left(\frac{m+kb}{m+ka}\right)}$$

and for a conducting surface we have

$$\rho_s = \epsilon_0 \hat{n}\cdot\bar{E}$$

$r = a$

$$\hat{n} = \hat{a}_r$$

$$\rightarrow \rho_s(a) = \frac{\epsilon_0 k V_0}{(m+ka)\ln\left(\frac{m+kb}{m+ka}\right)}$$

$r = b$, $\hat{n} = -\hat{a}_r$

$$\rightarrow \rho_s(b) = -\frac{\epsilon_0 k V_0}{(m+kb)\ln\left(\frac{m+kb}{m+ka}\right)}$$

not the same surface density and it does also not diminish simply with the geometry

-> there must be space charge between the cylinders

volume charge $a < r < b$

We have

$$V(r) = \frac{I}{2\pi L k}\ln\left\{\frac{m+kb}{m+kr}\right\}$$

$$= \frac{V_0 \ln\left\{\frac{m+kb}{m+kr}\right\}}{\ln\left\{\frac{m+kb}{m+ka}\right\}}$$

We could use

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2 V(r) = \frac{1}{r}\partial_r\left(r\,\partial_r V(r)\right)$$

$$\nabla^2 V = \frac{V_0}{\ln\left\{\frac{m+kb}{m+ka}\right\}}\,\frac{-km}{(kr+m)^2 r}$$

$$\rightarrow \rho(r) = +\frac{\epsilon_0 V_0}{\ln\left\{\frac{m+kb}{m+ka}\right\}}\,\frac{km}{(kr+m)^2 r}$$

the volume charge

$$\lim_{k\to 0} R(k) = \lim_{k\to 0}\frac{1}{2\pi L k}\ln\left\{\frac{m+kb}{m+ka}\right\}$$

$$= \frac{b-a}{m}\cdot\frac{1}{2\pi L}$$

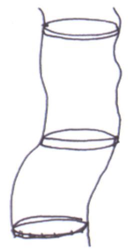
(1)

Viðbót við skiladæmi

Um einsleitan leiðara með föstu þversniði gildir

$$R = \frac{l}{\sigma A}$$

Ef við getum „snitt" niður leiðara með breytilegt þversnið og eðlisleiðni þ.a. straumþéttleikinn sé jafn innan hverrar sneiðar, sem þýðir t.d. að J er hornrétt á sneiðarnar og σ getur aðeins breyst þvert á sneið þá má leggja saman viðnám sneiðanna



$$R = \sum_{i=1}^{n} R_i = \sum_{i=1}^{n} \frac{dl_i}{\sigma_i A_i}$$

(2)

Sem yfirfæra má í heildi (ferilheildi!)

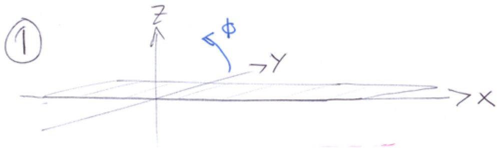
$$R = \int_C \frac{dl}{\sigma A} \qquad (*)$$

Því fengist

$$R = \int_a^b \frac{dr}{(m+kr)2\pi L} = \frac{1}{2\pi L k} \ln\left\{\frac{m+kb}{m+ka}\right\}$$

fyrir sívalnings leiðara. Hér er mikilvægt að σ er aðeins fall af r, og að (almennt) R_i leggjast saman. Íhugið hvernig heildið væri sett upp fyrir G. Hafið í huga hve heildisjafnan $(*)$ er sterklega takmörkuð. Myndin mín af leiðara er dæmi um leiðara sem ekki væri hægt að nota $(*)$ á!

(1)



long strip with current density $\bar{J}_s = \hat{a}_x J_{so}$

Find the magnetic flux density $\bar{B}$ at any point.

The strip is composed of wires with width dy' and current $dI = J_{so} \cdot dy'$. Each one gives

$$d\bar{B}_\phi = \hat{a}_\phi \frac{\mu_0 dI}{2\pi r}$$

where ϕ is measured in the y-z-plane from y-axis, with r in the same plane

$$\hat{a}_\phi = (-\hat{a}_y \sin\phi + \hat{a}_z \cos\phi)$$

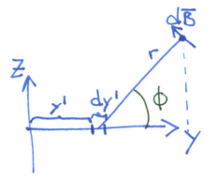
$$r = \sqrt{((y-y')^2 + z^2)}$$

$z'=0$, and $\bar{B}$ is independent of x ← long strip

(2)

We sum up the contribution from each wire using that

$$\hat{a}_\phi = (-\hat{a}_y \sin\phi + \hat{a}_z \cos\phi)$$



$$= \left(-\hat{a}_y \frac{z}{r} + \hat{a}_z \frac{y-y'}{r}\right)$$

$$B_z = -\frac{\mu_0 J_{so}}{2\pi} \int_0^w dy' \frac{(y-y')}{r^2} = -\frac{\mu_0 J_0}{2\pi} \int_0^w dy' \frac{(y-y')}{(y-y')^2 + z^2}$$

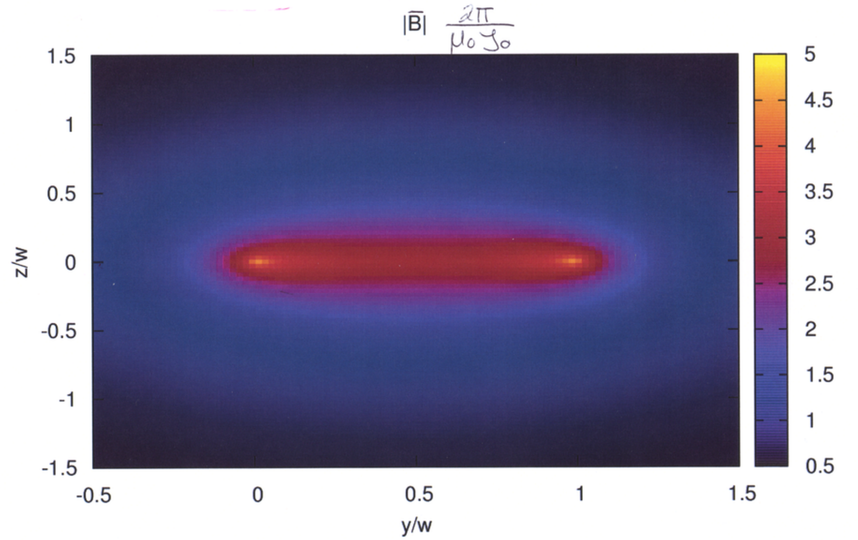
$$B_y = +\frac{\mu_0 J_{so} z}{2\pi} \int_0^w dy' \frac{1}{r^2} = \frac{\mu_0 J_{so} z}{2\pi} \int_0^w dy' \frac{1}{(y-y')^2 + z^2}$$

③

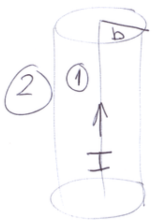
$$B_z = -\frac{\mu_0 J_0}{4\pi}\left\{\ln\left[\frac{z^2+y^2}{z^2+(y-w)^2}\right]\right\}$$

$$B_y = \frac{\mu_0 J_0}{2\pi}\left\{\arctan\left[\frac{y}{z}\right]-\arctan\left[\frac{y-w}{z}\right]\right\}$$

④



⑤



In Ex 6.1 Ampères Law is used to derive

$$\bar{B}_1 = \hat{a}_\phi \frac{\mu_0 r I}{2\pi b^2} \qquad r<b$$

$$\bar{B}_2 = \hat{a}_\phi \frac{\mu_0 I}{2\pi r} \qquad r>b$$

Can we find directly $\bar{A}$?

We know $\nabla^2 \bar{A} = -\mu_0 \bar{J}$

here $\bar{J}$ only has $\hat{a}_z$ component, so we need the z-component of the vector laplacian in Cilynder coordinates

$$\boxed{\left\{\frac{\partial^2}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}+\frac{\partial^2}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\right\}A_z = -\mu_0 J_z}$$

⑥

We remember also

$$B_\phi = \left(\frac{\partial A_r}{\partial z}-\frac{\partial A_z}{\partial r}\right)$$

$$J_z = J_z(r) = \begin{cases} J_0 & \text{if } r<b \\ 0 & \text{if } r>b \end{cases}$$

depends only on r

$\rightarrow A_z = A_z(r)$ and

$$\left\{\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right\}A_z(r) = -\mu_0 J_z(r)$$

<u>r<b</u>

$$A_z^{I}(r) = C_3\left(\frac{r}{b}\right)^2 + C_1$$

<u>r>b</u>

$$A_z^{I}(r) = C_4 \ln\left(\frac{r}{b}\right) + C_2$$

7

$r<b$

To fulfil the eq.

$$\left\{\partial_{r^2}^2 + \frac{1}{r}\partial_r\right\} A_z^{II}(r) = \frac{4C_3}{b^2} = -\mu_0 J_z = -\mu_0\left(\frac{I}{b^2\pi}\right)$$

$$\rightarrow \boxed{C_3 = -\frac{\mu_0 I}{4\pi}}$$

We need continuity in $r=b$

$$A_z^{II}(b) = A_z^{I}(b)$$

$$\rightarrow \boxed{-\frac{\mu_0 I}{4\pi} + C_1 = C_2}$$

and can not be determined further due to the gauge freedom

$$\rightarrow A_z(r) = \begin{cases} -\frac{\mu_0 I}{4\pi}\left(\frac{r}{b}\right)^2 + C_1 & r<b \\ -\frac{\mu_0 I}{4\pi}\ln\left(\frac{r}{b}\right) + C_2 & r>b \end{cases}$$

to give correct $\overline{B}$ outside, $r>b$ we need

$$\boxed{C_4 = -\frac{\mu_0 I}{4\pi}}$$

8

Of course one might directly integrate

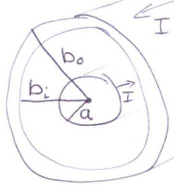
$$B_\phi = \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)$$

here, but I wanted to use the opportunity to look at the vector "$\nabla^2\overline{A}$".

This shows how convenient the integral theorems are, Gauß law and Ampères law.

1

① Coaxial transmission line, find the inductance per unit length. We consider current I, and find $\overline{B}$ in all regions by using Ampères law



$$\oint_C \overline{B}\cdot d\overline{l} = \mu_0 I$$

Cylinder symmetry

$\rightarrow \overline{B} = \hat{a}_\phi B$ in all regions.

$$2\pi r B = \mu_0 I_{enc}$$

$0 \leq r \leq a$

$$I_{enc}(r) = I\left(\frac{r}{a}\right)^2$$

$$\rightarrow \overline{B} = \hat{a}_\phi \frac{\mu_0 I r}{2\pi a^2}$$

$a \leq r \leq b_i$ Here I is a constant

$$\rightarrow \overline{B} = \hat{a}_\phi \frac{\mu_0 I}{2\pi r}$$

2

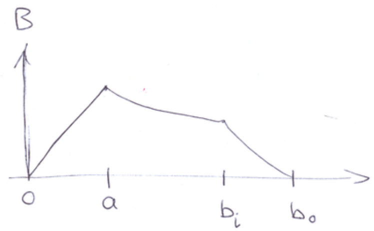
$b_i \leq r \leq b_o$

outer conductor

core conductor

$$I_{enc}(r) = -\left(\frac{r^2 - b_i^2}{b_o^2 - b_i^2}\right)I + I$$

$$= I\left(\frac{b_o^2 - r^2}{b_o^2 - b_i^2}\right) \rightarrow \overline{B} = \hat{a}_\phi\left(\frac{b_o^2 - r^2}{b_o^2 - b_i^2}\right)\frac{\mu_0 I}{2\pi r}$$



for $r > b_o$ $B = 0$ since then the current I through C is 0.

Now we use the energy of the field to calculate L

$$W_m = \frac{1}{2}\int_{V'} dv' \frac{B^2}{\mu_0}$$

(3)

$$W_m^{(I)} = \frac{1}{2\mu_0}\int_0^a 2\pi r\,dr\, B^2(r) = \frac{2\pi}{2\mu_0}\frac{\mu_0^2 I^2}{(2\pi)^2 a^4}\int_0^a dr\, r^3$$

$$= \frac{\mu_0 I^2}{16\pi}$$

$$W_m^{(II)} = \frac{2\pi}{2\mu_0}\frac{\mu_0^2 I^2}{(2\pi)^2}\int_a^{b_i}\frac{dr}{r} = \frac{\mu_0 I^2}{4\pi}\ln\left(\frac{b_i}{a}\right)$$

$$W_m^{(III)} = \frac{2\pi}{2\mu_0}\frac{\mu_0^2 I^2}{(2\pi)^2}\int_{b_i}^{b_o}\frac{dr}{r}\left(\frac{b_o^2-r^2}{b_o^2-b_i^2}\right)^2$$

$$= \frac{\mu_0 I^2}{2}\frac{1}{(b_o^2-b_i^2)^2}\left\{b_o^4\ln\left(\frac{b_o}{b_i}\right) - \frac{3b_o^4}{4} + b_i^2 b_o^2 - \frac{b_i^4}{4}\right\}$$

(4)

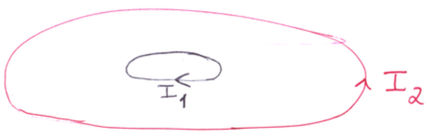
Compare to $W_m = \frac{1}{2}LI^2$ to get

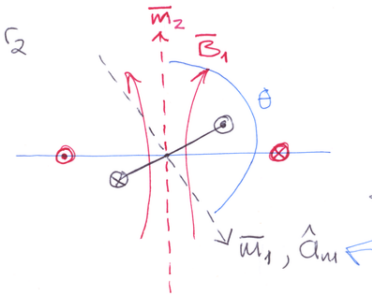
$$L = \frac{2W_m}{I^2} = \left\{\frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln\left(\frac{b_i}{a}\right) + \frac{\mu_0}{(b_o^2-b_i^2)^2}\left[b_o^4\ln\left(\frac{b_o}{b_i}\right) - \frac{3b_o^4}{4} + b_i^2 b_o^2 - \frac{b_i^4}{4}\right]\right\}$$

on length L

(5)

$r_1 << r_2$





Seen on edge out of scale

Example 6-6 in book gives us the on axis (z-axis) field

$$\bar{B}_2 = \hat{a}_z \frac{\mu_0 I_2 r_2^2}{2(z+r_2^2)^{3/2}}$$

$$\approx \hat{a}_z \frac{\mu_0 I_2}{2r_2}$$

at $z=0$, and valid for $r << r_2$ → for all of ring 1

Torque on the small ring 1

$$\bar{T}_1 = \bar{m}_1 \times \bar{B}_2$$

(assuming B_2 is constant over the area of ring 1)

$$\bar{m}_1 = \hat{a}_{m1} I_1 \pi r_1^2$$

I include the current direction in the direction of $\hat{a}_m$

(6)

$$\bar{T}_1 = (I_1\pi r_1^2)\left(\frac{\mu_0 I_2}{2r_2}\right)(\hat{a}_{m1}\times\hat{a}_z)$$

$$= \frac{\mu_0 I_1 I_2 \pi r_1^2}{2r_2}(\hat{a}_m\times\hat{a}_z)$$

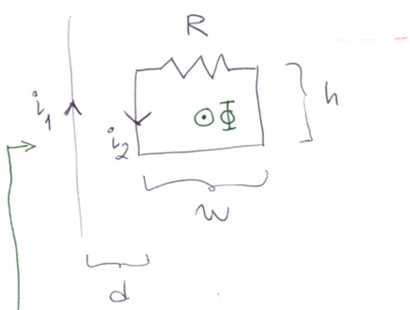
{Good $\sin\theta > 0$ for $0 \le \theta \le \pi$}

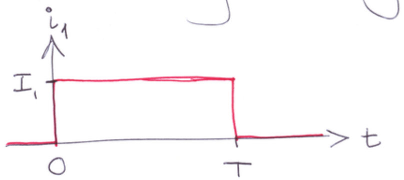
$$|\bar{T}_1| = \frac{\mu_0 I_1 I_2 \pi r_1^2}{2r_2}\sin\theta$$

$\bar{T}$ wants to align $\bar{m}_1$ and $\bar{m}_2$ as I have defined them.

P7.3 (1)

We define the currents as shown in the diagram. Square pulse is sent along the straight wire





a) find the induced current i_2 in the loop with L

b) find the energy dissipated in R if $T >> L/R$

i_1 and i_2 create flux in opposite directions

We will use Faraday's law of induction, but we need to define the flux through the loop:

$$\Phi_2 = \Phi_{12} + \Phi_{22}$$

$$= -L_{12} i_1 + L i_2$$

I define positive flux out of the page

(2)

$$\oint_{C_2} \bar{E} \cdot d\bar{l} = -\frac{d\Phi_2}{dt} = +L_{12}\frac{di_1}{dt} - L\frac{di_2}{dt}$$

$$= R i_2$$

$$\rightarrow \quad R i_2 = +L_{12}\frac{di_1}{dt} - L\frac{di_2}{dt}$$

$$\rightarrow \quad L_{12}\frac{di_1}{dt} - L\frac{di_2}{dt} - R i_2 = 0 \qquad (*)$$

L is given, but we need to find L_{12}

$$L_{12} = \frac{\Phi_{12}}{i_1} = \frac{h}{i_1}\int_d^{d+w} dr\, B_{12} = \frac{h}{i_1}\int_d^{d+w} \frac{\mu_0 i_1}{2\pi r} dr = \frac{\mu_0 h}{2\pi}\ln\left(1+\frac{w}{d}\right)$$

(3)

a) We need to evaluate

Heaviside step function

$$\frac{di_1}{dt} = I_1 \frac{d}{dt}\left\{\theta(t)\theta(T-t)\right\}, \qquad \theta(t) = \begin{cases} 0 & \text{if } t<0 \\ 1 & \text{if } t>0 \end{cases}$$

We have to use

$$\frac{d\theta(t)}{dt} = \delta(t) \leftarrow \text{Dirac-Delta function}$$

$$\rightarrow \quad \frac{di_1}{dt} = I_1\left\{\delta(t)\,\theta(T-t) - \theta(t)\,\delta(t-T)\right\}$$

$$= I_1\left\{\delta(t) - \delta(t-T)\right\}$$

So Faraday's law (*) gives us the differential equation

$$\boxed{\frac{di_2}{dt} + \frac{R}{L} i_2 = \frac{L_{12}}{L} I_1\left\{\delta(t) - \delta(t-T)\right\}}$$

(4)

Inhomogeneous first order equation of the form

$$y' + p(t)y = q(t)$$

has the solution

$$y(t) = y(t_0)e^{-P(t)} + e^{-P(t)}\int_{t_0}^{t} e^{P(s)} q(s)\,ds$$

Where

$$P(t) = \int_{t_0}^{t} ds\, p(s)$$

We have here

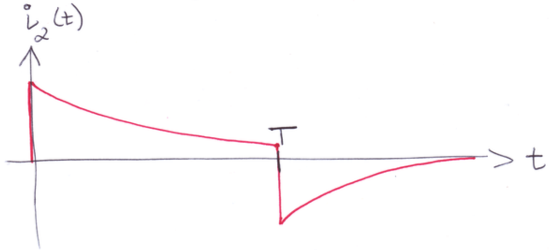
$$P(t) = \int_0^t ds\,\frac{R}{L} = \frac{Rt}{L}$$

and our solution of the inhomogeneous differential equation is

$$i_2(t) = i_2(0)\, e^{-\frac{Rt}{L}} + \frac{L_{12} I_1}{L}\left\{ e^{-\frac{Rt}{L}} - \theta(t-T)\, e^{-\frac{R}{L}(t-T)} \right\}$$

Use $i_2(0) = 0$

$$\rightarrow \quad i_2(t) = \frac{L_{12} I_1}{L}\left\{ e^{-\frac{Rt}{L}} - \theta(t-T)\, e^{-\frac{R}{L}(t-T)} \right\}$$



If $T \gg \frac{L}{R} \quad \rightarrow \quad e^{-\frac{RT}{L}} \ll 1$

and the solution after $t > T$ is thus

$$i_2(t) \approx -\frac{L_{12} I_1}{L}\, e^{-\frac{R}{L}(t-T)}$$

asymptotic form

b) The energy dissipated in R of $T \gg \frac{L}{R}$

$$E_{diss} = \int_0^\infty dt\; i_2^2(t) \cdot R \;\simeq\; \left(\frac{L_{12} I_1}{L}\right)^2 R \int_0^\infty e^{-\frac{2Rt}{L}}\, dt + \left(\frac{L_{12} I_1}{L}\right)^2 R \int_T^\infty e^{-\frac{2R(t-T)}{L}}\, dt$$

$$\approx 2\left(\frac{L_{12} I_1}{L}\right)^2 R \int_0^\infty dt\, e^{-\frac{2Rt}{L}} = \boxed{\frac{1}{L}\left(L_{12} I_1\right)^2}$$

P7-14

Línulegt einsleitt á svæðum efni

Byrjum með Ampère-Maxwell (A-M)

$$\bar\nabla \times \bar H = \bar J + \frac{\partial}{\partial t}\bar D, \qquad \bar H = \frac{1}{\mu}\bar B, \qquad \bar D = \epsilon \bar E$$

$$\rightarrow \quad \bar\nabla \times \left(\frac{\bar B}{\mu}\right) = \bar J + \epsilon \frac{\partial}{\partial t}\bar E$$

Notum nú $\quad \bar B = \bar\nabla \times \bar A \quad$ og $\quad \bar E = -\bar\nabla V - \frac{\partial}{\partial t}\bar A$

til þess að umskrifa A-M

$$\bar\nabla \times \left(\frac{1}{\mu}\bar\nabla \times \bar A\right) = \bar J + \epsilon\left(-\frac{\partial^2}{\partial t^2}\bar A\right) - \epsilon \bar\nabla\left(\frac{\partial V}{\partial t}\right)$$

eða

$$\mu\bar\nabla \times \left(\frac{1}{\mu}\bar\nabla \times \bar A\right) = \mu\bar J - \epsilon\mu\frac{\partial^2}{\partial t^2}\bar A - \epsilon\mu\bar\nabla\left(\frac{\partial V}{\partial t}\right)$$

notum Lorentz-kvörðun

$$\bar\nabla \cdot (\epsilon \bar A) + \mu\epsilon^2 \frac{\partial}{\partial t} V = 0 \quad \rightarrow \quad -\frac{\partial}{\partial t}V = \frac{1}{\mu\epsilon^2}\bar\nabla \cdot (\epsilon \bar A)$$

$$\mu\bar\nabla \times \left(\frac{1}{\mu}\bar\nabla \times \bar A\right) = \mu\bar J - \epsilon\mu\frac{\partial^2}{\partial t^2}\bar A + \epsilon\mu\bar\nabla\left[\frac{1}{\mu\epsilon^2}\bar\nabla \cdot (\epsilon \bar A)\right]$$

sem gefur bylgjujöfnuna

$$\boxed{-\bar\nabla \times \left(\frac{1}{\mu}\bar\nabla \times \bar A\right) + \epsilon\bar\nabla\left[\frac{1}{\mu\epsilon^2}\bar\nabla \cdot (\epsilon \bar A)\right] - \epsilon\frac{\partial^2}{\partial t^2}\bar A = -\bar J}$$

Byrjum með

$$\bar\nabla \cdot \bar D = \rho \qquad \text{og} \qquad \bar D = \epsilon \bar E, \qquad \bar E = -\bar\nabla V - \frac{\partial}{\partial t}\bar A$$

$$\bar\nabla \cdot (\epsilon \bar E) = \rho \quad \longrightarrow \quad \bar\nabla \cdot (\epsilon \bar\nabla V) + \frac{\partial}{\partial t}\bar\nabla \cdot (\epsilon \bar A) = -\rho$$

Notum aftur $\bar\nabla\cdot(\epsilon\bar A)+\mu\epsilon^2\frac{\partial}{\partial t}V=0$

$$\rightarrow \bar\nabla\cdot(\epsilon\bar\nabla V)-\mu\epsilon^2\frac{\partial^2 V}{\partial t^2}=-\rho$$

eða

$$\boxed{\frac{1}{\epsilon}\bar\nabla\cdot(\epsilon\bar\nabla V)-\mu\epsilon\frac{\partial^2}{\partial t^2}V=-\frac{\rho}{\epsilon}}$$

P7-28

Á svæði án hleðslu gildir (og strauma)

$\bar\nabla\cdot\bar E=0$, því er hægt að skilgreina

$\bar A_e$ þ.a. $\bar E=\bar\nabla\times\bar A_e$. Gera ráð fyrir tímaháðu ...

a) finna tengsl $\bar H$ og $\bar A_e$

b) Sýna að $\bar A_e$ sé lausn á óhliðraðri jöfnu Helmholtz

Maxwell jöfnur

$$\bar\nabla\times\bar E=-i\omega\mu\bar H$$
$$\bar\nabla\times\bar H=i\omega\epsilon\bar E$$

$\bar\nabla\cdot\bar E=0 \rightarrow$ höfum $\bar A_e$ þ.a. $\bar E=\bar\nabla\times\bar A_e$

Notum Faraday lögmálið

$$\hookrightarrow \bar H=\frac{i}{\omega\mu}\bar\nabla\times\bar E=\frac{i}{\omega\mu}\bar\nabla\times\bar\nabla\times\bar A_e$$
$$=\frac{i}{\omega\mu}\left\{\bar\nabla(\bar\nabla\cdot\bar A_e)-\nabla^2\bar A_e\right\} \quad (2)$$

Ampère-Maxwell

$$\bar\nabla\times\bar H=i\omega\epsilon\bar E=i\omega\epsilon\bar\nabla\times\bar A_e$$

$$\rightarrow \bar\nabla\times\left(\bar H-i\omega\epsilon\bar A_e\right)=0$$

því er hægt að finna skalarmætti V_m þ.a.

$$\bar H-i\omega\epsilon\bar A_e=-\bar\nabla V_m$$

$$\rightarrow \bar H=i\omega\epsilon\bar A_e-\bar\nabla V_m \quad (1)$$

saman gefa (1) og (2)

$$i\omega\epsilon\bar A_e-\bar\nabla V_m=\frac{i}{\omega\mu}\left\{\bar\nabla(\bar\nabla\cdot\bar A_e)-\nabla^2\bar A_e\right\}$$

nota „Lorentz-kvörðun"

$$\bar\nabla\cdot\bar A_e=i\omega\mu V_m$$

þá fæst

$$\boxed{\nabla^2\bar A_e+\omega^2\mu\epsilon\bar A_e=0}$$

sem svar við b-lið

Notum í

$$\bar H=i\omega\epsilon\bar A_e-\bar\nabla V_m$$

$$\rightarrow \boxed{\bar H=i\omega\epsilon\bar A_e+\frac{i}{\omega\mu}\bar\nabla(\bar\nabla\cdot\bar A_e)}$$

sem er svarið við a-lið.

(8-11) (1)

y-skautuð flöt bylgja í x-ætt $\epsilon = 2.5$

$f = 3\,GHz$ "Loss tangent $= 10^{-2}$"

$$\tan\delta_c = \frac{\epsilon''}{\epsilon'}$$

Ef $\epsilon = \epsilon' - i\epsilon''$ við $\epsilon' = 2.5$ og $\epsilon'' = +\frac{\sigma}{\omega}$

fá fæst $\tan\delta_c = \frac{\sigma}{\omega\epsilon'}$. Tengjum hetta við

$$\gamma = \alpha + i\beta \approx i\omega\sqrt{\mu\epsilon'}\left\{1 - i\frac{\epsilon''}{2\epsilon'} + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right\}$$

fyri efni við lítlum tapi

$$\alpha \simeq \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}}$$

$$\beta \simeq \omega\sqrt{\mu\epsilon'}\left(1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right)$$

(2)

a) Finna helmingunar vegalongd bylgjunnar

Bylgjan er við dopunarþátt $e^{-\alpha x}$

fyri helmingunar vegalengd $x_{1/2}$ gildir $e^{-\alpha x_{1/2}} = \frac{1}{2}$

$$\rightarrow x_{1/2} = \frac{\ln 2}{\alpha}$$

$$\alpha = \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}}$$

notum at $\sqrt{\mu\epsilon_0} = \frac{1}{c}$ $\mu \sim \mu_0$

$$\rightarrow \sqrt{\mu\epsilon_0} = \sqrt{\mu\frac{\epsilon'}{\epsilon_r}} \quad \text{því } \epsilon_0\epsilon_r = \epsilon'$$

$$= \frac{1}{c} \rightarrow \sqrt{\mu} = \sqrt{\frac{\epsilon_r}{\epsilon'}}\frac{1}{c}$$

$$\alpha = \frac{\omega\epsilon''}{2}\frac{\sqrt{\epsilon_r}}{\sqrt{\epsilon'}\,c}\frac{1}{\sqrt{\epsilon'}}$$

$$= \frac{\omega}{2}\left(\frac{\epsilon''}{\epsilon'}\right)\frac{\sqrt{\epsilon_r}}{c}$$

$$= \frac{\omega}{2}(\tan\delta_c)\frac{\sqrt{\epsilon_r}}{c} = \frac{\pi f}{c}(\tan\delta_c)\sqrt{\epsilon_r} \sim 0.497\ m^{-1}$$

(3)

$$\rightarrow x_{1/2} = 1.4\ m$$

b) Ákvarða η_c, λ, u_p og u_g

(8.50) gevur at
$$\eta_c \simeq \sqrt{\frac{\mu}{\epsilon'}}\left(1 + i\frac{\epsilon''}{2\epsilon'}\right)$$

$$\simeq \sqrt{\frac{\mu_0}{\epsilon_r\epsilon_0}}\left(1 + i\frac{\epsilon''}{2\epsilon'}\right)$$

$$= \frac{1}{\sqrt{\epsilon_r}}\eta_0\left(1 + \frac{i}{2}\tan\delta_c\right) = \frac{1}{\sqrt{2.5}}377\left(1 + \frac{i}{2}\cdot 10^{-2}\right)\Omega$$

$$\simeq 238\left(1 + i\cdot 0.5\cdot 10^{-2}\right)\Omega$$

(4)

$$\lambda = \frac{2\pi}{\beta}, \quad \beta = \omega\sqrt{\mu\epsilon'}\left(1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right)$$

$$\omega\sqrt{\mu\epsilon'} = \omega\sqrt{\frac{\epsilon_r}{\epsilon'}\epsilon'}\frac{1}{c} = \frac{\omega}{c}\sqrt{\epsilon_r}$$

$$= \frac{2\pi f}{c}\sqrt{\epsilon_r}$$

$$\rightarrow \lambda = \frac{c}{f\sqrt{\epsilon_r}\left(1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right)}$$

$$\simeq \frac{c}{f\sqrt{\epsilon_r}}\left(1 - \frac{1}{8}(\tan\delta_c)^2\right) \simeq 0{,}063\ m$$

5

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon'}\left(1+\frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right)} \approx \frac{c}{\sqrt{\epsilon_r}}\left(1-\frac{1}{8}(\tan\delta_c)^2\right)$$

$$\approx 1{,}9\cdot 10^8 \ \mathrm{m/s}$$

$$u_g = \left(\frac{d\beta}{d\omega}\right)^{-1} = \left(\sqrt{\mu\epsilon''}\left(1+\frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right)\right)^{-1} \approx \frac{c}{\sqrt{\epsilon_r}}\left(1-\frac{1}{8}(\tan\delta_c)^2\right)$$

$$\approx u_p$$

c) Ef í $x=0$

$\bar{E} = \hat{a}_y\, 50\sin\left(6\pi 10^9 t + \frac{\pi}{3}\right)\ \frac{V}{m}$ ~~hvað er~~ þá $\bar{H}$

6

$\bar{H} = \frac{1}{\eta_c}\hat{a}_x \times \bar{E}$ útbreiðslustefnan

fyrir fasorinn er því $\bar{E} = \hat{a}_y e^{i\pi/3}$

$$\bar{H} = \hat{a}_z \cdot \frac{50}{|\eta_c|} e^{i\left(\pi/3 - \arctan\left(\frac{\eta_c''}{\eta_c'}\right)\right)}$$

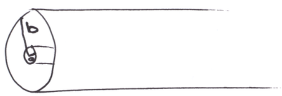
$$\approx \hat{a}_z \cdot 0{,}21 \cdot e^{i\left(\frac{\pi}{3} - 0{,}0016\pi\right)}$$

$$\bar{H}(x,t) \approx \hat{a}_z\, 0.21 \cdot e^{-\alpha x}\sin\left(6\pi 10^9 t - \beta x + \tfrac{\pi}{3} - 0{,}0016\pi\right)$$

$$\approx \hat{a}_z\, 0.21 \cdot e^{-0.497x}\sin\left(6\pi 10^9 t - 31.6\pi x + 0{,}332\pi\right)\ \frac{A}{m}$$

$\frac{\pi}{3} - 0{,}0016\pi$

P8-19



V_0 - dc milli kjarna og kápu
I flæðir til viðnáms

Finna aflflæði með $\bar{\mathcal{P}}$ í samása kapli

Gerum ráð fyrir línuhleðslu ρ_L á innri kjarna

Gauß lögmál gefur $\bar{E} = \hat{a}_r \frac{\rho_L}{2\pi\epsilon r}$

þurfum að losna við ρ_L og fá V_0 í staðinn

$$V_0 = -\int_b^a \bar{E}\cdot d\bar{r} = \frac{\rho_L}{2\pi\epsilon}\ln\left(\frac{b}{a}\right)$$

$$\boxed{\bar{E} = \hat{a}_r \frac{V_0}{r\ln(b/a)}}$$

Lögmál Ampères gefur

$$\bar{H} = \hat{a}_\phi \frac{I}{2\pi r}$$

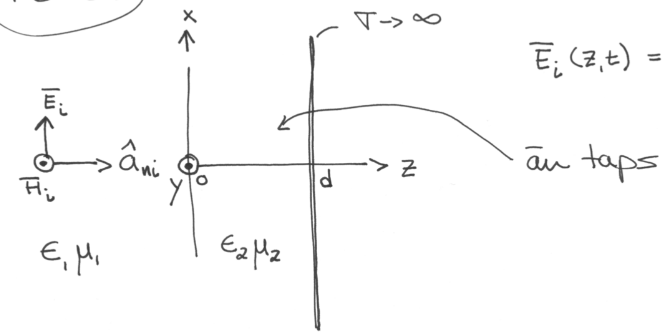
$$\bar{\mathcal{P}} = \bar{E}\times\bar{H} = \hat{a}_z \frac{V_0 I}{2\pi r^2 \ln(b/a)}$$

Nú þarf aflið sem flæðir um þversnið kapals

$$P = \int_S \bar{\mathcal{P}}\cdot d\bar{s} = \frac{V_0 I}{2\pi\ln(b/a)}\int_0^{2\pi} d\phi \int_a^b \left(\frac{1}{r^2}\right) r\,dr$$

$$= V_0 I$$

P8-32



$$\bar{E}_i(z,t) = \hat{a}_x E_{io} \cos\left[\omega\left(t - \frac{z}{u_p}\right)\right]$$

a) finna $\bar{E}_r(z,t)$

I i Efni 1

fyrir fasora

$$\bar{E}_1 = \hat{a}_x \left(E_{io} e^{-i\beta_1 z} + E_{ro} e^{i\beta_1 z}\right)$$

$$\bar{H}_1 = \frac{\hat{a}_y}{\eta_1}\left(E_{io} e^{-i\beta_1 z} - E_{ro} e^{i\beta_1 z}\right)$$

til tess at bylgjan stevnir út í -z-ætt

+ fasestilyndi →

$\beta_1 = \omega\sqrt{\mu_1 \epsilon_1}$

$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$

II

$$\bar{E}_2 = \hat{a}_x \left(E_2^+ e^{-i\beta_2 z} + E_2^- e^{i\beta_2 z}\right)$$

$$\bar{H}_2 = \hat{a}_y \left(E_2^+ e^{-i\beta_2 z} - E_2^- e^{i\beta_2 z}\right)$$

Í z=d er góður leiðari → $E_\parallel$ verður at vera samfelt

$\to E_2^- = -E_2^+ e^{-i2\beta_2 d}$ og tí fæst

$$\bar{E}_2 = \hat{a}_x\left(e^{-i\beta_2 z} - e^{i\beta_2(z-2d)}\right)E_2^+$$

$$\bar{H}_2 = \hat{a}_y\left(e^{-i\beta_2 z} + e^{i\beta_2(z-2d)}\right)\frac{E_2^+}{\eta_2}$$

tað gevur okkum y fl.b.-streymur í leiðara vegna $\bar{H}$

Í z=0 Eingin leiðsla, eingin leiðari

$H_1(0) = H_2(0)$

$E_1(0) = E_2(0)$

báðir sviðini eru samsíða skilfleti

fyrir rafsviðið fæst

$$E_{io} + E_{ro} = E_2^+\left(1 - e^{-2i\beta_2 d}\right)$$

og segulsviðið

$$\frac{1}{\eta_1}\left(E_{io} - E_{ro}\right) = \frac{E_2^+}{\eta_2}\left(1 + e^{-2i\beta_2 d}\right)$$

E_{io} var givið, og her eru jøfnur til tess at finna E_2^+ og E_{ro}

$$\to \quad E_2^+ = \frac{2\eta_2 E_{io}}{(\eta_1 + \eta_2) + (\eta_1 - \eta_2)e^{-2i\beta_2 d}} \qquad (*)$$

$$E_{ro} = -\left(\frac{\eta_1 - i\eta_2 \tan(\beta_2 d)}{\eta_1 + i\eta_2 \tan(\beta_2 d)}\right)E_{io} \qquad (**)$$

a) $\bar{E}_r(z,t)$?

$$\bar{E}_r(z) = \hat{a}_x E_{ro} e^{i\beta_1 z}$$

$$\bar{E}_r(z,t) = \mathrm{Re}\left(E_r(z) e^{i\omega t}\right)$$

$$= \hat{a}_x E_{io} \mathrm{Re}\left[-e^{i\omega t + i\beta_1 z}\left(\frac{\eta_1 - i\eta_2 \tan(\beta_2 d)}{\eta_1 + i\eta_2 \tan(\beta_2 d)}\right)\right]$$

$$= -\hat{a}_x E_{io}\left[\cos(\omega t + \beta_1 z)\frac{\eta_1^2 - \eta_2^2 \tan^2(\beta_2 d)}{\eta_1^2 + \eta_2^2 \tan^2(\beta_2 d)} + \sin(\omega t + \beta_1 z)\frac{2\eta_1\eta_2 \tan(\beta_2 d)}{\eta_1^2 + \eta_2^2 \tan^2(\beta_2 d)}\right]$$

Ef við skilgreinum

$$\cos\theta = \frac{\eta_1^2 - \eta_2^2\tan^2(\beta_2 d)}{\eta_1^2 + \eta_2^2\tan^2(\beta_2 d)}, \quad \sin\theta = \frac{2\eta_1\eta_2\tan(\beta_2 d)}{\eta_1^2+\eta_2^2\tan^2(\beta_2 d)}$$

þá fæst

$$\bar{E}_r(z,t) = -\hat{a}_x E_{i0}\left[\cos(\omega t+\beta_2 d)\cos\theta + \sin(\omega t+\beta_2 d)\sin\theta\right]$$

$$= -\hat{a}_x E_{i0}\cos(\omega t + \beta_2 d - \theta)$$

og

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\eta_1\eta_2\tan(\beta_2 d)}{\eta_1^2-\eta_2^2\tan^2(\beta_2 d)}$$

$$\rightarrow \quad \theta = \arctan\left[\frac{2\eta_1\eta_2\tan(\beta_2 d)}{\eta_1^2-\eta_2^2\tan^2(\beta_2 d)}\right]$$

b) $\bar{E}_1(z,t) = \bar{E}_i + \bar{E}_r = \hat{a}_x E_{i0}\left[\cos(\omega t-\beta_1 z) - \cos(\omega t+\beta_2 z-\theta)\right]$

c) $\bar{E}_2 = \hat{a}_x E_2^+\left(e^{i(\omega t-\beta_2 z)} - e^{i(\omega t+\beta_2 z)}\right)$

$$\bar{E}_2(z,t) = \hat{a}_x 2\eta_2 E_{i0}\left[\frac{2(\eta_1-\eta_2)\sin(2\beta_2 d)\sin(\beta_2 d)\cos(\omega t)}{\left(\eta_2+\eta_1+(\eta_1-\eta_2)\cos(2\beta_2 d)\right)^2+\left((\eta_1-\eta_2)\sin(2\beta_2 d)\right)^2} - \frac{2\left(\eta_1+\eta_2+(\eta_1-\eta_2)\cos(2\beta_2 d)\right)\sin(\beta_2 d)\sin(\omega t)}{(\quad \cdots \quad)}\right]$$

d og e) fylgja með $\bar{\mathcal{P}}_{ave} = 0$

f) fyrir hvaða þykkt d hverfa áhrif efnis 2 ?

það gerist þegar $E_r = -E_{i0}$

$(**) \rightarrow \tan(\beta_2 d) = 0$

eða $d\beta_2 = n\pi$

$$\frac{d 2\pi}{\lambda_2} = n\pi \qquad n = 0,1,2,\ldots$$

$$n\lambda_2 = d\cdot 2$$

10-4

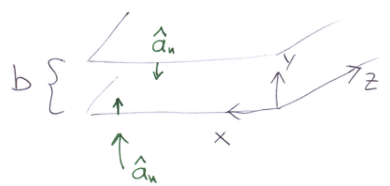
TM$_n$-háttir milli samsíða platna

Jöfnur (10-63→65)

$$E_z^0(y) = A_n\sin\left(\frac{n\pi y}{b}\right)$$

$$H_x^0(y) = \frac{i\omega\epsilon}{h}A_n\cos\left(\frac{n\pi y}{b}\right)$$

$$E_y^0(y) = -\frac{\gamma}{h}A_n\cos\left(\frac{n\pi y}{b}\right)$$



Yfirborðshleðslur

Lægriplata

$$\rho_{sl} = \hat{a}_n\cdot\bar{D}(0) = \epsilon E_y^0(0) = -\frac{\gamma\epsilon}{h}A_n$$

Efriplata

$$\rho_{su} = \hat{a}_n\cdot\bar{D}(b) = -\epsilon E_y^0(b) = (-1)^n\frac{\gamma\epsilon}{h}A_n$$

$\cos(n\pi)$

2

yfirborðsstraumar

H_x

logri $\quad \bar{J}_{sl} = \hat{a}_y \times \bar{H}(0) = -\hat{a}_z \frac{i\omega\epsilon}{h} A_n$

efri $\quad \bar{J}_{su} = -\hat{a}_y \times \bar{H}(b) = \hat{a}_z \frac{i\omega\epsilon}{h} A_n \cos(n\pi)$

$$= \hat{a}_z (-1)^n \frac{i\omega\epsilon}{h} A_n$$

$$= \begin{cases} \bar{J}_{sl} & \text{ef } n = 1,3,5\dots \\ -J_{sl} & \text{ef } n = 0,2,4,6,\dots \end{cases}$$

P10-5

yfirborðsstraumþéttleiki á plötum fyrir TE_n hátti milli samsíða platna

Jöfnur (10-83, 85)

$$H_z^0(y) = B_n \cos\left(\frac{n\pi y}{b}\right)$$

$$H_y^0(y) = \frac{\gamma}{h} B_n \sin\left(\frac{n\pi y}{b}\right)$$

$$E_x^0(y) = \frac{i\omega\mu}{h} B_n \sin\left(\frac{n\pi y}{b}\right)$$

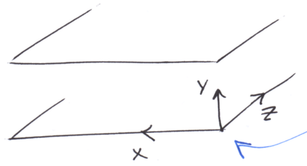
Efri plata

$$\bar{J}_{su} = -\hat{a}_y \times \bar{H}(b) = \hat{a}_x B_n \cos(n\pi)$$

$$= \begin{cases} \bar{J}_{sl} & \text{ef } n = 1,3,5,\dots \\ -\bar{J}_{sl} & \text{ef } n = 2,4,6 \end{cases}$$

Logriplata

$$\bar{J}_{sl} = \hat{a}_y \times \bar{H}(0) = \hat{a}_x B_n$$



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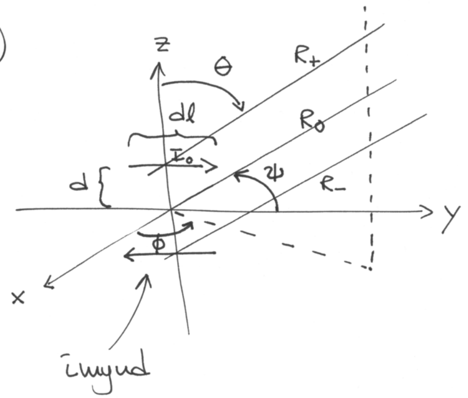

Notum því (11-19b) með ψ sem hefur tekið hlutverk θ

Einnig er fasa þáttur með $e^{-i\beta R_\pm}$ þar sem hliðrunin um $d \ll R_0$ leiðir til

$\to R_\pm \approx R_0 \mp d\cos\theta$

En í nefnara nálgum við $R_\pm \sim R_0$

$$R_+ = \sqrt{R_0^2 + d^2 - 2R_0 d\cos\theta} = R_0\left(1 + \frac{d^2}{R_0^2} - 2\frac{d}{R_0}\cos\theta\right)^{1/2} \approx R_0 - d\cos\theta + o\left(\left(\frac{d}{R_0}\right)^2\right)$$

því fæst

$$E_\psi^+ = \frac{i I_0 dl \eta_0 \beta}{4\pi R_0} e^{-i\beta(R_0 - d\cos\theta)} \sin\psi$$

$$E_\psi^- = -\frac{i I_0 dl \eta_0 \beta}{4\pi R_0} e^{-i\beta(R_0 + d\cos\theta)} \sin\psi$$

spegilmynd

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Frá rúmfræði fæst: $\quad \hat{a}_R \times \hat{a}_y = 1 \cdot 1 \cdot \sin\psi$

$$\to \sin\psi = |\hat{a}_R \times \hat{a}_y| = |(\hat{a}_x \sin\theta\cos\phi + \hat{a}_y \sin\theta\sin\phi + \hat{a}_z\cos\theta) \times \hat{a}_y|$$

$$= |\hat{a}_z \sin\theta\cos\phi - \hat{a}_x \cos\theta|$$

$$= \sqrt{\sin^2\theta\cos^2\phi + \cos^2\theta} = \sqrt{\sin^2\theta(1-\sin^2\phi) + \cos^2\theta}$$

$$= \sqrt{1 - \sin^2\theta\sin^2\phi}$$

$$E_\psi = E_\psi^+ + E_\psi^- = i\frac{I_0 dl}{2\pi}\left(\frac{e^{-i\beta R_0}}{R_0}\right)\eta_0\beta \sin(\beta d\cos\theta)\sqrt{1-\sin^2\theta\sin^2\phi}$$

Mynsturfallið er því

$$F(\theta,\phi) = |\sin(\beta d\cos\theta)|\sqrt{1-\sin^2\theta\sin^2\phi}$$

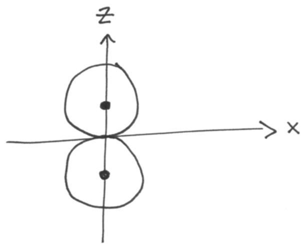
a) í xy-sléttu $\theta = \frac{\pi}{2}$ $\quad F_{xy}\left(\frac{\pi}{2}, \phi\right) = 0$

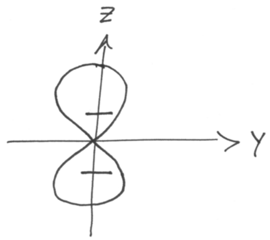
b) í xz-sléttu $\phi = 0$ $\quad F_{xz}(\theta, 0) = |\sin(\beta d \cos\theta)|$

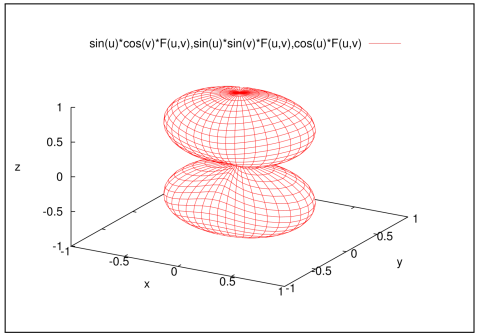
c) í yz-sléttu $\phi = \frac{\pi}{2}$ $\quad F_{yz}\left(\theta, \frac{\pi}{2}\right) = |\sin(\beta d \cos\theta) \cdot \cos\theta|$

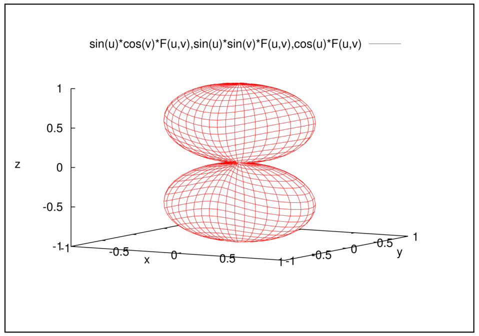
d) $d = \frac{\lambda}{4} \rightarrow \beta d = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$

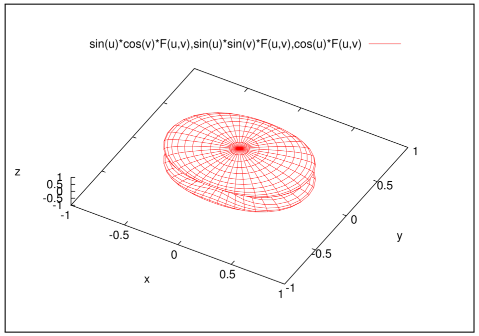
$F_{xz}(\theta, 0) = \left|\sin\left(\frac{\pi}{2}\cos\theta\right)\right|$ $\qquad F_{yz}\left(\theta, \frac{\pi}{2}\right) = \left|\sin\left(\frac{\pi}{2}\cos\theta\right)\cos\theta\right|$

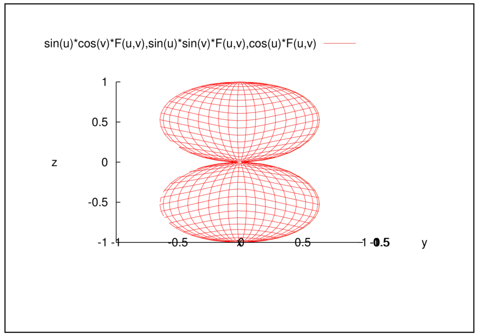








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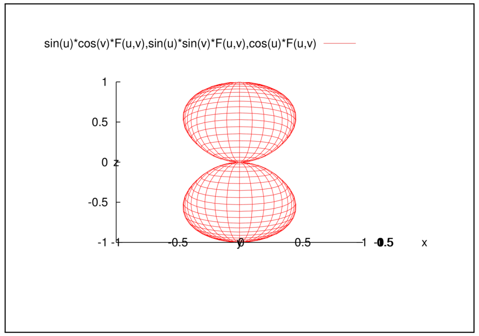
z
y


sin(u)*cos(v)*F(u,v),sin(u)*sin(v)*F(u,v),cos(u)*F(u,v)
z
x